Abbas Sheikh Aboumasoudi

Selection of an Optimal Network-Ranking Model to Achieve the Optimal Production Line Value Chain: a Case Study in the Textile Industry

Department of Industrial Engineering, Najafabad Branch, Islamic Azad University,
Isfahan, Iran
E-mail: a_sh_edu@yahoo.com

DOI: 10.5604/12303666.1228158

Abstract
Fulfilling needs and organisational resources with the least cost and highest quality is the main reason to achieve the optimal value chain. Application of most of the current techniques has merely been intended to choose the best scenario. However, industrial units need to build an ideal scenario as a value chain which focuses on intangible interstitial and hidden factors: good (good nature), bad (bad nature), fixed (obligatory nature) and free (not identifying their nature) and creates value. Therefore the model presented in this article answers this issue. First of all, we present a model based on the network approach of data envelopment analysis, then assess and rank the stages based on the scenarios for the stages forming the value chain, and finally the ideal decision unit is presented. For this reason, the general efficiency is designed with two natures; 1. input-centered (concentration on the costs) and 2. output-centered (concentration on the incomes).

Key words: *optimal value chain, data envelopment analysis (DEA), network-ranking models, ideal decision making unit.*

Introduction

Meeting needs and organisational recourses optimally is the main reason for forming a value chain in which the manager is able to guarantee the survival of the organisation, obtain profit and improve it gradually. In organisations, one of the most important issues of decision makers is to select the optimal value chain. Even organisations which are separated legally are considered connected from the viewpoint of material, information and financial flow, forming the value chain. However, a flexible supplier-manufacturer relationship is the key enabler in supply chain management as without flexibility from the vendor's side the supply chain cannot respond fast. Therefore the relationship with the supplier should be flexible enough to meet the changing market needs [4]. In many techniques used for evaluating the value chain, organisations are evaluated separately. Then, based on this approach, the value chain is ranked. There are different techniques available to identify and select the best unit and value chain. In this matter, different articles explain suitable techniques, of which the ANP fuzzy technique can be highlighted to select the best value chain through pair comparison [28]. Also by means of ANP we may evaluate decision making concerning the selection of the best value from different aspects. TOPSIS (technique of packaging selection in multi-criterion decision making issues) is one of the super-efficient evaluation techniques for selecting the best supplier (the optimal value chain) [9]. Data envelopment analysis (DEA) is one of the techniques which is used for selecting the optimal value chain, comparing the units in order to compute their proportional efficiency. In comparison to other techniques, DEA has different advantages, one of which is the ability to compare the input weights and outputs in the pair comparison processes based on their importance for each decision unit [10]. Nevertheless there are different criticisms of DEA Classic models. Generally in DEA Classic models the units are considered as Black Boxes and the internal processes of units are not taken into account in computing their proportional efficiency. To compensate for this weak point, different approaches are suggested which are able to cover the weak points of the classic model and to develop them. Liang (2005)[18], in their thesis, presented a two-stage level via a non-linear method based on an exploratory search to compute the efficiency of unit networks. In the present article, by means of the network approach, the goal programming – data envelopment analysis model is improved and developed. Any value chain includes separated units forming a chain. In the network approach, each of the units is considered as one stage. Therefore in the efficiency approach, any value chain will form the total efficiency of the forming units of the chain. This network and careful viewpoint will give the ability to compute the efficiency of the value chain for decision makers in the best way. Consequently decision makers are able to compare different value chains and select the best. The techniques presented in this article have a special notion among the different stages. It should be mentioned that the final decision maker will be able to make more decisions with a broader view and more information by means of this technique, since, while they are making a decision concerning the factors, they are evaluating the efficiency of the value chain in total based on the effects of each stage on each other. Therefore in the model presented in this article, all important and efficient dimensions in the decision making stages are noted. The changes in the global economic scenario have posed considerable

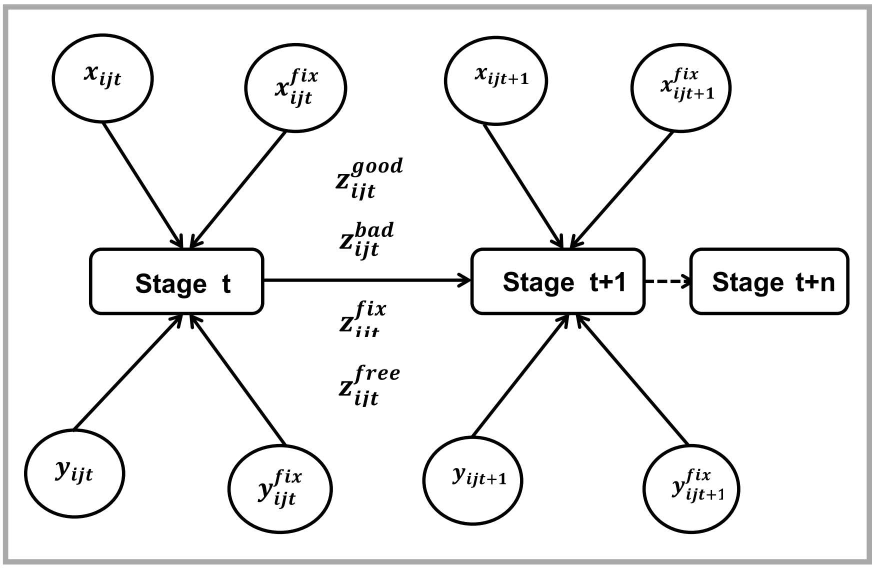

Figure 1. *Network model.*

threats to many companies, especially SMEs as they strive to stay competitive in world markets. This change in paradigms demands more flexibility in product designs. These challenges combined with increased variety and very short lead times have a great impact on the business of small to medium companies in securing a significant proportion of the markets in which they operate [5].

In the present article, firstly data envelopment analysis is described. The greatest attention is paid to the communication between stages. In the case study part, the efficiency of different scenarios is computed. Then, based on the output of the models presented, a new ranking is proposed for chain values. Finally, based on the evaluation of the value chain, the best one (best scenario) is analysed. This value chain is formed by the combination of different value chains.

Literature review

The first model of DEA was proposed in [10], which works under constant returns to scale (CRS). Then the CCR model was adjusted in [6] by adding a convexity constraint to calculate variable returns to scale (VRS). The DEA has been applied in many different settings such as agricultural economics [3], supply chain management [13], sports [22], universities [1], healthcare [8], banking [21], [23], etc.

Network DEA (NDEA) was developed to fill the void of Total Data Envelopment Analysis (TDEA) models when considering the internal structure of DMUs. In other words, TDEA models consider the whole production process as a black box. TDEA takes into account only initial inputs and final outputs. NDEA has been employed in many settings such as banking [7, 12, 26, 2], sport [19], and transportation [27, 29]. [27] employed NDEA to measure passenger and freight technical efficiency, service effectiveness, and the technical effectiveness of 20 railways. [19] used the NDEA approach to measure the efficiency scores of baseball teams. [17] created a NDEA model which distributes the system inefficiency into its components. [11] studied the open multistage process to estimate the overall performance of the network. [20] (in press) proposed a novel NDEA model to evaluate green supply chain management in the presence of flexible factors, bad outputs, and fuzzy data.

[25] created an interval DEA model in which efficiency was calculated within the range of an interval. The upper limit of the interval was set to one and the lower established by introducing a virtual ideal DMU whose performance was superior to any. [15] developed two ranking methods using positive ideal DMU. They ranked 20 Iranian bank branches by two ranking methods. [14] provided a four-phase fuzzy DEA framework based upon the theory of the displaced ideal, In which they made two hypothetical DMUs, namely the ideal and nadir DMUs as reference points to rank the DMUs. [16] proposed an interval DEA model to attain an efficiency interval including evaluations from both the optimistic and pessimistic perspectives. In their method, the lower limits of the DMUs are increased to obtain the maximum value one. The points derived from this method are called ideal points. Then the ideal points are employed to rank DMUs. [24] developed new DEA models for cross-efficiency evaluation by introducing a virtual ideal DMU (IDMU) and virtual anti-ideal DMU (ADMU), the purpose of which was to measure the cross-efficiencies in a neutral and more logical way.

Proposed methodology

To evaluate scenarios based on the communication between stages. In this part, firstly a model based on data the envelopment analysis approach network is analysed, and then the ideal decision unit is presented. In this model, it has been attempted to present a combination of a network model and ranking model (network-ranking model); what action is then required, and then a solution analysis and discussion of a real problem related to the textile industry based on the original model designed using Lingo software are conducted**.** The model is described via the following diagram ***Figure 1:***

x_{ijt} = controllable input (i = 1, 2, 3, ..., m)
x_{ijt}^{fix} = un-controllable fixed input and (i = 1, 2, 3, ..., p)
y_{ijt} = controllable output (i = 1, 2, 3, ..., s)
y_{ijt}^{fix} = un-controllable output (i = 1 , 2 , 3, ..., r)
j = 1, 2, 3, ..., n = (or the ability to produce a scenario)
n = number of scenarios
t = production stage and (t = 1, 2, ..., T)
i = number of inputs or outputs (i^{th} input and or i^{th} output)

Referring to ***Figure 1***, we describe the production ability collection n as the decision unit n (or n scenarios) in which (n = 1,…, N), and we evaluate them in t stages (t = 1,…, T), in each of which (scenario) the decision units hold a controllable input m (i = 1, 2,…, m), uncontrollable fixed input p (i = 1,…, p),controllable output s (i = 1,…, s) and uncontrollable fixed output r (i = 1,…, r). Also we describe four types of communications with Z^{good}, Z^{fix}, Z^{free}, Z^{bad}. On this basis, for example, signs such as (t = 1,…, T), (j = 1,…, n) (i = 1,…, n_{good}) Z^{good}_{ijt} are used to show good communication. In other words, based on the communication of intermediate factors of good, bad, free and fixed, which are described as follows, the model evaluates n stages of the value chain.

- z_{ijt}^{good} = intermediate factors of good nature, trying to maximise the factors such as high-quality thread which is produced at the t stage and entered at the weaving stage (t + 1 stage), which is a good and valuable factor
- z_{ijt}^{bad} = intermediate factors of bad nature which are going to minimise themselves, such as low-quality thread in the sample above
- z_{ijt}^{fix} = intermediate factors of fixed nature and fixed input and outputs which are necessary for different stages. For example, it is necessary to send 1000 kg of thread as an output of the weaving unit to the weaving stage.
- z_{ijt}^{fre} = intermediate factors of no nature which are not recognizable, such as the types of abrasions which are made on fabric at the weaving stage, and shall be studied whether or not the investment in them is suitable for improving them at the completion and dying stages, and or they shall be sold at lower prices on the market as grade 2 materials.

Therefore the production ability collection is described (a set of constraints) as follows, and a complete model including sub-collections is registered and repeated according to the number of scenarios we have had:

- $x_{it} \geq \sum_{j=1}^{n} x_{ijt}\,\lambda_j^t$
  $(i = 1, \ldots, m\,;\, t = 1, \ldots, T)$ (1)
- λ_j^t = *Benchmark of j^th scenario at t^th stage*
- $x_{it}^{fix} = \sum_{j=1}^{n} x_{ijt}^{fix}\,\lambda_j^t$
  $(i = 1, \ldots, p\,;\, t = 1, \ldots, T)$ (2)
- $y_{it} \leq \sum_{j=1}^{n} y_{ijt}\,\lambda_j^t$
  $(i = 1, \ldots, s\,;\, t = 1, \ldots, T)$ (3)
- $y_{it}^{fix} = \sum_{j=1}^{n} y_{ijt}^{fix}\,\lambda_j^t$
  $(i = 1, \ldots, r\,;\, t = 1, \ldots, T)$ (4)
- $z_{it}^{good} \leq \sum_{j=1}^{n} z_{ijt}^{good}\,\lambda_j^t$
  $(i = 1, \ldots, n\ good\,;\, t = 1, \ldots, T)$ (5)
- $z_{it}^{bad} \geq \sum_{j=1}^{n} z_{ijt}^{bad}\,\lambda_j^t$
  $(i = 1, \ldots, n\ bad\,;\, t = 1, \ldots, T)$ (6)
- z_{it}^{free} : free
  $(i = 1, \ldots, n\ free : t = 1, \ldots, T)$ (7)
- $z_{it}^{fix} = \sum_{j=1}^{n} z_{ijt}^{fix}\,\lambda_j^t$
  $(i = 1, \ldots, n\,fix\,;\, t = 1, \ldots, T)$ (8)
- $\lambda_j^t \geq 0\ (j = 1, \ldots, n : t = 1, \ldots, T)$ (9)
- Relation (9): Benchmark of j^th scenario related to t^th stage
- $\sum_{j=1}^{n} \lambda_j^t = 1 \quad (t = 1, \ldots, T)$ (10)

The final equation is the weighted average, which is equal to 1. This limitation shows the output which is changeable based on the scale. If the limitation is omitted, a model with an outcome in proportion to the fixed scale is obtained.

- λ = shows inefficient unit benchmarks; benchmarks (scenarios) are repeated in line with the number of stages in which $\lambda^t \in (t = 1, \ldots, T)$
- R^n = intensity vector at the *t*^th stage
- R^n = weight vector at the *t*^th stage (intensity vector at the *t*^th stage)
- *n* fix = number of fixed connections
- *n* bad = number of bad connections
- *C n* free = number of free connections

If the limitation is omitted, a model of fixed scale proportion is obtained. Note that the right sides of the formulas listed above (x_{ijt}, x_{ijt}^{fix}, y_{ijt} y_{ijt}^{fix}, z_{ijt}^{good} z_{ijt}^{bad}, z_{ijt}^{fix}) are positive. The left sides of the formula (x_{it}, x_{it}^{fix}, y_{it}, y_{it}^{fix}, z_{it}^{good}, z_{it}^{bad}, z_{it}^{fix}, z_{it}^{free}) are connected to each other by λ_{jt}. The continuity of the connection flow between the *t*^th and *t* + 1 stages is guaranteed under the following conditions:

- $\sum_{j=1}^{n} x_{ijt}^{\propto}\lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{\propto}\lambda_j^{t+1}$
  $(\forall i;\, t = 1, \ldots, T-1)$ (11)

This formula uses good, free, bad and fix instead of α and repeats them at each time stage. The presence of these limitations is important to the network model, since they relate the *t*^th stage to the *t* + 1 stage and there is no guarantee of the creation of a national network. Considering these variants, the decision unit (scenario) is described as follows:

- $x_{iot} = \sum_{j=1}^{n} x_{ijt}\lambda_j^t + s_{it}^-$
  $(i = 1, \ldots, m\,;\, t = 1, \ldots, T)$ (12)

Relation (12) is as the same as relation (1), to which a slack has been added for standardisation(and hence relations (13-19) represent relations (2-8). The left side of the relation shows the number of controllable inputs for the scenarios under study at the *t*^th stage.

0 = scenarios under study

$$x_{iot}^{fix} = \sum_{j=1}^{n} x_{ijt}^{fix}\lambda_j^t$$
$$(i = 1, \ldots, p\,;\, t = 1, \ldots, T) \qquad (13)$$

$$y_{iot} = \sum_{j=1}^{n} y_{ijt}\,\lambda_j^t - s_{it}^+$$
$$(i = 1, \ldots, s\,;\, t = 1, \ldots, T) \qquad (14)$$

$$y_{iot}^{fix} = \sum_{j=1}^{n} y_{ijt}^{fix}\,\lambda_j^t$$
$$(i = 1, \ldots, r\,;\, t = 1, \ldots, T) \qquad (15)$$

$$z_{iot}^{good} = \sum_{j=1}^{n} z_{ijt}^{good}\,\lambda_j^t - s_{it}^{good}$$
$$(i = 1, \ldots, n\ good\,;\, t = 1, \ldots, T) \qquad (16)$$

$$z_{iot}^{bad} = \sum_{j=1}^{n} z_{ijt}^{bad}\,\lambda_j^t + s_{it}^{bad}$$
$$(i = 1, \ldots, n\ bad\,;\, t = 1, \ldots, T) \qquad (17)$$

$$z_{iot}^{free} = \sum_{j=1}^{n} z_{ijt}^{free}\,\lambda_j^t + s_{it}^{free}$$
$$(i = 1, \ldots, n\ free\,;\, t = 1, \ldots, T) \qquad (18)$$

$$z_{it}^{fix} = \sum_{j=1}^{n} z_{ijt}^{fix}\lambda_j^t$$
$$(i = 1, \ldots, n\ fix\,;\, t = 1, \ldots, T) \qquad (19)$$

$$\sum_{j=1}^{n} \lambda_j^t = 1 \quad (t = 1, \ldots, T) \qquad (20)$$

Relation (20) is limitation (10), which is repeated.

$$s_{it}^{free} : \text{free } (\forall i, t),\ s_{it}^{bad} \geq 0,$$
$$s_{it}^{good} \geq 0,\ s_{it}^{+} \geq 0,\ s_{it}^{-} \geq 0 \qquad (21)$$

Here, *s* is the variant that results in standardisation of the limitations. In addition, s_{it}^-, s_{it}^+, s_{it}^{good}, s_{it}^{bad} *and* s_{it}^{free} are the side variants that show extra input, output shortage, ideal connection shortage, undesirable extra connections and connection deviation. More concisely, s_{it}^{free} : free $(\forall i, t)$ is related to relation (18), which describes the variant. If relation (16) is negative, the factor is changed to good. If positive, extra factor $= s_{it}^{bad}$ is the bad factor of relation (17); if it is (0), it is the fixed factor located in relation (19) (It is the undesirable connection in relation (17) and its selection as a negative larger or equal item will neutralize the (+) sign.)

s_{it}^{good} = desirable connection shortage
s_{it}^{+} = output shortage (income shortage)
s_{it}^{-} = output extra (cost excessiveness)
s_{it}^{free} = connection deviation

All five items mentioned above are undesirable and should be minimised in the global function:

λ_j^t = repetition of relation (9)

Goal and efficiency function

Total efficiency has two forms: efficiency of an input-based nature (stress cost), and efficiency of an output-based nature (stress income). Models of input-based natures maintain the current output while decreasing inputs. Dynamic SBM

(DSBM) is a side variant for the inputs; side variants for the bad connections are maximised. Models with output-based natures maintain the current input while maximising outputs. In the DSBM model, the variants for output increase simultaneously. The difference in the two models is their effective goal function.

Model with input-based nature

Total efficiency in a model of an input-based nature of is represented by relation (22):

$$\theta_0^* = \min \sum_{t=1}^{T} w^t \cdot \left[1 - \frac{1}{m+n\ bad}\left(\sum_{i=1}^{m} \frac{w_i^- s_{it}^-}{x_{iot}} + \sum_{i=1}^{n\ bad} \frac{s_{it}^{bad}}{z_{iot}^{bad}}\right)\right] \quad (22)$$

in which:

θ_0^* = determines efficiency of the j^{th} scenario (scenario under study)

m = number of inputs in the scenario under study

n bad = number of bad connections of the stage under study

w_i^- = weight of i^{th} input

w^t = weight of t^{th} stage

$\left(\sum_{i=1}^{m} \frac{w_i^- s_{it}^-}{x_{iot}} + \sum_{i=1}^{n\ bad} \frac{s_{it}^{bad}}{z_{iot}^{bad}}\right)$ = bad cost-making factor

The goal function is based on a non-radius model of an input-based nature that contains undesirable connections in addition to extra inputs. The limitations of (1-10) and (12-21) in this goal function are w_i^- and w^t, variants that indicate the ith input weight and t^{th} stage. If all weights are equal, w_i^- and w^t can be considered equal to 1. In model (22), the amount of efficiency is 0 to 1 ($0 \le \theta^* \le 1$). For example, if the efficiency of the production stages of four textile industries equal 0.5, 0.4, 0.9 and 1, the total efficiency of the network will be the sum of the weights by the efficiency of each stage:

Total network efficiency = $W * 0.5 + w * 0.4 + w * 0.9 + w * 1$

It is clear that the only scenario that can obtain a total network efficiency of 1 is that which has an efficiency of 1 at all forming stages. The result of bad n in the denominator of relation (22) is that the model will possess an input-based nature. The goal is for the inside to present items inclined toward zero (s_{it}^- and s_{it}^{bad} incline to 0). Here the efficiency of the stage will incline toward 1 and the same goal will be obtained. The only scenario to obtain an efficiency of 1 for the total network is that which obtains an efficiency of 1 at all forming stages.

Note that the simultaneous presence of variants s_{it}^- and s_{it}^{bad} in the goal function results from the common specifics of these two items, i.e., the lower the value of these two variants, the better. Undesirable connections are mediators between courses and are not inputs. Each course inside the bracket of model (22) indicates the efficiency of the t^{th} stage; if all side variants incline toward 0, the inside bracket will equal 1. Therefore model (22) is the average symmetric efficiency of the time period for all courses and varies from 0 to 1 ($0 \le \theta^* \le 1$). The optimum amount (*) is the efficiency of the t^{th} stage of an output-based nature as follows:

$$\theta_{ot}^* = 1 - \frac{1}{m+n\ bad}\left(\sum_{i=1}^{m} \frac{w_i^- s_{it}^-}{x_{iot}} + \sum_{i=1}^{n\ bad} \frac{s_{it}^{bad}}{z_{iot}^{bad}}\right) \quad (t = 1, \dots, T) \quad (23)$$

Relation (23) is the amount of the inside bracket of relation (22) and identifies the efficiency of the t^{th} stage of the j^{th} scenario.

A model is presented based on the importance of each scenario to achieve a suitable weight based on its importance. The weights are selected for managers, but to eliminate the effect of the human factor of the results, model (24) uses the input-based approach to select weights. Note that the W_p weights show proportional importance at each stage in comparison to all stages. The method of determining W_p is to calculate the total mass of the resources allocated to the p^{th} stage of all stages. This shows the proportional importance of that stage, i.e., the t^{th} stage input/total inputs that enter the stages of the scenario.

At other stages, different connections are available besides the inputs for each stage; this means that the inputs of the stage may include bad, fixed or free connections. Relation (24) is used to compute the weights for each stage of the W_p model. If formula $\sum_{j=1}^{n} x_{ioo}^{\propto} \lambda_j^t + \sum_{j=1}^{n} z_{ioo}^{\propto} \lambda_j^{t+1}$ is a fraction, it will represent the symmetric total unit inputs (scenarios) at the stage under study. The weights of other stages for each decision unit (scenario) are denoted as W_p in the input-based approach:

$$W_p = \frac{\sum_{j=1}^{n} x_{ioo}^{\propto} \lambda_j^t + \sum_{j=1}^{n} z_{ioo}^{\propto} \lambda_j^{t+1}}{\sum_{j=1}^{n} x_{ijt}^{\propto} \lambda_j^t + \sum_{j=1}^{n} z_{ijt}^{\propto} \lambda_j^{t+1}} \quad (24)$$

The first stage of the W_1 formula for computing the weights in special cases is:

$$W1 = \frac{\sum_{j=1}^{n} x_{ioo}^{\propto} \lambda_j^t}{\sum_{j=1}^{n} x_{ijt}^{\propto} \lambda_j^t + \sum_{j=1}^{n} z_{ijt}^{\propto} \lambda_j^{t+1}} \quad (25)$$

Note that the difference between stage (1) and the other stages is that no intermediate factor enters the first stage. At other stages, at least one of the following factors is available:

$$\sum_{j=1}^{n} x_{ijt}^{\propto} \lambda_j^t + \sum_{j=1}^{n} z_{ijt}^{\propto} \lambda_j^{t+1} \quad (\forall i; t = 1, \dots, T-1) \quad (26)$$

Formula (26) is the denominator of fraction (25), which includes all inputs studied in the different scenarios. But the numerator of fraction (25) will include only the inputs of stage (1) which are in proportion. In this case, α denotes the bad, fixed or free relations. Based on the nature of the connection of inputs, α may denote controllable and uncontrollable inputs. If the expression $\sum_{j=1}^{n} x_{ioo}^{\propto} \lambda_j^t$ is a fraction, it will show the total input weights used for the decision unit (scenario) at the related stage. Note that no connection will enter the model in the first course and the model will only have controllable and uncontrollable inputs. In the first course, α denotes the total symmetric input for the uncontrollable and controllable inputs.

The total efficiency of each scenario is computed based on the total symmetric efficiency at different stages using the weights exploited. In this sample, the total efficiency denotes that for the input-based nature at the t^{th} stage. Total efficiency denotes the total symmetric efficiency of courses, as mentioned in formula (27):

$$\theta_0^* = \sum_{t=1}^{T} W_p \theta_{ot}^* \quad (27)$$

in which:

θ_0^* = total efficiency

W_p = weight at the p^{th} stage

θ_{ot}^* = efficiency of the t^{th} stage of the **j**th scenario

Right side of relation (27) = relation (24) × relation (23)

If the optimal answers for model (22) are applied as $\theta_{ot}^* = 1$, the related (decision unit) scenario for the input-based nature at the t^{th} stage is an efficient scenario. It means that side variants of s_{it}^- and s_{it}^{bad} at the t^{th} stage in model (23) all equal zero, i.e., their undesirable connections equal zero. If $\theta_0^* = 1$, the input-based (decision unit) scenario is efficient and s_{iot}^{*-} and s_{iot}^{*-} variants equal zero at all stages.

Model with ouput-based nature

The total efficiency for the output-based nature is as follows:

$$\frac{1}{\tau_o^*} = \max \sum_{t=1}^{T} w^t \cdot \left[1 - \frac{1}{s+n\ good}\left(\sum_{i=1}^{m} \frac{w_i^+ s_{it}^+}{y_{iot}} + \sum_{i=1}^{n\ good} \frac{s_{it}^{good}}{z_{iot}^{good}}\right)\right] \quad (28)$$

The portion inside the bracket shows the income shortage (bad). Based on the limitations of relations (1-10) and (12-21), w_i^+ is the i^{th} output, as in condition (29):

$$\sum_{i=1}^{s} w_i^+ = s \qquad (29)$$

In the fraction denominator, the goal function of (28) is to deal with output shortage and desirable connections as the variants. Please note that in this goal function there is a variant for shortage of good connections and one for shortage of output. Since these are naturally similar to the output and have common specifics, as they increase, they become ideal. This is shown in relation (5).

The good connections are not outputs, but they play the role of connectors of two stages. Any expression inside the brackets corresponds to the goal function of (28) with the efficiency of the t^{th} stage. If the side variants inside the expression equal zero, the amount inside the bracket will equal 1. The goal function is steady in relation to the measuring unit and is ≥ 1. Therefore the average symmetry efficiency of the t^{th} stage of an output-based nature for τ_{ot}^* is shown as (30)

$$\tau_{ot}^* = \frac{1}{1-\frac{1}{s+n\ good}\left(\sum_{i=1}^{m}\frac{w_i^+ s_{it}^{+*}}{y_{iot}}+\sum_{i=1}^{n\ good}\frac{s_{it}^{good\,*}}{z_{iot}^{good}}\right)}, \quad (t = 1, \dots, T) \qquad (30)$$

A model is next presented based on the importance of each stage to obtain suitable weights for each stage:

$$W_p = \frac{\sum_{j=1}^{n} y_{ioo}^{\propto}\lambda_j^t + \sum_{j=1}^{n} z_{ioo}^{\propto}\lambda_j^{t+1}}{\sum_{j=1}^{n} y_{ijt}^{\propto}\lambda_j^t + \sum_{j=1}^{n} z_{ijt}^{\propto}\lambda_j^{t+1}} \qquad (31)$$

in which:

- Nominator: output of related t^{th} stage of related scenario
- Denominator: All outputs which quit all related scenario stages

The W_p weights denote the proportional importance of each stage in comparison with all stages. Based on the output-centered model, the proportional criteria of the stage is to select the W_p amount of decision units (scenario) in the output of each stage to the total outputs at the levels studied. In the W_p model, the nominator of (31) – $\sum_{j=1}^{n} y_{iot}^{\propto}\lambda_j^t + \sum_{j=1}^{n} z_{iot}^{\propto}\lambda_j^{t+1}$ denotes the total output weight of each stage and includes outputs, good connections, and free and fixed connections. The denominator of the fraction of $\sum_{j=1}^{n} y_{ijt}^{\propto}\lambda_j^t + \sum_{j=1}^{n} z_{ijt}^{\propto}\lambda_j^{t+1}$ shows the total output weight and includes the total output as well as the good, fixed and free connections that exit the stages. The amount of controllable and non-controllable output used instead of α is based on the nature of the connection. The total efficiency of each scenario (decision unit) is based on the symmetric average efficiency at different stages from the weights. To compute τ_{ot}^* in formula (30), it is repeated T times. The total efficiency for the output-based nature for the τ_{ot}^* stage is the symmetric total efficiency, defined as:

$$\tau_o^* = \sum_{t=1}^{T} W_p\, \tau_{ot}^* \qquad (32)$$

Relation (32) = relation (31) × relation (30)

Since there is no intermediate factor sent for the next stage, to compute the weight of the final stage weight (T stage), the only outputs used are those that can quit the scenario, i.e., formula $\sum_{j=1}^{n} z_{ioo}^{\propto}\lambda_j^{t+1}$ is omitted from the nominator of formula (31).

Case study

In this part, a textile industry value chain is evaluated. This value chain is formed of 4 stages which are located in different directions from each other. The forming stages are shown in ***Figure 2*** and also 10

Table 1*. Number of connections, inputs and outputs for 4 stages of the value chain (costs and prices in US \$ – weight in grams).*

Scenarios			A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Spinning	Inputs	w/p percent	45/55	45/55	45/55	45/55	45/55	20/80	20/80	20/80	20/80	20/80
		Worker's cost	1.6	1.6	1.8	2	2	0.7	0.7	0.9	1.1	1.1
		Energy cost	0.9	0.9	1.1	1.2	1.2	0.4	0.4	0.5	0.6	0.6
		Maintenance cost	0.5	0.5	0.6	0.6	0.6	0.2	0.2	0.3	0.3	0.3
	Good Connection	Thread quality	14	16	18	20	20	10	12	14	16	18
		Quality	35	35	40	55	64	30	30	35	50	54
	Bad Connection	Thread swing	11.3	11.3	12.6	14.1	14.1	4.5	4.5	6	7.4	7.4
	Fixed connection	Point	40	40	48	60	60	40	40	48	60	60
	Free Connection	Price	16.2	16.2	18	20.2	20.2	6.4	6.4	8.6	10.6	10.6
Weaving	Inputs	Worker's Cost	1.1	0.9	1.1	1.2	1.1	0.4	0.3	0.4	0.5	0.4
		Energy Cost	0.8	0.5	0.7	0.8	0.6	0.2	0.2	0.2	0.3	0.2
		Maintenance Cost	0.3	0.2	0.3	0.4	0.3	0.1	0.1	0.1	0.2	0.1
	Good Connection	One meter weight	400	330	330	330	280	400	330	330	330	280
		Quality	60	65	70	75	80	50	55	60	65	70
	Free Connection	One meter price	10.5	8.6	11.2	12.2	10.5	3.8	3.1	4	4.4	3.8
Completion and Dying	Inputs	Worker's Cost	0.7	0.6	0.8	0.8	0.7	0.2	0.2	0.3	0.3	0.2
		Raw material cost	1.5	1.2	1.6	1.7	1.5	0.5	0.4	0.6	0.6	0.5
		Energy cost	1	0.9	1.1	1.2	1	0.4	0.3	0.4	0.4	0.4
		Color and side material Cost	0.4	0.3	0.5	0.5	0.4	0.2	0.1	0.2	0.2	0.2
	Good Connection	Quality	60	65	70	75	80	50	55	60	65	70
	Free Connection	Price of each meter	15	12.3	16	17.4	14.9	5.4	4.4	5.6	6.2	5.4
Garment	Inputs	tweedy fabric cost	45	36.9	48	52.2	44.7	16.2	13.2	16.8	18.6	16.2
		Worker's Cost	63	51.6	67.2	72.9	62.6	22.7	18.4	23.5	26	22.7
		Control Cost	9	7.4	9.6	10.4	8.9	3.2	2.6	3.4	3.7	3.2
		Energy Cost	4.2	3.4	4.5	4.8	4.2	1.5	1.2	1.5	1.7	1.5
	Outputs	Quality	60	65	70	75	80	50	55	60	65	70
		Suit price	210	172.2	224	243	208.6	75.6	61.5	78.3	86.7	75.6

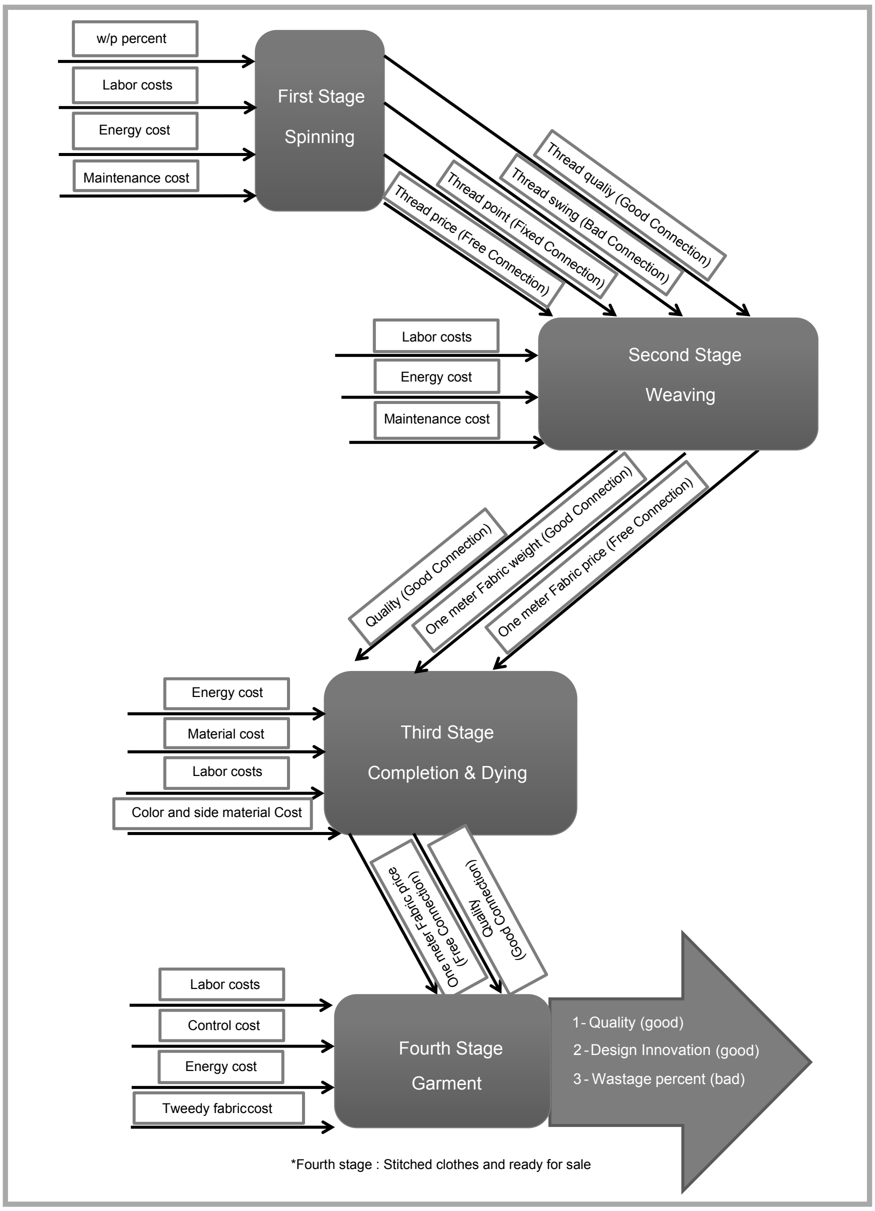

Figure 2. *Connection between different production stages.*

Table 2. *Results of solving 10-stage scenario of four-stage garment production stages based on model. Note: θ_1, θ_1, θ_3, θ_4 – efficiency amount of each of the four stages of production for each of the ten scenarios. W_1, W_2, W_3, W_4 – represent the relative importance of each production the whole process.*

DMU	θ_1	θ_2	θ_3	θ_4	W_1	W_2	W_3	W_4	Overall score
DMU_1	0.928	0.91	0.856	0.854	0.268	0.238	0.246	0.248	0.8876
DMU_2	0.943	0.925	0.878	0.866	0.234	0.217	0.307	0.242	0.9005
DMU_3	0.937	0.923	0.836	0.875	0.285	0.315	0.153	0.247	0.9018
DMU_4	0.963	0.89	0.812	0.871	0.245	0.354	0.273	0.128	0.8841
DMU_5	1	0.821	0.897	0.882	0.224	0.239	0.266	0.271	0.8978
DMU_6	0.897	1	0.941	0.899	0.242	0.347	0.201	0.21	0.942
DMU_7	0.923	0.991	1	0.992	0.227	0.229	0.313	0.231	0.9786
DMU_8	0.921	0.983	0.961	0.957	0.21	0.193	0.321	0.276	0.9557
DMU_9	0.963	0.977	0.972	0.918	0.317	0.22	0.263	0.2	0.9554
DMU_{10}	0.975	0.796	1	1	0.264	0.339	0.208	0.189	0.9242

Table 3. *Results of ideal scenarios for 4 stages of value change in textile industries.*

Stages	First stage	Second stage	Third stage	Fourth stage
Ideal scenario of value chain (1)	Scenario 5	Scenario 6	Scenario 7	Scenario 10
Ideal scenario of value chain (2)	Scenario 5	Scenario 6	Scenario 10	Scenario 10

scenarios (decision unit) suggested by scientific-experimental experts are evaluated. p^{th} stage connections are entered at the p + 1 stage as the input. Also at each stage, interstitial factors (good, bad, fixed and free) are imported and are defined as input. In ***Table 1***, the amount of each factor for 10 scenarios is observed in the value chain.

Finding (ideal decision making unit)

As shown in ***Table 2***, the tenth scenario is introduced as an optimal scenario at the stages of completion and dying (third stage) and garment (forth stage); nevertheless, in the symmetric average, the proportional efficiency ranks slower than for the 7^{th} scenario. In this ranking, scenario (7) obtains the rank of the best scenario, followed by scenarios (8), (9) and (6). The evaluation is presented in ***Table 2***, indicating that the ideal scenario for the value chain is based on a combination of the 10 scenarios that it is shown in ***Table 3***. If the two ideal chain value scenarios are solved with the previous 10 scenarios using the model by Lingo software, they will prove to be strictly efficient.

If these two ideal chain value scenarios are solved with 10 previous scenarios by means of model (1), these two ideal scenarios are highly efficient. In the present article, in addition to ranking of the available scenarios, and since no scenario was able to be introduced as an efficient value chain based on the establishment of an ideal decision making unit with a network approach, the ideal scenario is established. The scenario established that is not available between scenarios is introduced as an artificial scenario; but on the the other hand, this scenario is formed of different stages which are among the main scenarios, and therefore it is real.

Conclusions

- This model is formulated as a network-ranking through linear programming in a way that net profit management strategy is assessed and evaluated through efficiency according to the connections between stages and interstitial factors.
- In designing this model, n factory types (in the case study: 4 types: spinning, weaving, finishing & dyeing and clothing) that usually operate separately are considered both in discretion as well as in combination and integration.

- This model is focused mostly on the connections between the stages constituting the intended value chain. In this mode, 4 connection types (good, bad, fixed and free) are introduced for the stages of the value chain and the scenarios presented are assessed and ranked based on this. At the end of this part an ideal scenario is introduced for the value chain that is created with a general outlook and attention towards the constituting stages and connections between stages. In fact the model evaluates different scenarios based on process-oriented management according to the connection between different stages of production (4 stages in textile industries).
- In n-stage production industries, the formulation of competitive strategies are getting more critical for survival, profit and growth therein, and major decision-makers and planners in production units have urged these industries towards rational management of costs and incomes as well as interstitial factors, and finally towards effective efficiency , this model being a tool for achieving this goal.
- Data envelopment analysis (DEA) was used to determine the relative efficiency of the decision making units (DMUs). One application of DEA is to set benchmarks for inefficient DMUs, helping inefficient DMUs to find improvement strategies. This paper introduces a new approach for ranking efficient DMUs using a network structure. The ideal decision making unit (DMU) constructed offers real and practical solutions for improvement in efficiency.

With respect to the results of this paper, the following research topics are proposed for the future:

- Using goal programming, the goals of managers and experts can be incorporated into the ideal DMU constructed.
- The same approach can be repeated to determine ideal networks in sustainable supply chain management.

References

1. Abramo G, Cicero T, and Andrea D'Angelo C. A field-standardized application of DEA to national-scale research assessment of universities. *Journal of Informatics* 2011; 5, 4: 618-628.
2. Akther S, Fukuyama H and Weber W L. Estimating two-stage network Slacks-based inefficiency: An application to Bangladesh banking. *Omega* 2013; 41, 1: 88-96.
3. André F J, Herrero I and Riesgo L. A modified DEA model to estimate the importance of objectives with an application to agricultural economics. *Omega* 2010; 38, 5: 371-382.
4. A. Jayant, Wadhwa S, Gupta P and Garg SK. Simulation modeling of outbound logistics of supply chain. *International Journal of Industrial Engineering*, 2012; 19(2): 90-100.
5. Ayyaz Ahmad, Muhammad Ilyas Mazhar and Ian Howard. A framework for the adoption of rapid prototyping for SMEs: from strategic to operationa. *International Journal of Industrial Engineering* 2012; 19(3): 161-170.
6. Banker R, Charnes A and Cooper WW. Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science* 1984; 30, 9: 1078-1092.
7. Barth W and Staat M. Restructuring the branch network of a bank: the dynamic perspective. *International Journal of Business and Systems Research* 2008; 2, 3: 272-284.
8. Caballer-Tarazona M, Moya-Clemente I, Vivas-Consuelo D and Barrachina-Martínez I. A model to measure the efficiency of hospital performance. *Mathematical and Computer Modelling* 2010; 52, 7–8: 1095-1102.
9. Chamodrakas I and Martakos D. A utility-based fuzzy TOPSIS method for energy efficient network selection in heterogeneous wireless networks. *Applied Soft Computing* 2011; 11, 4: 3734-3743.
10. Charnes A, Cooper WW and Rhodes E. Measuring the efficiency of decision making units. *European Journal of Operational Research* 1978; 23, 1: 429-444.
11. Cook D, Zhu J, Bi G, and Yang F. Network DEA: additive efficiency decomposition. *European Journal of Operational Research* 2010; 207, 2: 1122-1129.
12. Deville A. Branch banking network assessment using DEA: A benchmarking analysis – A note. *Management Accounting Research* 2009; 20, 4: 252-261.
13. Farzipoor Saen R. Restricting weights in supplier selection decisions in the presence of dual-role factors. Applied Mathematical Modelling 2010; 34, 10: 2820-2830.
14. Hatami-Marbini A, Saati S and Tavana M. An ideal-seeking fuzzy data envelopment analysis framework. Applied Soft Computing 2010; 10, 4: 1062-1070.
15. Jahanshahloo G R, Hosseinzadeh Lotfi F, Khanmohammadi M, Kazemimanesh M and Rezaie V, Ranking of units by positive ideal DMU with common weights. Expert Systems with Applications 2010; 37, 12: 7483-7488.
16. Jahanshahloo G R, Hosseinzadeh Lotfi F, Rezaie V and Khanmohammadi M. Ranking DMUs by ideal points with interval data in DEA. Applied Mathematical Modelling 2011; 35, 1: 218-229.
17. Kao C and Hwang SN. Efficiency measurement for network systems: IT impact on firm performance. *Decision Support Systems* 2010; 48, 1: 437-446.
18. Liang F. Bayesian neural networks for nonlinear time series forecasting. *Statistics and Computing* 2005; 15(1): 13-29.
19. Lewis H F, Lock K A and Sexton T R. Organizational capability, efficiency, and effectiveness in Major League Baseball: 1901–2002. *European Journal of Operational Research* 2009; 197, 2: 731–740.
20. Mirhedayatian S M, Azadi M and Farzipoor Saen R. A novel network data envelopment analysis model for evaluating green supply chain management. *International Journal of Production Economics* 2014: 147, Part B: 544-554.
21. Nandy D. Efficiency study of Indian public sector banks – an application of data envelopment analysis and cluster analysis. *International Journal of Business Performance Management* 2012; 13, No. ¾: 312-329.
22. Ramón N, Ruiz J L and Sirvent L. Common sets of weights as summaries of DEA profiles of weights: With an application to the ranking of professional tennis players. *Expert Systems with Applications* 2012; 39, 5: 4882-4889.
23. Schaefer A, Burger A and Moormann J. Sophisticating business performance management for banks: using data envelopment analysis on business process level. *International Journal of Business Performance Management* 2012; 13, ¾: 227-243.
24. Wang Y-M, Chin K-S and Luo Y. Cross-efficiency evaluation based on ideal and anti-ideal decision making units. *Expert Systems with Applications* 2011; 38, 8: 10312-10319.
25. Wang N-S, Yi R-H and Wang W. Evaluating the performances of decision-making units based on interval efficiencies. *Journal of Computational and Applied Mathematics* 2008; 216, 2: 328-343.
26. Yang C and Liu H-M. Managerial efficiency in Taiwan bank branches: A network DEA. *Economic Modelling* 2012; 29, 2: 450-461.
27. Yu M M and Lin E T J. Efficiency and effectiveness in railway performance using a multi-activity network DEA model. *Omega* 2008, 36, 6: 1005-1017.
28. Zhao Y, Triantis K, Murray-Tuite, P and Edara P. Performance measurement of a transportation network with a downtown space reservation system: A network-DEA approach, Transportation Research Part E: Logistics and Transportation Review 2011, 47, 6: 1140-1159.
29. Zhu J. Airlines performance via two-stage network DEA approach. *Journal of CENTRUM Cathedra: The Business and Economics Research Journal* 2011; 4, 2: 260-269.

Received 19.08.2015 Reviewed 01.02.2016