Wei Tian,
***Chengyan Zhu,**
Shanyuan Wang

Geometric Model of Three Dimensional Integrated Cellular Woven Structures

College of Textiles,
Donghua University,
Shanghai 200051, P. R. China

***College of Materials and Textiles,**
Key Laboratory of Advanced Textile Materials
and Manufacturing Technology,
Ministry of Education,
Zhejiang Sci-Tech University(ZSTU),
Hangzhou 310033, P. R. China.
E-mail: cyzhu@zstu.edu.cn

Abstract
Following the geometric pattern of the constituent yarn system in 3D integrated cellular woven structures, a general purpose geometric cell model was developed. The length and orientation angle of the constituent yarns in each system within a unit cell were calculated with the purpose of predicting the volume fraction of the fiber and the cellular part. In order to verify the geometric model, samples of 3D integrated cellular woven composites with 6 different binding parameters were selected, and the volume fractions of each part in the composites were measured. The experimental results were in good agreement with the results of the model. Furthermore, the geometric model was used to discuss the effects of the binding parameters on the volume fraction of the cellular part and the fiber. The investigation showed that the volume fractions of the cellular part and the fiber varied with the binding length of section E.

Key words: *composites, integrated cellular structures, geometric model, binder.*

3D integrated cellular woven structures are formed by interlacing binding threads (hereafter termed as binders) in the thickness direction to join layers of typical 3D woven structures together [1]. This kind of structure, with cellular parts, has a large share in the world market. As 3D integrated cellular structures are composed of three kinds of yarn systems, the mechanical properties of the composites are not only related with the properties of the fibers, matrix, the volume fraction of the fiber and the cellular part, but are also affected by the geometrical shapes of the yarns. By building a geometric model, the geometrical shape of the yarn in each system can be described accurately. Based on such models, analysing the contribution of each yarn system to the composite mechanical property can make the designability of the composite property eventually possible.

Till now, many studies have been conducted on the geometric model of typical 3D wovenstructures. However, there very few have reported on the geometric model of 3D integrated cellular woven structures, as most works focus on the properties of such materials [2 - 5]. In recent years, some efficient models have been created for typical 3D woven structures. As regards the classical woven structure model proposed by Peirce, Byun [6, 7], they assumed that the yarn section was lentoid and tried to establish a geometric model for 3D woven structures under compacted conditions for the first time. Later, Ko [8] recommended a geometric model when analysing the relationship between a fabric structure and fiber volume fraction. In this model, structure parameters were considered, and the character of the 3D woven structure in the balance system was studied. Pochiraju [9] used a geometric model to describe the distribution of fiber in a woven structure when testing the stiffness of special 3D composites. This model set the yarn number in the thickness and the physical dimension of the yarn as the structure parameters, using the sinusoid to represent the shape of of the binder yarn axes. and obtained the relationship between the structure parameters and the yarns was obtained. Ding Xin [1] developed a general purpose geometric model for a 3D woven structure and analysed the how the parameters affect the molding of the fiber structure. Yang Caiyun [10] assumed the section of warp and weft as rectangle and lentoid, respectively. In their model, the micro-design of the elastic property of the interlock structure composite was realised primarily.

In this study a geometric model of a 3D integrated cellular woven structure was established on the basis of the researches above . The model can predict the volume fraction of each part of the composite, and analyse changes in the volume content for each part of the structure when the structure parameters are changed. Furthermore, it can guide the property design of 3D integrated cellular composites.

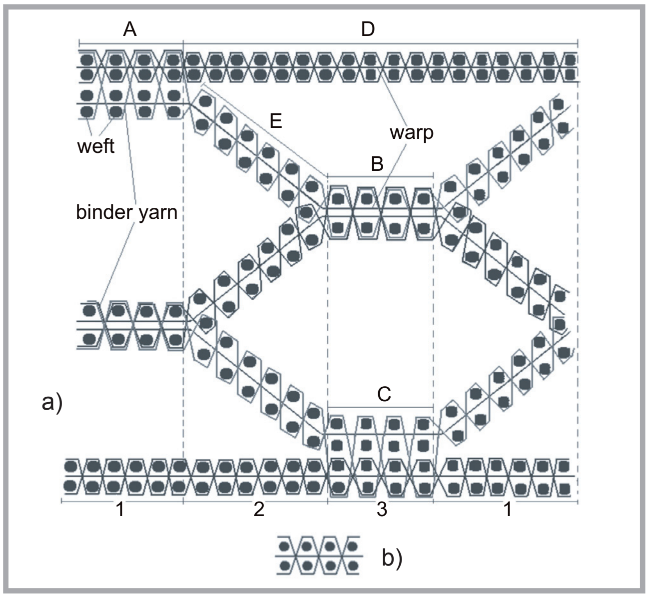

***Figure 1.** Sectional drawing of the integrated cellular structure (a) and orthogonal structure (b).*

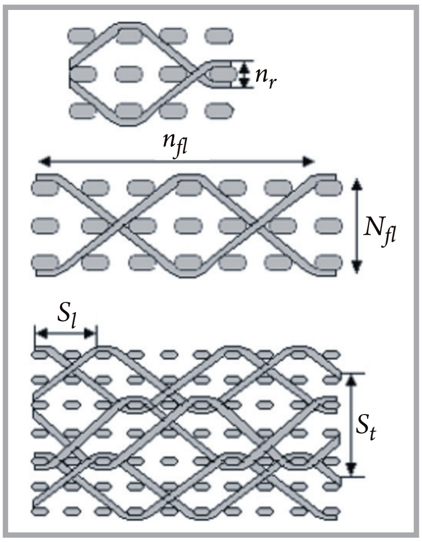

Figure 2. *Structure parameters of a binding pattern.*

Structure parameters of a 3D integrated cellular woven structure

3D Integrated Cellular Woven Structures

A two-layer 3D woven structure was selected as the basic cell of 3D integrated cellular woven structures in which the binding style was orthogonal interlock/ through thickness binding. A section in the warp direction of the 3D integrated cellular woven structure and its basic cell can be seen in ***Figure 1.***

Because of the special structure, the structure parameters may be different at different binding stations for the same binder. In order to simplify the definition of the parameter, the structure was divided into five kinds of sections according to the binding style, e.g. section A - panel binding with sandwich-layer, section B - sandwich-layer binding with sandwich-layer, section C - sandwich-layer binding with panel, section D - panel and section E - sandwich-layer, see ***Figure 1.a***.

Constitutions of yarn system

A 3D integrated cellular woven structure is made up of three groups of yarn systems, e.g. warps in the panel and sandwich part, wefts in the panel and sandwich part, binders in the interior of each layer and binders interlacing two layers, as shown in ***Figure 1.a***. In the yarn systems above, binders are the most particular yarns and a change in the binding length at different sections will lead to a change in the volume fraction of the fiber and the cellular part. Hence the key issue is how to describe the character of the binders during the design-process of 3D integrated cellular woven structures.

Definition of structure parameters

In order to develop a good geometric model for 3D integrated cellular woven structures, the types of structure parameters must be defined. Through the relationship between the parameters and the structure, a representation of the geometric structure can be obtained. ***Figure 2*** is a sketch of certain structure parameters.

According to the characterization of the paragraphs above, the most important part of creating a model is to give a numeric description of the binders and binding styles for the representation of the geometric structure. For the 3D integrated cellular woven structure in this paper, 8 groups of structure parameters were used to depict the binders, e.g. the binding-depth in each section (N_A, N_B, N_C, N_D, N_E) the binding-distance in each section (n_{Al}, n_{Bl}, n_{Cl}, n_{Dl}, n_{El}), the depth of the hook joint strength in each section (n_{Ar}, n_{Br}, n_{Cr}, n_{Dr}, n_{Er}), the step number in each section along the warp, weft and thickness direction ((S_{Al}, S_{Bl}, S_{Cl}, S_{Dl}, S_{El}), (S_{At}, S_{Bt}, S_{Ct}, S_{Dt}, S_{Et}) and (S_{Ac}, S_{Bc}, S_{Cc}, S_{Dc}, S_{Ec})), the binding-length in each section (n_{Ad}, n_{Bd}, n_{Cd}, n_{Dd}, n_{Ed}) and the total number of wefts along the thickness direction in each section (n_{Af}, n_{Bf}, n_{Cf}, n_{Df}, n_{Ef}).

Geometric models of 3D integrated cellular woven structures

In order to precisely assess the influence of the parameters of the 3D integrated cellular woven structure on the mechanical properties of the composite, it is necessary to establish a model to describe the geometric shape of the yarns in the structure. According to the observation of the yarns in the composites, the following assumptions can be made: firstly, the yarn section is racetrack, the yarns have the same section along the axial direction, and the array of fibers is homogeneous; secondly, yarns in the weft-system are in a straight state; thirdly, the gather density of the yarns in the preform is the same, established as ρ = 0.8 [8, 11].

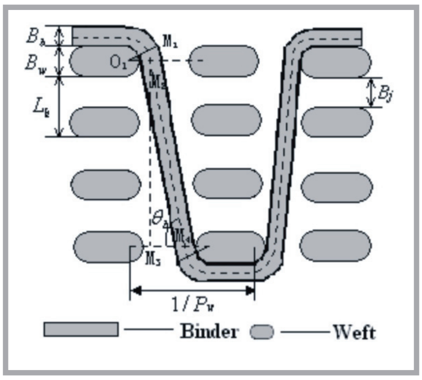

Figure 3. *Orientation angle of binders.*

Orientation Angle of Binders and Warps

Orientation angle of binder **θ_b**

In this paper, the binding style is orthogonal interlock/ through thickness binding, as shown in ***Figure 3***. From the geometric relationship in the figure, the orientation angle of binder θ_b can be calculated as equation (1).

$$\begin{cases} \theta_b = \arcsin\left(\dfrac{-\mu_1 + \sqrt{\mu_1^{\,2} - 4\mu_0\mu_2}}{2\mu_0}\right) \\ P_w \le \dfrac{1}{A_w} + \dfrac{1}{B_b} \end{cases} \tag{1}$$

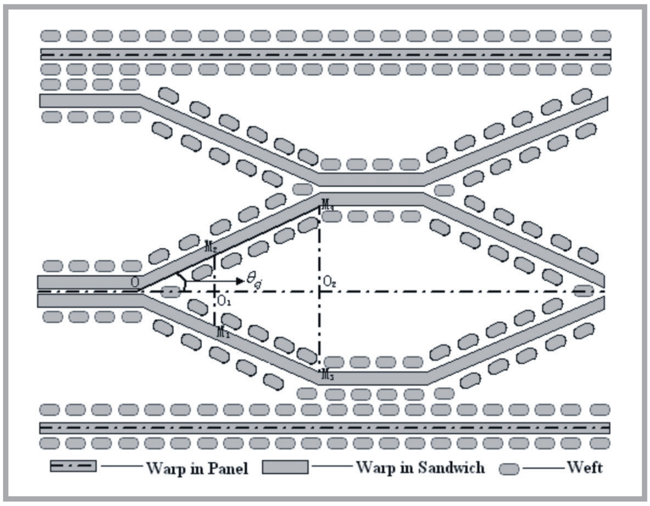

Figure 4. *Orientation angle of warps.*

Where

$$\mu_0 = (\frac{1}{P_w \cdot A_w} - 1)^2 + [(\frac{B_b}{A_w} + \frac{B_w}{A_w}) \cdot (N_f - 1)]^2$$

$$\mu_1 = 2 \cdot \frac{B_b}{A_w} \cdot (1 - \frac{1}{P_w A_w})$$

$$\mu_2 = (\frac{B_b}{A_w})^2 - [(\frac{B_b}{A_w} + \frac{1}{P_w A_w})(N_f - 1)]^2$$

And N_{ft}, A_W, B_W, B_b and P_W are the binding depth of the binder, the major axis and minor axis of the weft, the minor axis of the binder and the weft density, respectively.

Orientation angle of the warp θ_j
The warps are not interlaced with other yarns in the 3D integrated cellular woven preform. In ***Figure 4***, warps in the panel and the sandwich part are all drafted.

According to the design scheme, the orientation angle of the warp in the panel is approximately equal to zero. For the sandwich part, the angle of orientation can be calculated by the following equation:

$$\theta_b = \arccos(\frac{n_{Dd} - n_{Bd}}{2 \cdot n_{Ed}}) \qquad (2)$$

Where, n_{Bd}, n_{Dd} and n_{Ed} are the binding length of each section, respectively.

Length of each yarn system

In a steady state, the yarn shape of the 3D integrated cellular woven preform is displayed in ***Figure 5***.

Length of the binder within one repeat of the pattern
According to ***Figure 5*** (A),

$$\overline{P_0 P_1} = A_w$$

$$\overline{P_1 P_2} = \overline{P_3 P_4} = \theta_b \cdot (B_w + B_b) / 2$$

$$\overline{P_2 P_3} = \frac{(B_w + B_b) \cdot N_f}{\cos\theta_b}$$

In which, A_W and B_W are the major minor axes of the weft, respectively. B_b, N_{ft} and θ_b are the minor axis, binding depth and orientation angle of the binder, respectively.

From the formula above, it is obvious that the total length of the binder (L_b) can be calculated as follows:

$$L_b = Weft \cdot \sum_{m=0}^{7} \overline{P_m P_{m+1}} = Weft \cdot [A_w + \frac{N_f (B_w + B_b)}{\cos\theta_b} + \theta_b \cdot \frac{B_w + B_b}{2}] \qquad (3)$$

In the equation, *Weft* is the number of weft in one repeat of the pattern.

Length of the warp within one repeat of the pattern
Based on ***Figure 5*** (C), the length of the warp in one cycle (L_j) is:

$$L_j = \frac{Weft}{2}(A_w + B_w) \qquad (4)$$

Length of the weft within one repeat of the pattern
On the basis of ***Figure 5*** (B), within one pattern, the length of one weft (L_W) can be obtained:

$$L_W = 100 \cdot S_C / P_j$$

In the formula, S_C is the column number of the yarn across which one binder moves from the place of one binder pair to another. P_j is the number of reeds (tooth/10 cm).

Hence the total length of the weft for one repeat of the pattern is:

$$L'_w = 200 \cdot (n_{Ad} + n_{Bd} + n_{Dd} + 3 \cdot n_{Ed}) \cdot S_c / P_j \qquad (5)$$

The relationship above between the fabric structure and geometric parameters is the theoretical foundation for the design of 3D integrated cellular woven composites.

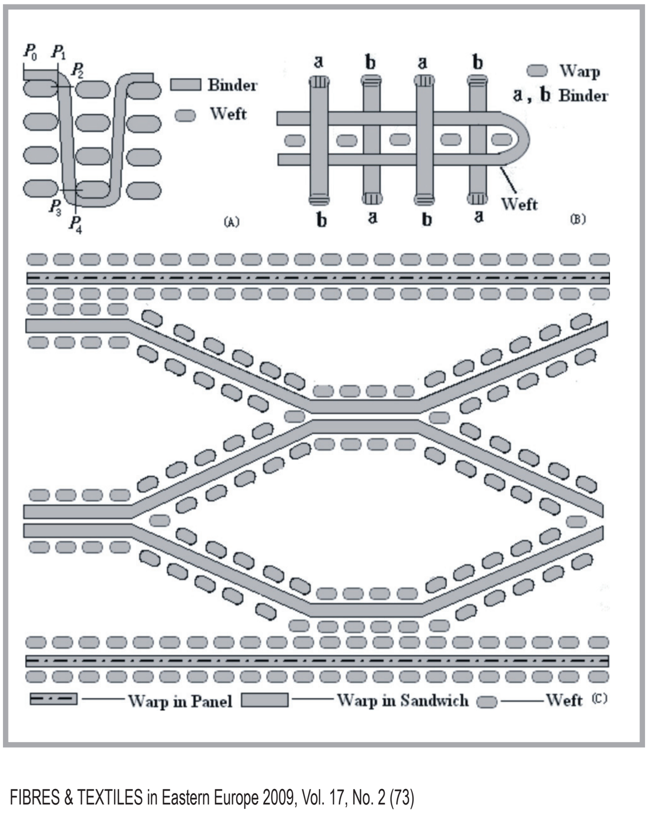

Figure 5. *Yarn shape of 3D integrated cellular woven preform (A) binder (B) weft (C) warp.*

Volume fractions of the fiber and cellular part

Volume Fraction of the Fiber in 3D Integrated Cellular Woven Composites

For 3D integrated cellular woven composites, the volume fraction of the fiber can be obtained by:

$$V_f = \frac{(V_b + V_j + V_w) \cdot \rho}{V_{unit}} \times 100\% \qquad (6)$$

In equation (6), V_b, V_j, V_W and V_{unit} are the volume of the binder, the warp, weft and composite panel, respectively.

Volume fraction of the cellular part in 3D integrated cellular woven composites

A 3D integrated cellular woven composite is composed of three parts. The third part is the cellular, the others being the fiber and matrix. Although the third part is vacant, it is still very important for the mechanical property of the composites, and it is also the basis for distinguishing the 3D integrated cellular structure from other 3D structures. ***Figure 6*** shows the scheme of the cellular part in the composites.

According to the relationship shown in ***Figure 6***, it can be seen that volume of the cellular part is

$$V_c = 3L_W[n_{Bd} \cdot n_{Ed} \cdot \sin\theta_{cj} (A_W + B_b) + n_{Ad} \cdot n_{Ed} \cdot \sin\theta_{cj} + 2n2Ed \,.\, \sin\theta_{cj} \cos\theta_{cj}] \qquad (7)$$

In which, n_{Bd}, n_{Ed} and n_{Ed} are the binding lengths of each section. From equation (7), the volume fraction of the cellular part in the 3D integrated cellular woven composite can be calculated.

Comparison of the theoretical value and testing value

In order to verify the model above, 6 composites with different structure parameters were selected to test their fiber content and cellular part. The panel was solidified by Vacuum Assisted Resin Infusion with epoxy vinyl ester resin. A comparison of the calculations and test results is in ***Table 1***.

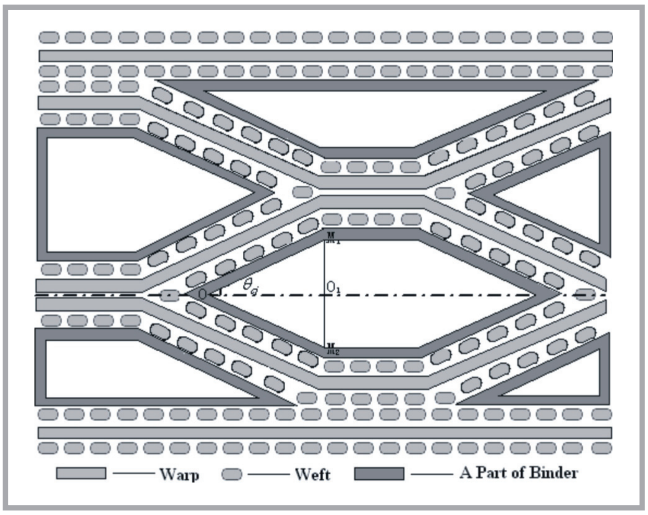

Figure 6. *Cellular part in the 3D integrated cellular woven preform.*

Table 1. *Theoretical value and testing value for the composites.*

Samples	Hollow volume percent, %		Fiber volume percent, %	
	Testing value	Theoretical value	Testing value	Theoretical value
1#	0	0	33.80	33.79
2#	22.5	20.11	24.6	27.00
3#	26.9	23.59	28.5	25.82
4#	28.9	26.76	25.80	24.75
5#	28.0	27.46	20.4	24.51
6#	29.2	32.04	27.9	22.97

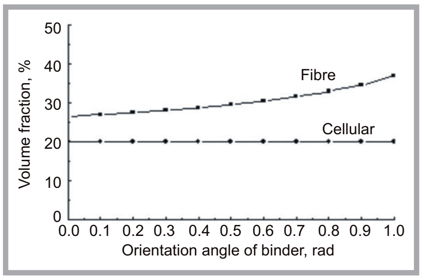

Figure 7. *Effect of the orientation angle of the binder on the volume fraction.*

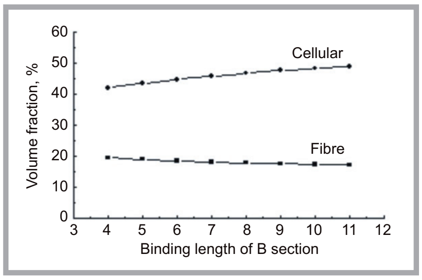

Figure 8. *Effect of the binding length of section b on the volume fraction.*

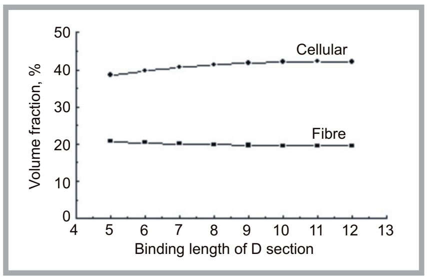

Figure 9. *Effect of the binding length of section D on the volume fraction.*

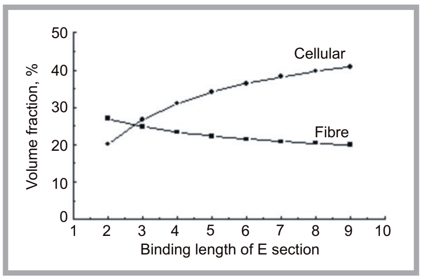

Figure 10. *Effect of the binding length of section E on the volume fraction.*

From ***Table 1***, it can be seen that the theoretical values are in good agreement with the testing results. It illustrates that the calculation for the model is right.

Effect of structure parameters on the volume fraction of the fiber/cellular part

As regards the fiber strength, the volume fraction of the fiber and cellular part are very important factors to decide the impact resistance of the composites. Therefore this section will discuss the elements that may influence the volume fraction of these two parts. ***Figures 7 - Figure 10*** express the effect of each parameter on the volume fraction of the fiber and cellular part.

In ***Figure 7***, it can be seen that some influences on the volume fraction of the fiber are exist when other parameters are the same; however, the orientation angle of the binder is different. It is possible to state that changing the angle will lead to the alteration of the binder length. However, the influence on the volume of the panel and cellular part is not obvious.

Conclusions

Little work has been done on the geometric modelling of 3D integrated cellular woven structures. On the premise that the yarn section of the 3D integrated cellular woven structures was racetrack, a geometric model was established. From this geometric model, both the volume fractions of the fiber and the cellular part can be calculated. This work also comprised a uniform foundation for analysing the mechanical behaviour of microstructure composites.

From the comparison between the results calculated and the test results, it proves that the geometric model proposed in this paper was satisfactory and could be used to calculate the volume fraction of each part of 3D integrated cellular woven composites.

Furthermore, the geometric model was used to discuss the effects of binding parameters on the volume fraction of the cellular part and the fiber. The results discussed show that enlarging the binding length of section E can cause a reduction in the volume fraction of the cellular part; however, the volume fraction of the fiber is augmented. Thus it can be concluded that, with the weaving and molding being satisfactory, enhancing the binding length of section E is one of the most efficient ways of lightening the unit mass of the composites, improving their impact performance per mass .

Acknowledgments

The authors would like to express their sincere gratitude for the support from the Key Laboratory of Advanced Textile Materials and Manufacturing Technology, Zhejiang Sci-Tech University.

References

1. *Ding Xin, Yi Honglei. A geometric model of three dimensional woven structures [J]. Acta Materiae Compositae Sinica, 2003, 20 (5) pp. 108-113.*
2. *Zic I, Ansell MP, Newton A, Price RW. Mechanical properties of composite panels reinforced with integrally woven 3-D fabrics. The Journal of the Textile Institute 1990; 81 pp. 461–79.*
3. *Tan X, Chen X. Parameters affecting energy absorption and deformation in textile composite cellular structures. Materials Design 2005, 26 (5) pp.424–38.*
4. *Tian Wei, Zhu Chengyan. Impact Performance of 3D Integrated Cellular Woven Composite Panel [J]. Journal of Donghua University, 2006(5) pp. 161-164.*
5. *Tian Wei, Zhu Chengyan, Wang Shanyuan. Quasi-static impact behavior and energy absorption of 3D integrated cellular woven composites.2007 International Forum on Biomedical Textile Materials, Shanghai, China, 2007.*
6. *Byun J H, Chou T W. Elastic properties of three-dimensional angle-interlock fabric preforms [J]. The Journal of the Textile Institute, 1990, 81 (4) pp. 538- 548.*
7. *Peirce F T. The geometry of cloth structure [J]. The Journal of the Textile Institute, 1937, 28 (3) pp. T45- 96.*
8. *Ko F K, Du G W. Processing of textile p reform s. In: Gutow ski T G, ed. Advanced Composites Manufacturing [M]. New York: John Wiley & Sons Inc, 1997. pp. 182- 187.*
9. *Pochiraju K, Chou T W. Three-dimensionally woven and braided composites: A model for anisotropic stiffness prediction [J]. Polymer Composites, 1999, 20 (4) pp. 565- 580.*
10. *Yang Caiyun, Tian Ling-ling, Liu Yong. Micro-design of Elastic Property for 3D Angle-interlock Structure Composites [J]. Journal of materials Engineering, 2007, (8) pp. 3-11.*
11. *Chou T W. Microstructural design of fiber composite [M]. Cambridge: Cambridge University Press, 1992, pp. 390-391.*

Received 08.01.2008 Reviewed 28.11.2008