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Determination of the Elastic Constants of Plain Woven Fabrics by a Tensile Test in Various Directions

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Abstract
In this paper the values of elastic constants of woven fabrics for different angles of extension direction were analysed. Four types of fabric samples of different raw material composition and the same type of weave were tested under tensile forces in seven directions oriented with a 15° increment with respect to the weft direction. The elasticity modulus and Poisson's ratio of the woven fabrics were determined experimentally in a laboratory. Based on the experimentally obtained values, theoretically calculated elastic constants for arbitrarily chosen fabric directions were calculated. A good agreement between the experimental results and values of the elastic constants calculated was shown, hence the theoretical equations can be used with high accuracy to calculate the elastic constants of fabric in various directions. Therefore the measurements need to be undertaken when the tensile force is acting on the fabric only in the warp, weft and at an angle of 45°.

Key words: *woven fabric, anisotropic, modulus of elasticity, Poisson's ratio, elastic constants.*

Introduction

Anisotropy is a characteristic of most materials, especially woven fabrics. The impact of the direction of action of an external load (tensile force) on the properties of the fabric is enormous, and is frequently examined [1]. In a biaxial woven structure two main directions are defined: longitudinal (warp) and transverse (weft). When the load acts in one of the main directions, the extension which causes a rupture is minimal, and the associated modules assume the maximum value [2]. When the angle of the external load (tensile force) changes, the elastic constants change as well.

Although measurements of stress and strain at fabric rupture during a force action in the longitudinal or transverse direction are dealt with in most experimental studies, the behavior of the fabric and its deformation are equally important when changing the inclination angle of the tensile force (load), especially in technical textiles [3, 4]. For example, in parachute materials as well as in composites, stress and strain can occur in different directions of the force action. When the action angle of the external load changes, the elastic constants of the fabric also vary. The amount of breaking force and elongation of the fabric are determined experimentally in laboratory tests of the fabric. Using these experimentally obtained values, the modulus of elasticity is determined. Therefore it is necessary to better understand the mechanical properties of fabric behaviour. The interaction between yarns in woven fabrics under a tensile force play a significant role. Although the application of textiles in different industries is growing, especially in the development of composite materials, understanding the behaviour of the mechanical properties of fabric is still limited. Kilby was among the first to start studying the mechanical properties of material under the action of a tensile load, 1937 [5]. He begins with the classical theory of elasticity with the assumption that fabric is an anisotropic material with two planes of symmetry [6]. He defined Poisson's ratio and gave the dependence of the modulus of elasticity of fabric in relation to the direction of action of tensile forces. The tensile properties of fabrics in an arbitrary direction of the tensile force were also measured. The method for measuring anisotropic tensile properties of fabrics is called the „uniaxial test method" or the method of force action in only one direction [7 - 10].

The aim of this work was to analyse the influence of anisotropy on the elastic constants of plain woven fabrics in various directions and to determine the degree of agreement between experimental results and calculated values.

Elastic constants of anisotropic materials

Fabric is an elastic anisotropic material. As anisotropic materials fabrics are a special type of porous material and can be treated as non-uniform mixtures of fibers and air [11]. The impact of external forces on fabrics causes them to change their shape and volume.

Theoretical analysis of fabric behaviour is often very complex, hence experimental verification of theoretical predictions for them is more important than for other materials [12].

Woven fabrics as anisotropic materials

Woven fabrics and textile materials are generally inhomogeneous, anisotropic and discontinuous objects so that even under moderate loads they have large deformations and displacements in propagation planes [13]. Determination of mechanical properties and prediction of fabric behavior during the production process and finally in use is a very important part of textile science.

These issues have been dealt with over the years by many researchers, and many complex measurement systems for measuring various mechanical characteristics of fabric have been designed [14, 15]. As these fabric features are mainly explored regarding elasticity, specifically under low loads, it is very difficult to achieve sufficient accuracy in determining the degree of deformation and spatial displacement of material [16]. It is necessary to take care that the measuring system has no impact on the test specimen; for this purpose optical measurement methods or processing video recordings are used nowadays [17]. Such a procedure was applied in the experimental part of this work.

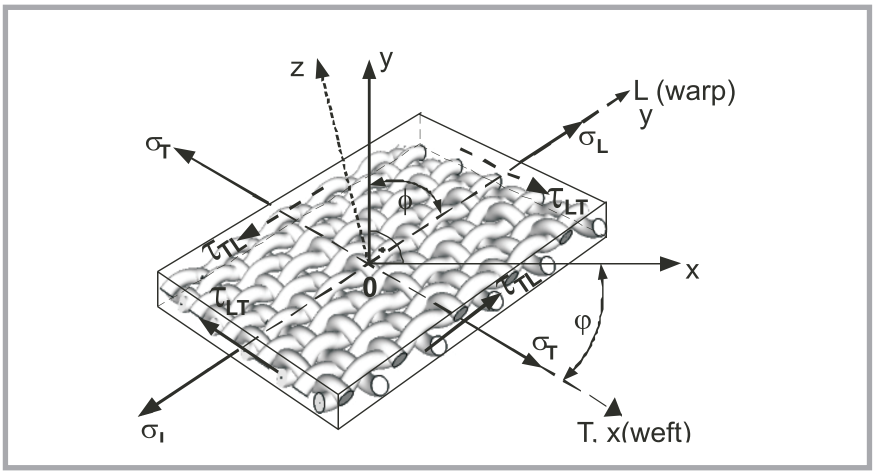

Figure 1. *Element of an orthotropic plate - woven fabric.*

Elastic constants of anisotropic material in the direction of the major axes

A special type of anisotropic material with two mutually perpendicular planes of elastic symmetry is called orthotropic material. These planes of elastic symmetry are planes of orthotropy, and their cross sections are axes of orthotropy. The most widely used orthotropic materials are wood, plywood, glass reinforced plastics, laminates and other composite materials. These materials are mainly used in the form of plates with a plane state of stress [18, 19]. The element of an orthotropic plate (woven fabric) with a plane state of stress is shown in ***Figure 1***.

The x- and y-axes coincide with those of orthotropy T, L, and the z axis is perpendicular to the plane of the element. To get the relationship between stress and strain, we must determine the modulus of elasticity (Young's modulus) E_i and Poisson's ratio ν_{ij}, which are determined experimentally in laboratory tests. Constants E_i and ν_{ij} are also called elastic constants (constants of elasticity), while technical literature uses the term engineering elastic constants. For the plane state of stress of an elastic, homogeneous and isotropic material in a linear region (linear relationship between stress and strain), Hooke's law is applied, which is:

$$\begin{Bmatrix} \varepsilon_T \\ \varepsilon_L \\ 2\varepsilon_{TL} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_T} & -\dfrac{\nu_{LT}}{E_T} & 0 \\ -\dfrac{\nu_{TL}}{E_L} & \dfrac{1}{E_L} & 0 \\ 0 & 0 & \dfrac{1}{G_{LT}} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_T \\ \sigma_L \\ \tau_{LT} \end{Bmatrix} \qquad (1)$$

Equation 1 represents the matrix form of Hooke's law. In these expressions index T refers to the transverse or transverse axis (weft), with $\varphi = 0°$, and index L in the longitudinal or longitudinal axis (warp), at $\varphi = 90°$, ***Figure 1***. The transverse and longitudinal elasticity modulus are denoted by E_T and E_L, the shear modulus by G_{TL} and G_{LT} and Poisson' ratios by ν_{LT} and ν_{TL}, respectively. Furthermore in expression (1) σ_T represents normal stress on the plane which is perpendicular to the direction of the transverse axis T, σ_L the normal stress on the plane perpendicular to the direction of the longitudinal axis L, and $\tau_{i,j}$ is the shear (tangential) stress. The first index i indicates the direction of the vertical line to the plane in which the stress acts, and the second index j indicates the direction of stress, where relation $\tau_{ij} = \tau_{TL} = \tau_{LT}$ applies. A detailed description of normal σ_i and shear $\tau_{i,j}$ stresses is given in [20]. Denotation ε_T represents the relative length deformation in the direction of the T-axis, and ε_L is that in the direction of the L-axis. In the case of a uniaxial state of stress Hooke's law can be written as follows:

$$E = \frac{\sigma}{\varepsilon} = \frac{F}{\varepsilon \cdot b \cdot d} \text{ MPa} \qquad (2)$$

where: F - tensile force in N, ε – elongation in %, b – width in mm, d – fabric thickness in mm.

Poisson's ratio of a woven fabric

When a tensile force acts in one direction on the fabric, in this direction the fabric stretches, and in that perpendicular to the direction of force, the fabric contracts. This phenomenon is described by Poisson's ratio. Poisson's ratio is one of the fundamental properties of any structural material including fabric. This coefficient determines the important mechanical characteristics of fabrics in many applications, including a variety of composite systems containing textiles as a structural element. Values of Poisson's ratio for fabrics are different from those of standard engineering materials, and they can reach values outside the range 0.1 - 2 [21]. Knowing the value of the coefficient is also necessary when creating computer simulations of fabrics and garments. To determine the Poisson's ratio of fabrics, devices for measuring tensile strength are used, and the coefficient is determined in the linear part of the diagram of Hooke's law. Researchers determined the Poisson's ratio in the warp and weft direction of fabric based on geometric models thereof and excluded the impact of the Poisson's ratio on the yarn. In this way, they came to the conclusion that the Poisson's ratio of fabrics results from the interaction between the warp and weft, and can be expressed in terms of the structural and mechanical parameters of the system [22].

The physical meaning of Poisson's ratio is shown by expression (3). Longitudinal and transverse strains have an opposite sign.

$$\nu = -\frac{\varepsilon_p}{\varepsilon} \qquad (3)$$

Poisson's ratio is defined as a negative value of the ratio of transverse ε_p and longitudinal extension strain ε in relation to the arbitrary action direction of the tensile force on the fabric sample.

Elastic constants of anisotropic material for arbitrarily oriented axes

For an orthotropic and elastic material when the x- and y-axes do not coincide with those of orthotropy, the anisotropic behaviours under tensile loads can be written in matrix form by means of Hooke's law (4):

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ 2\varepsilon_{xy} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_x} & -\dfrac{\nu_{yx}}{E_x} & \alpha_x \\ -\dfrac{\nu_{xy}}{E_y} & \dfrac{1}{E_y} & \alpha_y \\ \alpha_x & \alpha_y & \dfrac{1}{G_{xy}} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \qquad (4)$$

Elastic constants E_x, E_y, G_{xy}, ν_{xy}, α_x & α_y in coordinate system x, y are estimated by ***Equations 5 - 10*** for the transformation of the elastic constants as in [23] where: $E_{45} = E_x$, a $\nu_{45} = \nu_{xy}$, when the x-axis closes an angle of 45° with the T-axis.

The shear modulus [23] is shown in ***Equation 11***:

$$G_{TL} = \frac{E_{45}}{2 \cdot (1 + \nu_{45})} \tag{11}$$

Experimental determination of elastic constants of woven fabrics

In the experimental part of the work the magnitudes of tensile forces were determined which are used to calculate the modulus of elasticity, elastic coefficients and Poisson's ratio of fabric depending on the direction of the tensile force on the fabric. For this purpose, classical methods and instruments for testing the tensile properties of fabrics were used.

The experiment was conducted to confirm that the calculation of the elastic constants for anisotropic material is entirely true for woven fabrics or textile surface materials. The experiment was carried out by measuring fabric spatial deformation under the action of a tensile force till rupture, specifically in the warp and weft directions, and at angles of 15°, 30°, 45°, 60°, 75° to the weft.

Samples and testing apparatus

To carry out this study, four different fabrics of different raw material composition (cotton, wool, wool + lycra, PES) and of the same weave (plain weave) were obtained. Raw material and structural properties of the fabrics tested are shown in ***Table 1***.

The yarn count was determined by the gravimetric method according to Standard ISO 2060:1994. Number of threads per unit length was determined according to Standard ISO 7211-2:1984. Standard ISO 5084:1996 describes a method for the determination of the thickness of fabric.

Tests of tensile forces of fabric in specific directions were done on samples of four woven fabrics of different raw material composition and different warp and weft densities, woven in plain weave. Before testing, all samples were conditioned in standard atmosphere (relative air humidity 65 ± 2%, at a temperature of 20 ± 2°C). For the purposes of this testing standard samples with dimensions 350 × 50 mm were placed in the clamps of the instrument at a distance of 200 mm and exposed to uniaxial tensile stress till rupture.

$$\frac{1}{E_x} = \frac{\cos^4\varphi}{E_T} + \frac{\sin^4\varphi}{E_L} + \left(\frac{1}{G_{TL}} - \frac{2 \cdot \nu_{LT}}{E_T}\right) \cdot \cos^2\varphi \cdot \sin^2\varphi \tag{5}$$

$$\frac{1}{E_y} = \frac{\sin^4\varphi}{E_T} + \frac{\cos^4\varphi}{E_L} + \left(\frac{1}{G_{TL}} - \frac{2 \cdot \nu_{LT}}{E_T}\right) \cdot \cos^2\varphi \cdot \sin^2\varphi \tag{6}$$

$$\frac{1}{G_{xy}} = \frac{1}{G_{TL}} + \left(\frac{1}{E_T} + \frac{1}{E_L} + \frac{2 \cdot \nu_{LT}}{E_T} - \frac{1}{G_{TL}}\right) \cdot 4\cos^2\varphi \cdot \sin^2\varphi = \frac{1}{G_{yx}} \tag{7}$$

$$\nu_{xy} = \frac{E_x}{E_T}\left[\nu_{LT} - \left(1 + 2\nu_{LT} + \frac{E_T}{E_L} - \frac{E_T}{G_{TL}}\right) \cdot \cos^2\varphi \cdot \sin^2\varphi\right] \tag{8}$$

$$\alpha_x = 2\left[\left(\frac{1 + 2\nu_{LT}}{E_T} + \frac{1}{E_L} - \frac{1}{G_{TL}}\right) \cdot \sin^2\varphi + \frac{1}{2G_{TL}} - \frac{\nu_{LT}}{E_T} - \frac{1}{E_L}\right] \cdot \cos\varphi \cdot \sin\varphi \tag{9}$$

$$\alpha_y = 2\left[\left(\frac{1 + 2\nu_{LT}}{E_T} + \frac{1}{E_L} - \frac{1}{G_{TL}}\right) \cdot \cos^2\varphi + \frac{1}{2G_{TL}} - \frac{\nu_{LT}}{E_T} - \frac{1}{E_L}\right] \cdot \cos\varphi \cdot \sin\varphi \tag{10}$$

Equations 5 - 10.

The samples were cut in seven different directions: the warp direction (φ = 90°), weft direction (φ = 0°), and at angles 15°, 30°, 45°, 60°, 75° to the weft. Three tests were done for each mentioned direction of force action on the fabric sample. Tensile properties of all samples were tested in accordance with Standard ISO 13934-1:1999 using the strip method for measuring fabric strength on a tensile strength tester.

A TEXTECHNO Statimat M Tensile Tester, shown in ***Figure 2***, was used for tests. The Statimat M tensile tester is an automatic, microprocessor-controlled instrument operating on the principle of constant deformation speed. The follow-

Table 1. *Structural characteristics of the samples tested.*

Material	100% cotton	100% wool	95% wool 5% lycra	100% polyester (PES)
Warp linear density, tex	32	50.6	28	32
Weft linear density, tex	30	47	30	22
Warp density, cm^{-1}	22	26	32	31
Weft density, cm^{-1}	22	18	29	26
Fabric weight, g/m²	150.3	234.8	178.2	164.6
Fabric thickness, mm	0.318	0.568	0.328	0.252

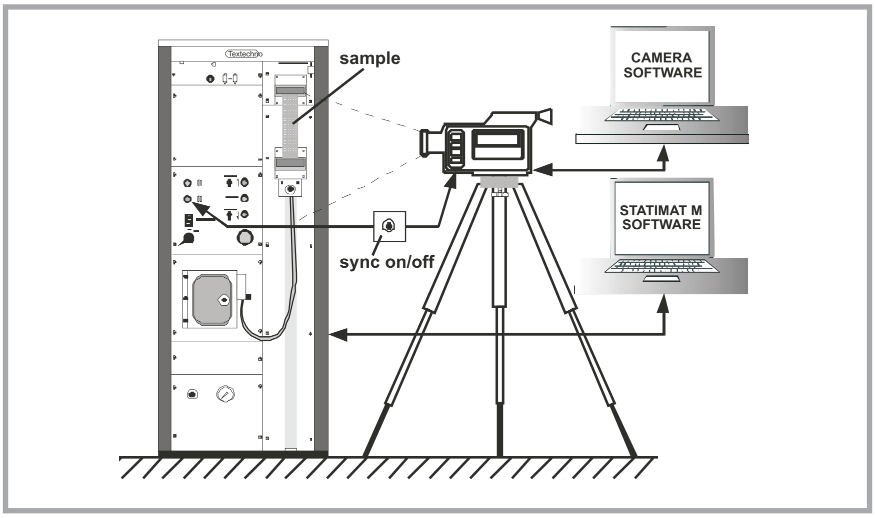

Figure 2. *Schematic view of the experiment.*

***Table 2**. Tensile forces and elongation in the elastic range.*

Angle φ, °	Cotton		Wool		Wool + lycra		PES	
	ε, %	F, N	ε, %	F, N	ε, %	F, N	ε, %	F, N
0	1.10	5.69	1.06	6.58	7.76	0.32	2.06	1.34
15	2.06	1.32	2.98	0.69	11.04	0.22	3.66	0.76
30	6.08	1.01	5.92	0.54	11.84	0.17	5.36	0.56
45	5.56	0.73	8.50	0.41	13.40	0.17	6.46	0.39
60	5.54	1.07	6.80	0.47	9.14	0.22	5.68	0.61
75	2.94	1.60	3.94	0.88	7.22	0.31	2.84	0.75
90	1.52	3.01	1.48	3.43	3.94	0.55	1.30	1.91

ing conditions were used for tests: clamp distance - 200 mm, and pulling speed - 100 mm/min. Measurement results were collected and stored on hard disk by the software package of the tensile tester.

Test method

For accurate recording and measurement of the spatial deformation of the fabric, a 1 ×1 grid pattern was mounted on the tensile tester immediately behind the test specimen, and the whole process of drawing the specimen till rupture was recorded by a Panasonic NV-GS500 Digital Video Camera placed on a tripod in front of the device, as shown in ***Figure 2***. The digital video camera, with a resolution of 720 × 576 pixels recording speed of 25 frames/s, is connected to the computer via an IEEE 1394 (FireWire) interface. The horizontal distance between the camera and sample is such that 1 mm on the grid amounted to 10 pixels. For measuring, white light is used. All the footage was stored on the computer's hard disk in MPEG-2 format. The tensile tester and camera are connected to a special assembly with simultaneous on/off, which fully ensures the exactness of video recording of the entire process of stretching the fabric to rupture. The mentioned grid pattern enables fast and accurate editing of the footage processed by the software package created for this purpose, which specifies the spatial deformation of samples on the basis of shifting in the direction of the x- and y-axis.

Test results

Diagrams of mean values of test results of the action of the tensile force F and the corresponding longitudinal strain (elongation) ε on samples when the force acts in the warp (φ = 90°) and weft directions (φ = 0°) as well as at angles 15°, 30°, 45°, 60° & 75° are shown in ***Figure 3***.

The related mean values of the tensile force F and elongation ε of samples in the elastic range in the warp (φ = 90°) and weft directions (φ = 0°) and at angles 15°, 30°, 45°, 60° & 75° are given in ***Table 2***.

From the diagrams presented in ***Figure 3***, values of the tensile force in the elastic range are used. We determined the modulus of elasticity (Young's modulus) from a particular region on the force – elongation curve, obtained by monitoring experimental data from the experimental

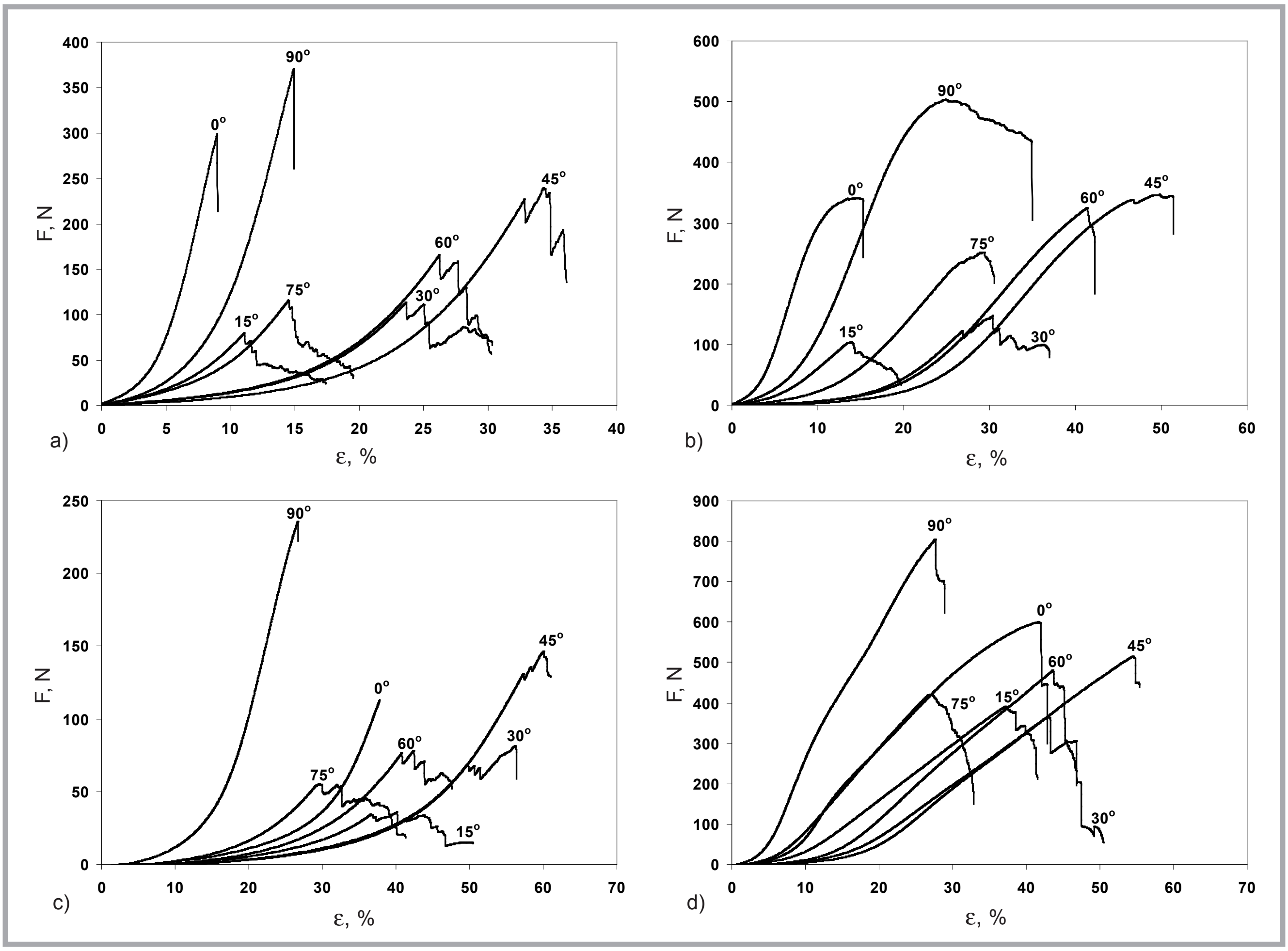

***Figure 3.** Force - elongation (F-ε) of: a)cotton fabric, b) wool fabric, c) wool + lycra fabric, d) PES fabric.*

***Table 3.** Experimentally obtained values of the modulus of elasticity (Young's modulus) E and Poisson's ratio v.*

Angle φ, °	Cotton		Wool		Wool + lycra		PES	
	E, MPa	ν	E, MPa	ν	E, MPa	ν	E, MPa	ν
0	32.559	0.566	21.860	0.705	0.250	0.071	5.152	0.381
15	4.030	0.845	0.815	0.805	0.122	0.374	1.648	0.741
30	1.040	0.869	0.319	1.399	0.085	0.552	0.824	1.312
45	0.821	1.136	0.170	1.377	0.076	0.599	0.478	1.366
60	1.211	1.013	0.242	1.146	0.149	0.641	0.846	1.114
75	3.423	0.767	0.786	1.058	0.261	0.551	2.096	1.078
90	12.436	0.243	8.149	0.277	0.851	0.196	11.674	0.779

set-up with a regression control chart [24]. Using these values and using ***Equations 2*** and ***3***, mean values of the initial modulus of elasticity *E* and Poisson's ratio ν in relation to the arbitrary direction of action of the tensile force on the fabric sample are calculated. The experimental values of the modulus of elasticity *E* and Poisson's ratio *v* obtained are shown in ***Table 3*** and are used to calculate the elastic constants with respect to the arbitrary direction of action of the tensile force on the fabric sample [25, 26].

For each direction of the force action three tests were done, and only the mean values of the experiment calculated are shown in ***Table 3***. The fabrics tested have a greater warp than weft density, or the same, and *E* values at complementary angles are not the same. Since fabrics are a special type of anisotropic material, Poisson's ratio values fall outside the interval from 0 to 0.5, which is given for homogeneous materials such as Al, Cu, Fe, glass, etc. For the plain weave fabrics tested Poisson's ratio assumes values from 0.071 to 1.399 depending on the direction of action of the tensile force, as shown in ***Table 3***.

Calculation of elastic constants in relation to an arbitrarily selected coordinate system

According to ***Equations 5 - 11*** and based on the data in ***Table 3***, the values of elastic constants E_x, $E_{y,}$, G_{xy}, v_{xy}, α_x & $\alpha_{y,}$ were calculated depending on the change in the action angle of the tensile force in relation to the weft. Diagrams of their calculation values for each 5° are shown in ***Figure 4 - 6***.

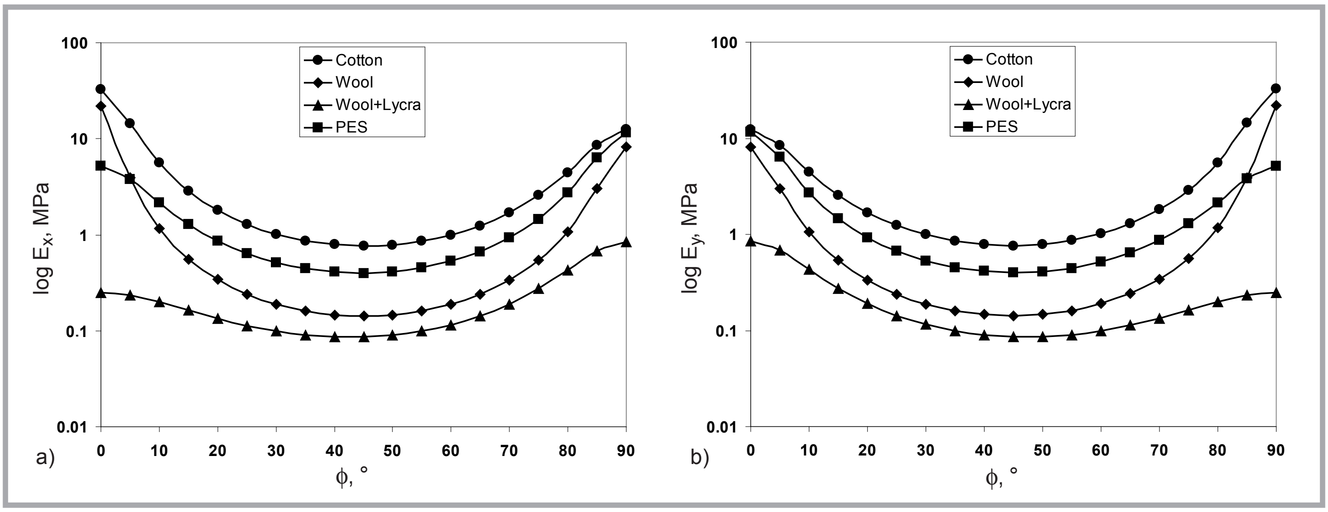

***Figure 4.** Elastic constant: a) E_x, b) E_y.*

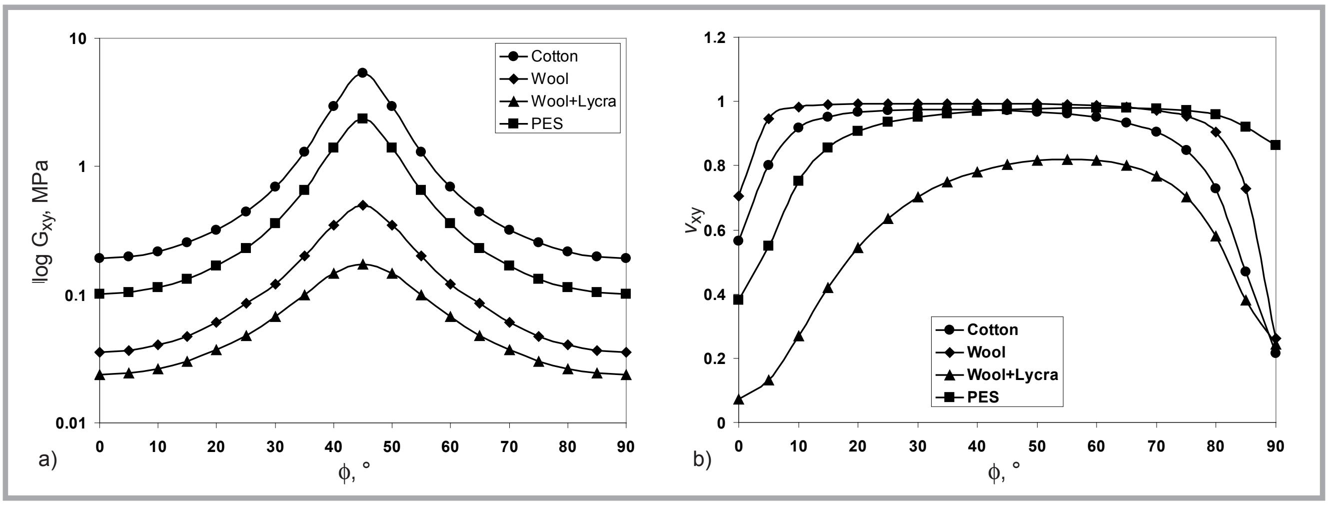

***Figure 5.** Elastic constant: a) G_{xy}, b) v_{xy}.*

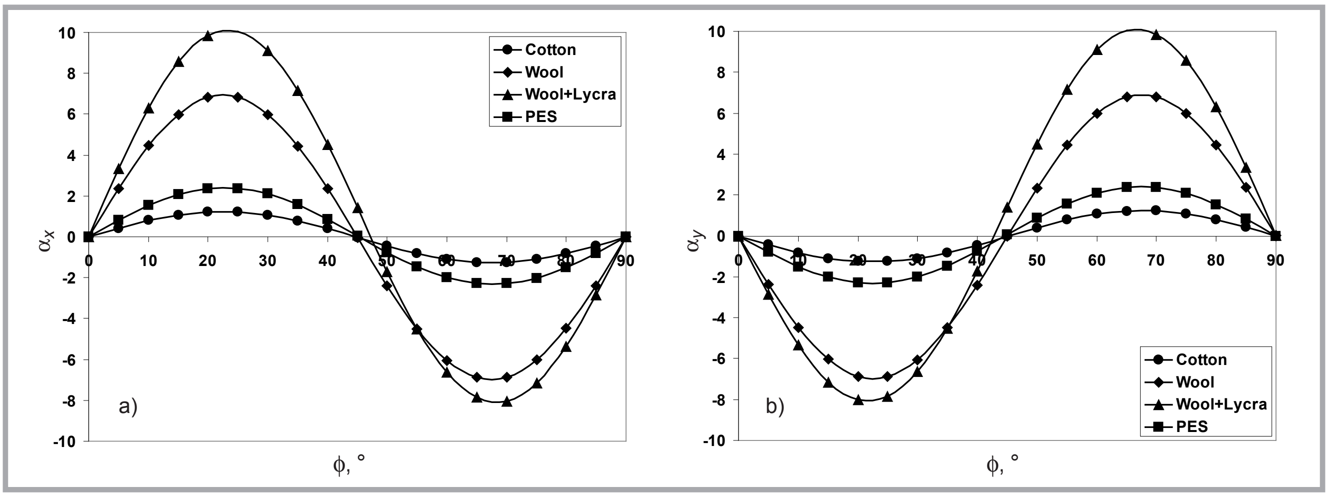

Figure 6. *Elastic constant: a) α_x, b) α_y.*

Diagrams of elastic constants (modulus of elasticity) E_x and E_y are shown in ***Figure 4***. The lowest value E_x is always at an angle of 45° and at that angle E_y is the same as E_x. By definition E_y is equal to E_x if one of the angles (φ) is complementary to another. The diagram also shows that the highest values E_x and E_y lie in the warp (φ = 90°) and weft directions (φ = 0°).

The values of elastic constant E_x decrease from the weft direction (φ = 0°), seen at angles 15° and 30°, while at an angle of 45° they assume the lowest value, increasing towards 90° (warp direction). E_x for cotton and wool fabric assumes the highest value when the stretching force acts in the weft direction. For PES and wool+lycra fabric it assumes the highest value when the stretching force acts in the warp direction, shown in ***Figure 4.a***.

Values of the elastic constant decrease from 0° when the stretching force acts in the weft direction; at 45° they assume the lowest value and E_y increases towards 90°. ***Figure 4.b*** shows diagram E_y, which assumes the highest value for cotton and wool fabric when the stretching force acts in the warp direction, and for PES and wool+lycra fabric it assumes the highest value when the stretching force acts in the weft direction.

The diagram of elastic constant G_{xy} (shear modulus) is shown in ***Figure 5.a***, which is a symmetric curve in relation to the angle of 45°. At that angle G_{xy} assumes the highest value for all fabric samples. When the stretching force acts in the warp (φ = 90°) and weft directions (φ = 0°), the elastic constants G_{xy} have the lowest value for all fabric samples. G_{xy} values in the warp and weft directions are mutually equal for each fabric sample or it is observable that elastic constants G_{xy} are mutually equal for complementary angles.

Table 4. *Calculated values of E_x and ν_{xy}.*

Angle φ, °	Cotton		Wool		Wool + lycra		PES	
	E_x, MPa	$\nu_{\xi y}$	E_x, MPa	ν_{xy}	E_x, MPa	ν_{xy}	E_x, MPa	ν_{xy}
0	32.559	0.566	21.860	0.705	0.250	0.071	5.152	0.381
15	2.857	0.952	0.559	0.990	0.164	0.420	1.284	0.855
30	1.009	0.974	0.190	0.994	0.099	0.702	0.515	0.952
45	0.758	0.971	0.142	0.993	0.086	0.803	0.399	0.974
60	0.984	0.950	0.188	0.987	0.115	0.816	0.530	0.979
75	2.544	0.848	0.539	0.954	0.275	0.703	1.460	0.972
90	12.436	0.216	8.149	0.263	0.851	0.244	11.674	0.864

Table 5. *Differences in % between experimental E & ν and calculated values E_x & ν_{xy}.*

Angle φ, °	cotton		wool		wool+lycra		PES	
	E, E_x	ν, ν_{xy}	E, E_x	ν, ν_{xy}	E, E_x	ν, ν_{xy}	E, E_x	ν, ν_{xy}
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	-29.1	12.7	-31.4	22.9	34.7	12.2	-22.1	15.4
30	-3.0	12.1	-40.5	-29.0	16.2	27.2	-37.5	-27.4
45	-7.7	-14.5	-16.2	-27.9	12.7	33.9	-16.6	-28.7
60	-18.7	-6.2	-22.3	-13.9	-23.0	27.3	-37.4	-12.1
75	-25.7	10.6	-31.4	-9.8	5.2	27.7	-30.3	-9.8
90	0.0	-11.0	0.0	-5.0	0.0	24.4	0.0	11.0

Elastic constants E_x, E_y & G_{xy} have the highest values for cotton fabric, followed by PES fabric and wool fabric, with wool+lycra fabric having the lowest value.

The values of elastic constant (Poisson's ratio) ν_{xy} are shown in ***Figure 5.b***. For cotton, wool, wool+lycra and PES fabrics, ν_{xy} gradually increases from 0° (weft), assumes the highest value at 45°, and then it falls when the tensile force acts in the warp direction (φ = 90°). Wool+lycra fabric has the lowest ν_{xy} and wool fabric has the highest ν_{xy}.

In ***Figure 6*** coefficients α_x and α_y assume maximum and minimum values between angles 0° and 90°, and their curve shape resembles the letter *S* horizontally positioned. It should be noted that the curve shape of coefficient α_x has mirror symmetry in relation to the curve shape of coefficient α_y.

Comparison of the calculated and experimental results

Table 4 shows the values of elastic constants calculated for various angles of action of the tensile force on fabric samples.

Table 5 shows a comparison, in percentage, between experimental values E and ν from ***Table 3*** and calculated values E_x and ν_{xy} from ***Table 4***.

In the warp (90°) and weft (0°) directions, differences in percentage between experimental values E and calculated values E_x are 0%, which follow from ***Equation 5*** due to the periodicity of the sin and cos functions for these values. For cotton, wool, wool+lycra and PES fabrics calculated values E_x are slightly lower than the experimental values E, which can be seen with a negative sign of percentage. From ***Equation 8*** it follows that the differences between experimental values ν and calculated values ν_{xy} are 0% in the weft direction. Calculated values ν_{xy} are slightly higher than the experimental values ν for wool+lycra fabric. These differences are in range from 0% to 33%. For other fabrics: cotton, wool and PES, differences in percentage are around 0%.

E_y was not taken into consideration concerning the correlation due to the values of trigonometric functions of complementary angles. The results of laboratory tests of elastic constants are almost equal to their calculated values, which confirm that the above-mentioned theoretical equations can be used to calculate the elastic constants of fabric with high accuracy

Conclusions

Fabrics can be defined as anisotropic elastomers if the tensile force acting on the fabric is low. Elastic constants vary depending on the angle φ (direction of action of tensile force). Elastic constants (modulus of elasticity) E_x and E_y assume the highest values when the stretching force acts at angles of 0° and 90°, with the minimum value being reached at an angle of 45°. The elastic constant (shear modulus) G_{xy} is symmetrical to the angle of 45°, with the maximum value being reached exactly at that angle. The values of the elastic constant (Poisson's ratio) ν_{xy} for cotton, wool, wool and lycra + PES fabric gradually increases from 0° (weft) to 45°, and then decreases in the warp direction (90°).

Coefficients α_x and α_y assume the maximum and minimum values between angles 0° and 90°, and their curve shape resembles the letter S horizontally positioned. If the angles under which the tensile force acts on the fabric are mutually complementary, then $E_x = E_y$ and $\alpha_x = \alpha_y$. Different materials have different values of elastic constants and Poisson's ratio, but the shape is similar to the corresponding curves. A good agreement between experimental results and the values of elastic constants calculated was shown. The above-mentioned theoretical equations, with high accuracy, can be used to calculate the elastic constants of fabrics for an arbitrarily chosen direction of action of the tensile force. Therefore measurements need to be undertaken when the tensile force acting on the fabric only in the warp and weft as well as at an angle of 45°.

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