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Application of Textile Items in Supports of Rotating Bodies

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Abstract
The study analysed the possibility of using an element with a textile structure for the design of supports for a spinning body that is not subject to external transverse forces. A numerical simulation was carried out and the phenomena that occur in the system were explained with the use of the energy function.

Key words: *nonlinear vibration, bearing supports, rotating body, fibres compression.*

Introduction

The study of the problem of the nonlinear vibration of a rotating body driven by an electric motor can be found in works [1 - 3]. It was found that to avoid a gradual decrease in the angular speed to the critical value, a nonlinear characteristic of supports was required. Stabilisation of the angular speed of the shaft was achieved by constraining its ends at the expense of the significant axial force acting on the bearings. In this paper an analysis of the application of a textile structure element for the design of resilient supports for a rotating body is presented. The experimental study of the behaviour of textiles subjected to compressive forces can be found in papers [4 - 6]. Computer simulation of the compression behaviour of fibre assemblies is presented in papers [7, 8]. A mathematical model of a layer of fibres was formulated in paper [9] and studied in work [10].

Equations of motion

The system considered is shown in ***Figure 1***. The rotating rigid body support contains a resilient element with a textile structure having a force–compression relationship as described in paper [9], and shown here in the second and third equation of set (3). In ***Figure 1***, C_0 is the body centre of gravity at rest, and the distance between points 0 and C_0 is the initial eccentricity (v_0, w_0). The transverse displacement (v, w) of the body from point C_0 to C results from the inertia forces causing the deflection of the support.

From ***Figure 1*** we find the components of body acceleration in form (1).

$$a_v = \frac{d^2 v}{dt^2} - 2\frac{d\alpha}{dt}\frac{dw}{dt} - \frac{d^2\alpha}{dt^2}(w_0 + w) + - \left(\frac{d\alpha}{dt}\right)^2 (v_0 + v)$$

$$a_w = \frac{d^2 w}{dt^2} + 2\frac{d\alpha}{dt}\frac{dv}{dt} + \frac{d^2\alpha}{dt^2}(v_0 + v) + - \left(\frac{d\alpha}{dt}\right)^2 (w_0 + w) \qquad (1)$$

Assuming that the result of all forces acting on the body, together with inertia forces, to be equal to zero, gives us a set of equations describing its motion. The summation of the moments of the forces gives ***Equation 2***, and that of the rectangular components of the forces gives us ***Equation 3***. The force of gravity is not included. Angle β is given by $\cos\beta = v/r$ and $\sin\beta = w/r$.

$$B\frac{d^2\alpha}{dt^2} - ma_v(w_0 + w) + + ma_w(v_0 + v) - M = 0, \qquad (2)$$

$$ma_v + \frac{c\,\mathrm{sgn}\left(\frac{dv}{dt}\right)\left(\frac{dv}{dt}\right)^2}{\left(1 - \frac{\sqrt{v^2 + w^2}}{H}\right)^3} + \frac{kv}{\left(1 - \frac{\sqrt{v^2 + w^2}}{L}\right)^3} = 0,$$

$$ma_w + \frac{c\,\mathrm{sgn}\left(\frac{dw}{dt}\right)\left(\frac{dw}{dt}\right)^2}{\left(1 - \frac{\sqrt{v^2 + w^2}}{H}\right)^3} + \frac{kw}{\left(1 - \frac{\sqrt{v^2 + w^2}}{L}\right)^3} = 0. \qquad (3)$$

In ***Equations 2, 3*** B is the mass moment of inertia of the body, m its mass, and k, c, L, H are material constants.

By substituting ***Equation 3*** into ***Equation 2*** and ***Equation 2*** into ***Equation 3***, one obtains a set of differential ***Equations 4*** (see page 98).

The motor torque M and its angular velocity $d\alpha/dt$ is a result of the mutual inter-

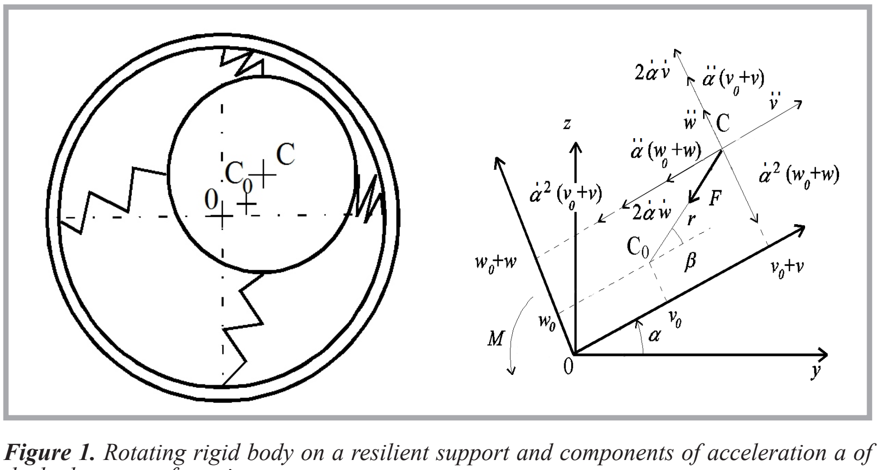

Figure 1. *Rotating rigid body on a resilient support and components of acceleration a of the body centre of gravity.*

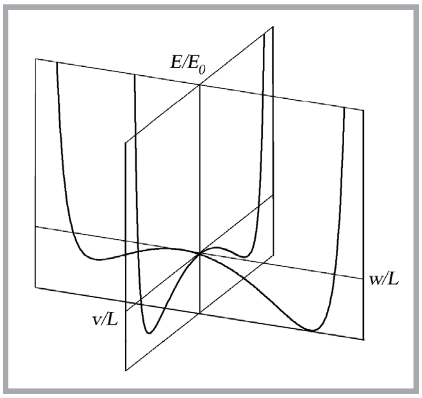

Figure 2. *Cross-sections of the energy surface E/E_0 versus v/L and versus w/L.*

action of system elements and can be calculated by the integration of ***Equation 5*** together with ***Equation 4***. In ***Equation 5*** Ω is the idle angular velocity, T the motor time constant and C denotes the stiffness of the motor characteristic.

$$T\frac{\mathrm{d}M}{\mathrm{d}t} = C\left(\Omega - \frac{\mathrm{d}\alpha}{\mathrm{d}t}\right) - M. \qquad (5)$$

We define an energy function $E = E(v,w)$ (6) by assuming that its partial derivatives with respect to the displacements are equal to the corresponding components of the forces ($\partial E/\partial v = F_v$, $\partial E/\partial w = F_w$) (7).

Numerical results and discussion

Calculations were performed for mass m = 5 kg, inertia B = 0.001 kgm^2, eccentricity v_0 = 0, w_0 = 0.001 m, motor constants C = 1 kgm^2/s, Ω = 100 rad/s, $T = 2\pi/\Omega$, material parameters H = 0.01 m, L = 0.01 m, c = 5 kg/m, $k = \Omega^2 m/25$, and initial conditions t = 0, α = 0, $\mathrm{d}\alpha/\mathrm{d}t=\Omega$, v = 0, $\mathrm{d}v/\mathrm{d}t$ = 0, w = 0, $\mathrm{d}w/\mathrm{d}t$ = 0, M = 0.

In ***Figure 2*** the energy cross-section planes are bound vertically by asymptotes at $v/L = \pm 1$, $w/L = \pm 1$, and horizontally by lines at the energy minimum and E/E_0 = 3000, where $E_0 = k(v_0^2 + w_0^2)/2$.

The energy surface shown in ***Figure 2*** is hat shaped with a hyperbolic brim, as opposed to the parabolic brim for a shaft with constrained ends [1, 2]. Similar to the shaft, the maximum on the energy surface is surrounded by an inclined concavity with a saddle and global minimum. The global minimum represents the state of stable rotation of the body. The maximum and saddle represent the states of unstable rotation of the body.

$$B\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} + \left(\frac{c\,\mathrm{sgn}\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)^2}{\left(1-\frac{\sqrt{v^2+w^2}}{H}\right)^3} + \frac{kv}{\left(1-\frac{\sqrt{v^2+w^2}}{L}\right)^3}\right)(w_0+w) +$$

$$-\left(\frac{c\,\mathrm{sgn}\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)^2}{\left(1-\frac{\sqrt{v^2+w^2}}{H}\right)^3} + \frac{kw}{\left(1-\frac{\sqrt{v^2+w^2}}{L}\right)^3}\right)(v_0+v) - M = 0,$$

$$\frac{\mathrm{d}^2v}{\mathrm{d}t^2} - 2\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\mathrm{d}w}{\mathrm{d}t} - \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}(w_0+w) - \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2(v_0+v) + \frac{c\,\mathrm{sgn}\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)^2}{m\left(1-\frac{\sqrt{v^2+w^2}}{H}\right)^3} + \frac{kv}{m\left(1-\frac{\sqrt{v^2+w^2}}{L}\right)^3} = 0, \qquad (4)$$

$$\frac{\mathrm{d}^2w}{\mathrm{d}t^2} + 2\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}(v_0+v) - \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2(w_0+w) + \frac{c\,\mathrm{sgn}\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)^2}{m\left(1-\frac{\sqrt{v^2+w^2}}{H}\right)^3} + \frac{kw}{m\left(1-\frac{\sqrt{v^2+w^2}}{L}\right)^3} = 0.$$

$$E = \frac{k}{2}\frac{v^2+w^2}{\left(1-\frac{\sqrt{v^2+w^2}}{L}\right)^2} - \frac{1}{2}m\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2\left((v_0+v)^2+(w_0+w)^2\right) \qquad (6)$$

$$F_v = \frac{\partial E}{\partial v} = \frac{kv}{\left(1-\frac{\sqrt{v^2+w^2}}{L}\right)^3} - \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 m(v_0+v),$$

$$F_w = \frac{\partial E}{\partial w} = \frac{kw}{\left(1-\frac{\sqrt{v^2+w^2}}{L}\right)^3} - \left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 m(w_0+w) \qquad (7)$$

Equation 4, 6, 7.

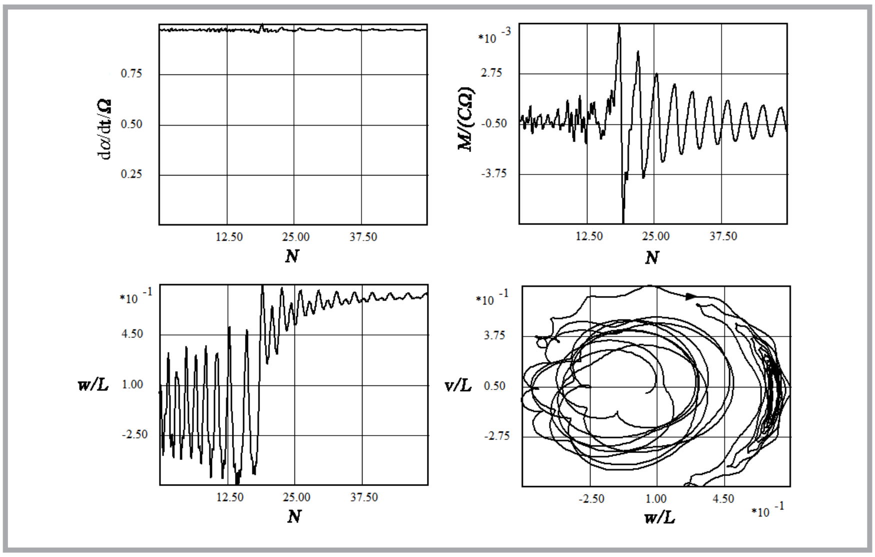

Figure 3. *Body angular velocity dα/dt, motor torque M and displacement w versus the number of body revolutions N, and the components (v,w) of the body displacement.*

The system when disturbed from any of these two unstable states goes to the energy minimum.

Comparing components (v,w) of the body displacement shown in ***Figure 3*** with the energy surface shown in ***Figure 2***, one can see that components (v,w) represent the motion on the energy surface. A similar observation was described in paper [1] for a shaft with axially constrained ends.

Conclusions

1. A resilient support with a hyperbolic characteristic stabilise the angular velocity of a rotating body that is driven by a motor, making the critical speed non-existent.
2. Textile structures can find application as elements of resilient supports, on condition that their fatigue strength be sufficient.

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