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Probability of the Critical Length of a Fatigue Crack Occurring at the Tooth Foot of Cylindrical Geared Wheels of the Drive System of a Fiomax 2000 Ring Spinner

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Abstract
In the present paper, we describe a method of determination of the probability of reaching the critical crack length at the tooth root of the cylindrical geared wheels of the drive system of the Fiomax 2000 ring spinner. The Paris-Erdogan formula was utilised for calculations of the fatigue crack length depending on the number of load cycles. Experimental investigations were performed on cylindrical geared wheels. The wheel specimens were manufactured from 1.6523 steel (UE) according to a technical specification relevant to the drive system of the ring spinner. The experiments were performed using a professional pulsator (pulsating test machine). Based upon the experiments (series of 12 tests), material constants and were calculated. These parameters were utilised in the Paris law of crack propagation for further calculations. Moreover it was also ascertained that these unknowns are related via the deterministic relationship. Therefore a function allowing for approximation of constant in dependence on exponent m was derived. In the next step, for the values of parameter chosen – belonging to the variability interval, established from experimental data – we determined the times of reaching the critical length of the fatigue crack. It was stated that the best approximation distribution describing the simulated random values of times of reaching the critical length of the tooth crack for the drive system of the ring spinner is the asymptotic Gumbel's distribution. Knowing the distribution and number of cycles until reaching the critical crack length at the tooth root, one can evaluate the fatigue life of the damaged wheel in the ring spinner (Fiomax) drive system for the assumed probability. The goal of the present paper is evaluation of the working time of the elements of the drive system of a ring spinner until the occurrence of damage. The highest fatigue life of geared wheels was achieved within the interval $(4.3 – 4.5)x\,10^5$ cycles. However, it is recommended to change of the geared wheel in case of the spotting of early symptoms of defect. For the stretching apparatus, the authors of the present paper suggest the exchange of the idler geared wheels at least once per year.

Key words: *tooth foot crack, Paris-Erdogan formula, probability distribution, cycle number, crack critical length, evaluation, fatigue life, cracked tooth, ring spinner, ring spinning frame.*

Nomenclature

a_0, a_1, a_2 constants of an equation determined based upon numerical simulation,

A_1, A_2, A_3, A_4, A_5 constants related to the equation for calculations of the intensity factor coefficient, determined for known technical specification of the geared cylindrical wheel and known loading conditions,

b width of the geared cylindrical wheel,
B_{EX} idler gear wheel,
C_{EX} idler gear wheel,
C the Paris formula material constant,
dl increase in crack length,
E_{EX} idler gear wheel,
F_{EX} idler gear wheel,
G constant utilized for determination of tooth shape coefficient,
Gv total stretch of spinning stream,
Hv main draft of fiber assembly,
Hz_{EX} idler gear wheel,
h_{fp} addendum height of tooth profile,
l crack length,
l_{kr} critical length of crack,
l_{min} introductory crack length,
m exponent of Paris curve (law),
m_n normal module,
m_y mean value of random value distribution,
N number of fatigue (loading) cycles,
Nw_{EX} idler gear wheel,

u expected value of random value distribution,
R reliability,
V_{EX} idler gear wheel,
Vv introductory stretch of spinning flow,
x nominal value of coefficient of displacement of tooth profile (correction coefficient),
z_i teeth number for standard cylindrical wheel,
z_n teeth number related to the transverse pitch diameter,
α_n normal pressure angle,
α probability estimated based upon random value distribution,
β helix angle,
ΔK coefficient of stress intensity,
σ maximal stress in the crack tip,
σ_y^2 variation of random value y,
ϑ auxiliary angle used for determination of tooth shape coefficient,
θ working time of gear until damage,
ρ_{hf} radius of the transition curve (filet radius) for the tangential tooth shape.

Introduction

The propagation of a fatigue crack at the tooth root of geared wheels in the ring spinner drive system (i.e. in its basic elements) can generate damage to the whole system or can decrease the quality of the end-product. The propagation of cracks reaching the critical length at the tooth root of the cylindrical geared wheel is connected with the phenomenon of exceeding the limit, and in consequence instant fatigue breakage occurs, precluding further work of the machine, which diminishes the output of the manufacturing process. In industrial practice, such damage has to be eliminated immediately. Knowledge of phenomena occurring during machine operation diminishes the risk of a breakdown of the whole machine [40]. Introductory initiation of tooth cracks in geared wheels in the normal service conditions of a particular machine can remain unidentified. Damage at the tooth foot – in rare cases – can happen very rapidly, damaging other working elements of a particular machine.

In the textile industry, there are several methods for yarn manufacture, depending on the structure of yarn required for a special end-product. During the spinning process, the manufacturer aims for achievement of a product of the highest physical properties. In the case of ring spinning the maximal ultimate rotational velocity of a spindle is equal to approx. 25 000 rev/min, whereas in the case of rotor spinning this velocity is equal to 150 000 rev/min. This essential increase in the rotational velocity of machine working elements has a negative influence on the reliability of the machine. Moreover in the case of ring spinning, the upper ultimate rotational velocity of spindles is limited by the dimensions of the machine working elements. Classi-cal ring spinning still has the dominant role in the spinning sector of the textile industry due to the quality of yarn achieved as well as its universality. During the spinning process, one of the essential factors influencing the production output is rupture of the spinning flow. This phenomenon causes the most essential problems within the whole technology of yarn production because it influences the production output, quality and number of defective products. The breakage or rapture resistance of yarn can be considered as one of the most important indicators which characterise the technological and organisation level of a spinning mill, and it comprises the basis for objective evaluation of its work. The guarantee of breakage resistance is not an easy task due to the complex mechanism of yarn breakage. The number of ruptures is influenced by the following factors [31-33]:

- yarn/thread tension and its variability,
- mass distribution of feeding and output product,
- strength of yarn,
- winding angle,
- mechanical defects of the stretching apparatus and spindles,
- damage to driving system, especially geared wheels,
- disruptions of operation of turning-winding system: shuttle-ring.

The number of ruptures can be evaluated only when the working conditions of yarn manufacturing are known as well as when we can assume that rupture occurs if its tension exceeds its strength. Unfortunately, although in industrial conditions the average yarn tension does not exceed the strength, yarn is subject to rupture. Due to this fact, for evaluating the number of ruptures expected, it is necessary to determine the variability of yarn tension and strength. However, the phenomenon of rapture itself should be analysed by means of probabilistic methods [34] i.e. based upon probability theory [35]. Yarn ruptures are also influenced by the uneven work of the machine drive system and changes in the velocity of machine working elements caused especially by the structure of the cop. Other reasons for wearing out machine working elements in ring spinning are the rotation of the cop in order to obtain yarn twist and strains on construction elements of the machine. Failure of the driving element of the spinning machine impacts the quality of the yarn, leading to destruction of the driving system in extreme cases. In ring spinners, the stretching apparatus is an essential element of the drive system. Frequent changes in rotational velocity have a direct influence on the dynamics of loading changes. The stretching apparatus consists of a series co-operating gears which are utilized for the setting of stretches. Geared teeth are subject mainly to high-cyclic fatigue phenomena.

Due to the cyclic loading of gear teeth (entering in and losing contact), there is a possibility of fatigue breakage of a particular tooth, which is usually connected to exceeding (at the tooth) the bending stresses of the foot – i.e. the so-called allowable/ultimate fatigue limit. Whereas the cause of too high values of stresses can be different e.g. a too low tooth module, over-loading, the notch effect as well as defects of the transit surface (filet zone) at the tooth foot. Tooth fatigue breakage is usually initiated on the transit surface and propagates along the arc placed along the tooth foot. The described shape of the crack forms invisibly on the outer surface of the tooth surface perpendicular to its main geometrical axis. Despite the propagation of the fatigue crack in its introductory phase, the operation of the gear is still possible and allowable. However,
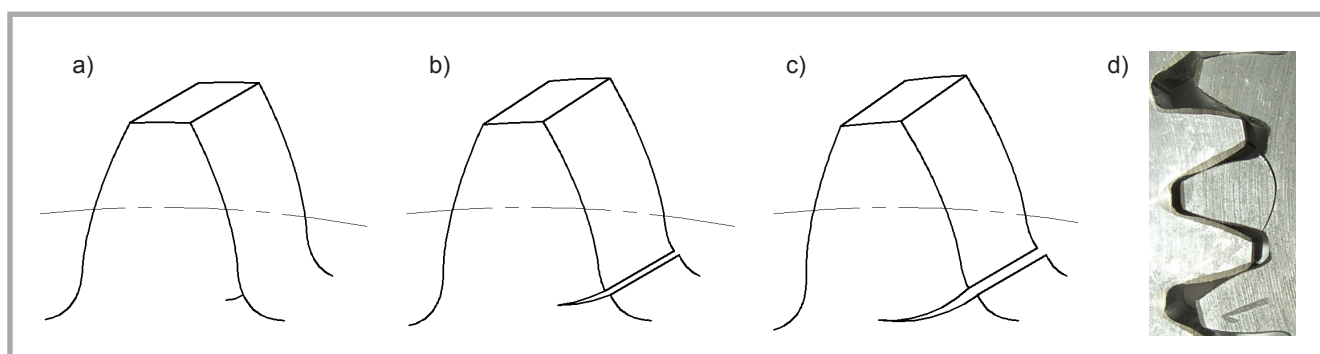

Figure 1. *Consecutive phases of propagation of a fatigue crack: a) initiation, b) propagation, c) critical/final crack which launches the process of rapid growth of the crack, so- called full break of the tooth, d) fatigue crack at the tooth root after full break.*
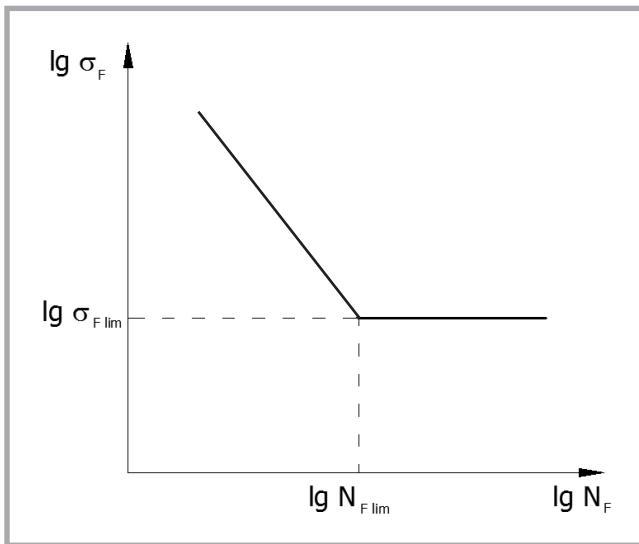
In the figure: vertical axis labeled $\lg \sigma_F$, with $\lg \sigma_{F\,lim}$ marked; horizontal axis labeled $\lg N_F$, with $\lg N_{F\,lim}$ marked.

Analysis of fatigue crack propagation for the sub-systems of the drive system of a Fiomax 2000 ring spinning machine

During design activities, the designers of machines and devices, including textile ones, pay special attention to immediate and fatigue strains and stresses [22]. However, nowadays, it is an insufficient approach to the design task because the user also considers such parameters of a product as its fatigue life and reliability. It is due to the fact that the last characteristics mentioned have a direct influence on costs connected with operation and maintenance procedures.

The fatigue life of the elements of textile machines and devices, especially of geared wheels, is mainly determined based on fatigue investigations. Based on these experiments, one can derivate a Woehler curve [30, 37, 38] which in turn can be utilized for further calculations taking into account fatigue hypotheses e.g. Palmgren – Miners, Haibach, Corten-Dolan or Szala [30, 37, 41]. In the present paper, a method of fatigue life determination based on fracture mechanics principles [37] has been described. The application of the boundary element method allowed to describe the propagation of a fatigue crack ($a = f(N)$). After inserting of this relationship into the Paris – Erdogan formula (or other), the fatigue life measured from the number of loading cycles was obtained.

In the case of fatigue investigations of geared wheels, we create fatigue charts utilizing the bi-logarithmic co-ordinate system $\lg \sigma_F - \lg N_F$ (***Figure 2***), where σ_F is the bending stress (at the tooth foot) in the case of pulsating loading (from zero level), whereas N_F is the relevant fatigue life measured by means of the number of cycles.

Based on this chart, one can determine the basic fatigue characteristics of a geared wheel-specimen i.e. average value of the unlimited fatigue strength of the tooth foot related to bending stresses $\sigma_{F\,lim}$ (for a probability of damage equal to 50%) according to the ISO 6336 standard. It is an approximate value because it depends on the assumed basic number of cycles. According to the ISO 6336 standard, for hardened steel, this value is equal to $N_{F\,lim} = 3 \cdot 10^6$ cycles. The chart discussed also allows determination
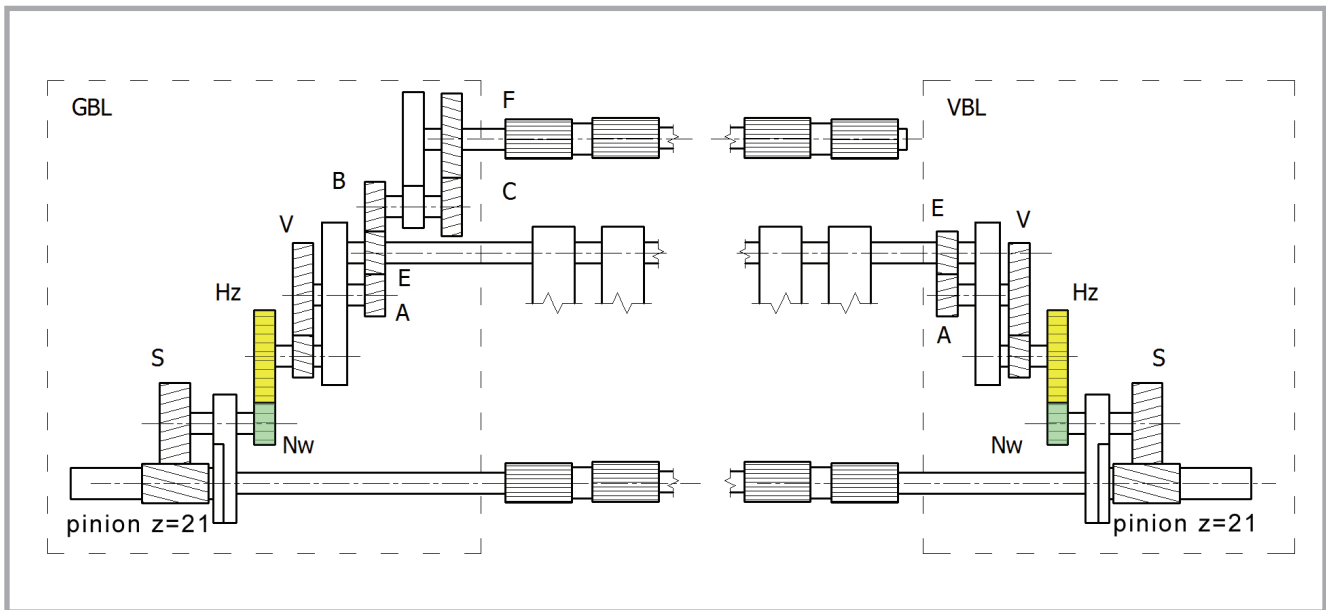
in the case of a continuously propagating crack, after exceeding of the fatigue limit or critical crack length, total breakage of the tooth can happen. In the majority of cases, the critical crack length of a crack located at the tooth foot is approximately half of the tooth width (***Figure 1***, see page 135). However, a detailed description of the situation depends on the loading character and geometrical shape of the toothing given via its technical specification. Early performed diagnostics of the apparatus or whole drive system consisting in registration of damage symptoms and their comparison with the standard (benchmark) signals allows for detection of incorrectness connected with the machine operation routine. Such an approach directly influences the fatigue life of the machine.

Aiming for proper and continuous machine operation, it is very important to operate it under recommended work conditions and according to the requirements as well as regular performance of maintenance tasks (especially repairs). Frequent checking of the drive and working systems of a spinner has a positive influence on the reduction of costs connected with the elimination of errors and defects occurring during the production process.

Evaluation of the technical state of spinning machines is usually performed based on diagnostic investigations [16-18], [23-26], [29]. These investigations consists in different aspects e.g. analysis of vibrations [16, 23, 25], state of surfaces of co-operating geared wheels [17, 24], pitting [18, 24, 27] or scoring [26-28]. Considering only the symptoms of damage to machine elements is not

sufficient and in some cases can lead to the opinion that the drive system is evaluated as sufficient or working properly. However, non-identified damage can appear suddenly, especially for such elements as geared wheels, bearings and shafts [19, 20, 22, 27, 28]. Therefore additional duties – besides possible damage identification via inspections – consist in the utilization of statistical and probabilistic methods and modelling routines [36, 37].

The characteristic feature of phenomena taking place during the wear of working elements of heavy textile machines (under variable loading conditions) is the complete randomness of these processes. In the case of the classic approach to evaluation of the fatigue life of working elements, mainly general statistical methods are used. Advanced probabilistic models are used very rarely due to really complicated mathematical modelling. Nevertheless in related references dedicated to the maintenance of textile machinery and devices, one can find probabilistic models which are based on the following types of random value distributions: exponential, Weibull, normal, Gumbel, Ferecht, Reyleigh, Gamma and log-normal distributions [36]. The majority of the distributions previously mentioned imply the application of advanced numerical methods and techniques as well as the conducting of investigations on a large number of specimens. Therefore during the identification of damage to a particular system, at the beginning it should be assumed that the probabilistic model applied allows for description of the phenomenon considered based on the operational investigations conducted.

Figure 3. *Scheme of the drive system of the stretching apparatus of the Fiomax 2000 ring spinner [11].*

of the average limited fatigue life, i.e. $\sigma_F > \sigma_{F\ lim}$ (for a probability of damage equal to 50%) relevant to the fatigue life assumed $N_F > N_{F\ lim}$. Moreover in the reverse approach it is also possible to determine the fatigue life N_F for the assumed level of stress σ_F.

Having determined the fatigue life (from the beginning of crack initiation, throughout the propagation until damage), it is possible to determine the reliability of the arbitrary drive system of the Fiomax 2000 spinner. The drive system of the stretching gear in the head of the ring spinner (***Figure 3***) has been analysed in the present paper. The geared wheels have been chosen arbitrarily for consideration, but only from those which allow determination of the total even stretch ***Equation (1)*** initial stretch, see ***Equation (2)*** and finally, main stretch, see ***Equation (3)***.

During the operation of machines, the geared wheels are subjected to variable loading which can cause damage, especially fatigue damage. The phenomena which occur most frequently are defects of the drive system e.g. fatigue cracks near the tooth foot.

As a crack, one considers this imperfection of the material structure to have a particular size and shape. When the material is loaded, the crack surfaces can open or be displaced in relation to each other, whereas in the unloaded state these surfaces can contact each other. Cracks can penetrate throughout an element

(***Figure 4.a***), can exist in its internal volume e.g. defect of the crystal structure, (***Figure 4.b***) or partly penetrate into the material interior (***Figure 4.c***).

Figure 4. *a) In-plane crack (throughout), b) internal crack (void), c) edge crack.*

In theoretical and practical considerations, the terms 'crevice', 'rupture' are also considered. In so-called fracture mechanics, a 'crack' represents a real defect
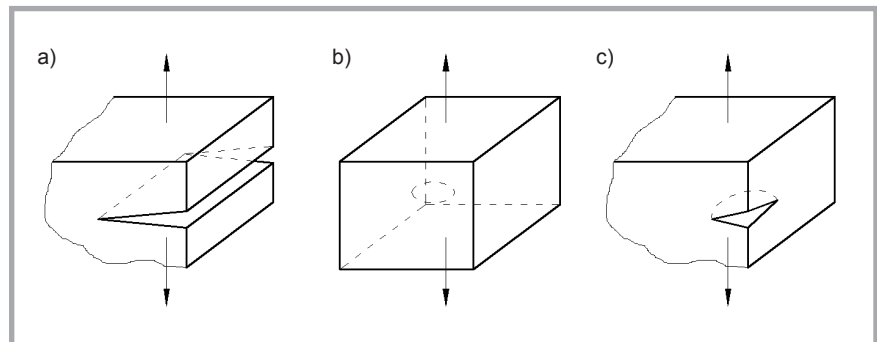
$$Gv = Hv \cdot Vv = 0{,}3560 \cdot \frac{Hz_{EX} \cdot B_{EX}}{Nw_{EX}} = 22{,}199, \qquad (1)$$

initial stretch:

$$Vv_{shaft\ II \to shaft\ III} = \frac{shaft\ II \cdot F_{EX} \cdot B_{EX}}{shaft\ III \cdot E_{EX} \cdot C_{EX}},$$

$$Vv_{shaft\ II \to shaft\ III} = \frac{31{,}5 \cdot 27 \cdot B_{EX}}{32{,}0 \cdot 37 \cdot 25} = 0{,}0287 \cdot B_{EX} = 1{,}0906, \qquad (2)$$

and finally, main stretch:

$$Vv_{shaft\ I \to shaft\ II} = \frac{shaft\ S \cdot V_{EX} \cdot E_{EX} \cdot Hz_{EX}}{shaft\ III \cdot E_{EX} \cdot C_{EX}},$$

$$Vv_{shaft\ I \to shaft\ II} = \frac{32{,}0 \cdot 85 \cdot 43 \cdot 37 \cdot Hz_{EX}}{31{,}5 \cdot 21 \cdot 24 \cdot 22 \cdot Nw_{EX}} = 12{,}390 \cdot \frac{Hz_{EX}}{Nw_{EW}} = 20{,}33. \qquad (3)$$
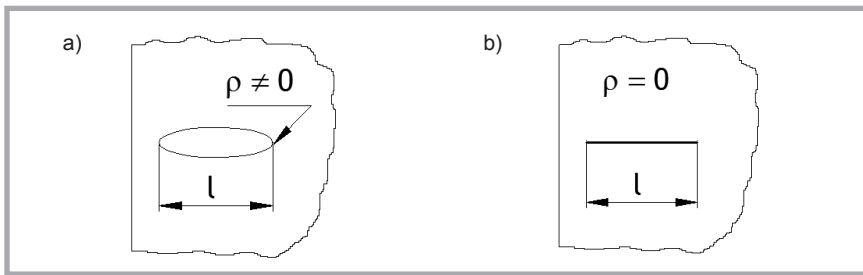
Equation (1), (2) and *(3)*.

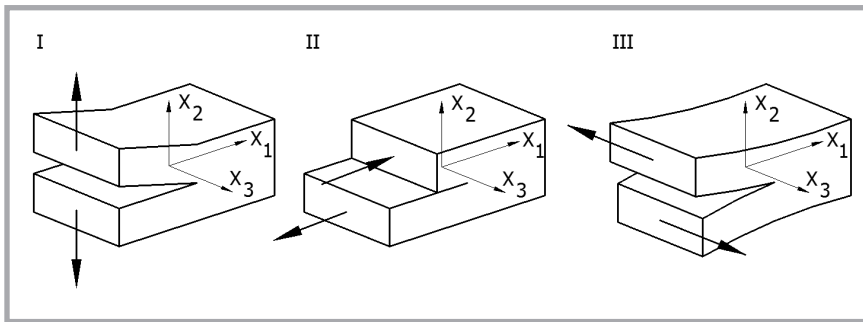*Figure 5. a) Crack ρ ≠ 0, b) crevice ρ = 0.*

Figure 6. Models of crack propagation: mode I – crack of opening, mode II – crack of (in-plane) longitudinal shear, mode III – crack of (out-of-plane) tangent shear.
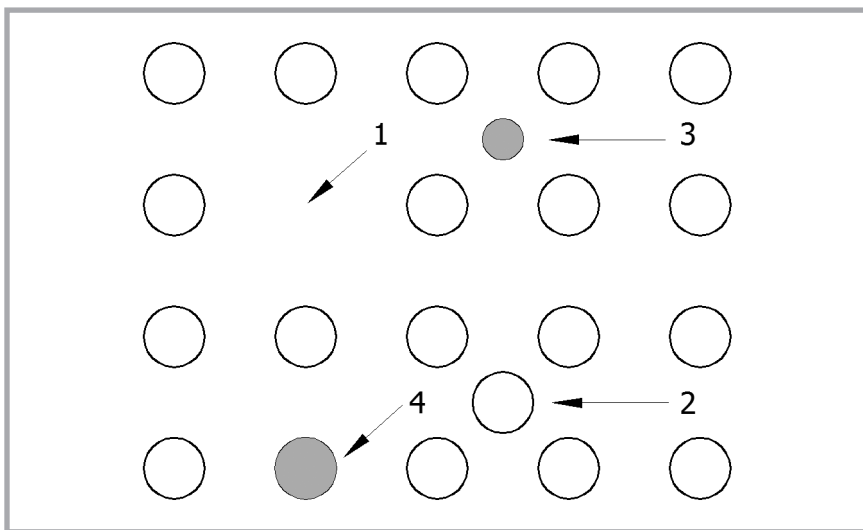

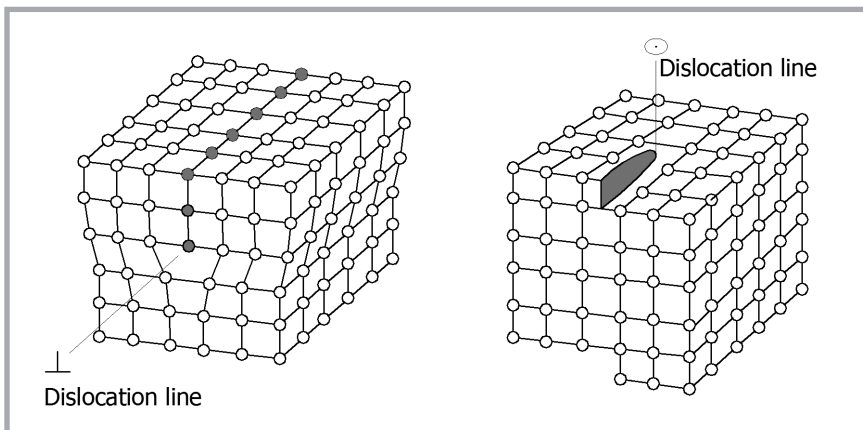*Figure 7. Point defects of crystals.*

Figure 8. Linear defects in material (dislocations) [9].

inside the material. A crevice has a theoretically radius of curvature equal to zero at its tip (***Figure 5***).

As a cracking process, we consider the growth of a crack to consist in an increase in its characteristic dimension i.e. length. Regarding the front (tip) of the crack, we consider its edge to be inside the material.

In general, three characteristic models of crack propagation – proposed by Irwin – are considered, i.e. a crack of mode I (opening) excited by tension stress, a crack of mode II (longitudinal shear) excited by shear loading acting parallel to the cracking plane and perpendicular to the crack front, as well as a crack of mode III (tangential shear) excited by shear loading parallel to the crack front (***Figure 6***).

The main factors which have an essential influence on the crack direction and propagation are the loading and material structure. The process of crack initiation can take place in the following structure forms [39]:
■ slip bands,
■ boundaries of grains,
■ undersurface inclusions (very rare).

Other indicators like heat-chemical and mechanical treatment of material surfaces, the work environment and the variability of loading also have an crucial influence.

Depending on the material structure, the causes of cracks arising/occurring can be divided into following types:
■ point defects (***Figure 7***) – in fact, one can distinguish four types of defects: 1 – vacancies (empty place –Schotthy's defect), 2 – inter-node atom, 3 – improper atom (inter-node), 4 – improper atom in the exact structure node,
■ dislocations (***Figure 8***) – along an edge (the plane of the additional plane exists in a crystal volume), screw dislocations (i.e. defect of the crystal structure caused by the displacement of part of the crystal volume around the particular axis).

Cracks passing along the grain boundaries frequently exist in the case of high stress amplitudes and at high temperatures. Sometimes even a relatively moderate force can cause that the inter-particle bonds are subject to damage. Weak bonds caused that molecular crystals are

susceptible to deformations. An exemplary crack propagation along the grain boundaries is presented in *Figure 9*.

Exemplary scenarios of crack propagations [10] are shown in *Figure 10*. Three curves representing crack propagation are drawn as a function depending on the following ratio: the number loading cycles *N* nin relation to the fatigue life *N* (until breakage). The points on the abscissa axis are related to the percentage, whereas the axis of ordinates represent an increase in the crack length (semi-log coordinate system).

As can be noticed, crack initiation is not always connected with its propagation. Similarly not every defect or initiated crack could be identified or detected.

Exemplary models of crack propagation are presented in *Figure 11*.

High stress concentration appears around the crack tip. Crack propagation is caused by an outbreak of slip planes, whereas the propagation and length increase of the crack occurs in the direction of the presence of tangent stresses (*Figure 11*, *level 1* and *2*). Under active loading, the crack opens consecutively and an increase in its length occurs simultaneously Δ*a* (*Figure 11*, *level 3*). Strengthening of the material and increasing stresses cause blunting of the crack tip (*Figure 11*, *level 4*). During one cycle of loading, the crack propagates by Δ*a*. Around the elastic surrounding of the crack tip, small plastic deformations occur. An increase in plastic deformations takes place along with an increase in loading. Assuming that the acting load has a variable character, then due to changes in the directions
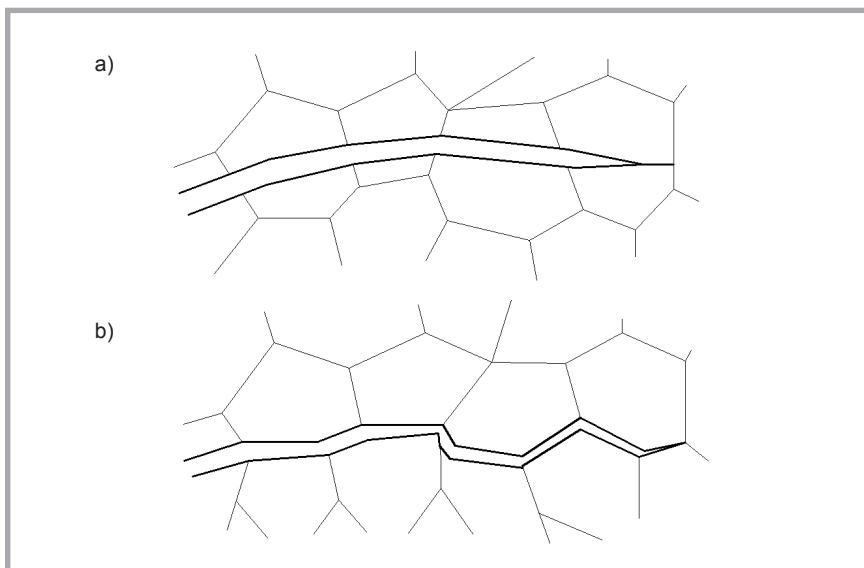

Figure 9. *Loss of cohesion of material: a)trans-crystal (crack crossing through grains); b) intercrystal (crack going along grain boundaries) [9].*
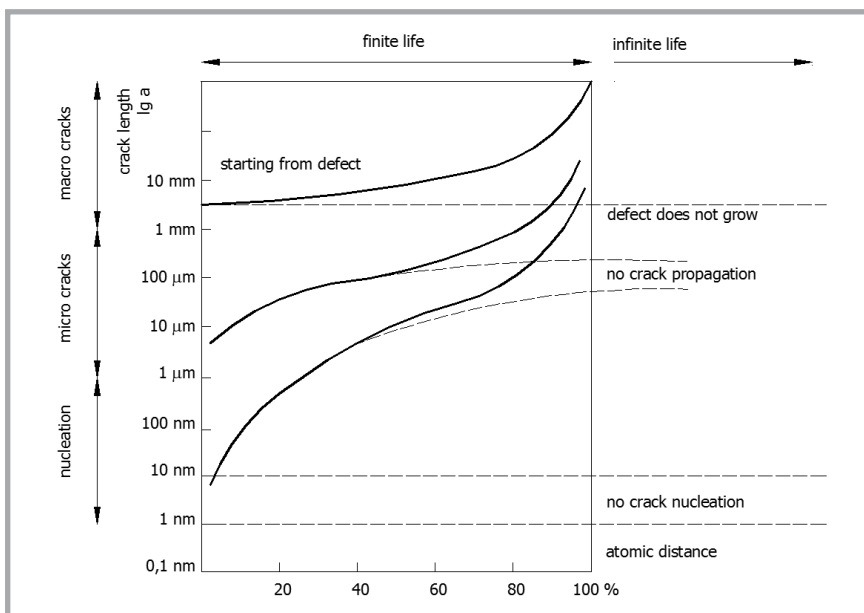

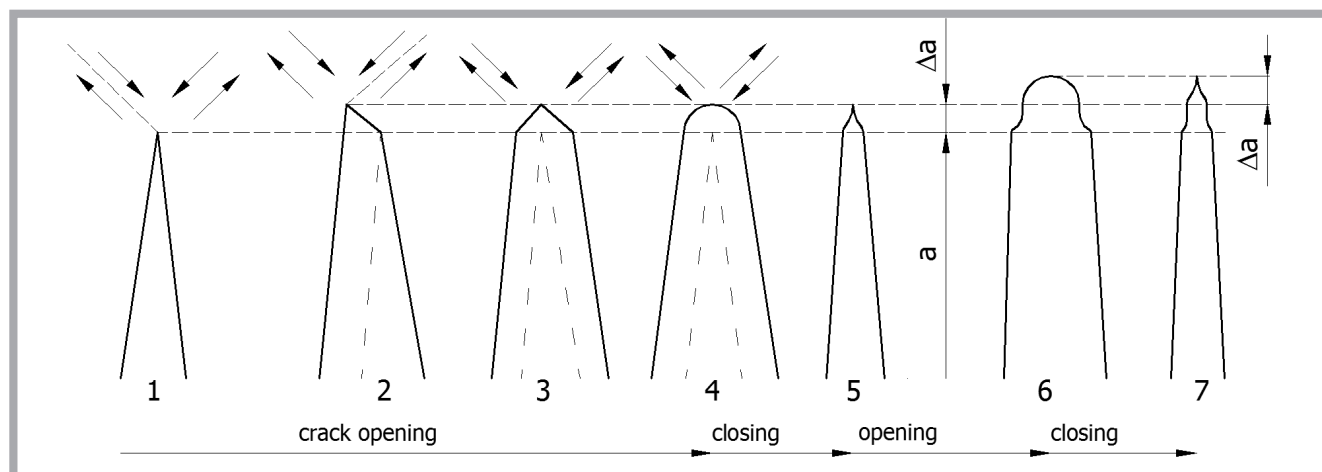*Figure 10.* *Different scenarios of fatigue crack propagation [10].*

Figure 11. *Exemplary model of crack propagation [9].*

Table 1. Equivalent materials for 20HNM steel – utilized for manufacturing of the geared wheel specimens.

Poland	EU	USA	France	Japan	China	Russia
20HNM	1.6523	8617	20NCD2	SNCM220	20CrNiMo	20HGNM

Table 2. Parameters of the geared wheels investigated [11].

Normal module	m_n	2.0
Pressure angle	α_n	20 deg
Number of teeth	z_1	39
Helix angle	β	0 deg
Correction coefficient	x	0
Radius of reference tooth shape	ρ_{hf}	0,5
Addendum height	h_{fp}	2,5

Table 3. Constants C and m for the Paris model determined for the experimental data shown in **Figure 12**.

Test no.	1	2	3	4	5	6
$C \times 10^{11}$	4.724	4.213	0.172	0.178	0.349	1.323
m	1.913	1.929	2.472	2.449	2.325	2.073
Test no.	7	8	9	10	11	12
$C \times 10^{11}$	5.149	5.562	2.645	3.688	1.079	1.523
m	1.872	1.848	2.364	1.928	2.132	2.041

Table 4. Constants of Paris law determined for the empirical data shown in **Figure 13**.

Test no.	1	2	3	4	5	6
$C \times 10^{11}$	1.381	1.081	0.908	0.573	0.437	0.437
m	2.1	2.14	2.168	2.24	2.28	2.28
Test no.	7	8	9	10	11	12
$C \times 10^{11}$	0.282	0.240	0.240	0.221	0.211	0.172
m	2.34	2.36	2.36	2.37	2.395	2.395

of the acting force, the closing of edges near the crack tip takes place as well as the arising of plastic deformations (**Figure 11**, **level 5**).

Prediction of the working time of the stretching gear of the Fiomnax 2000 ring spinner

Usually the detection of a fatigue crack at the tooth foot implies the immediate switching off of the drive system. The sudden suspension of the technological process is frequently connected with high economic losses. Therefore there is a need for the preparation of methodology for determination of the time when a particular crack reaches the critical length. The aims of these attempts are the assurance of continuous technological processes as well as minimisation of losses due to the sudden suspension of production.

A model which enables prediction of the time for reaching the critical crack length

(for crack growing at tooth foot) is derived by solving the modified Paris law [1, 12]:

$$\frac{dl}{dN} = C \cdot \left(\Delta K(l) \right)^m \quad (4)$$

Relationship (4) is a commonly applied equation describing the velocity of fatigue crack propagation. Paris, in his publication, stated that velocity of cracking depends on stress intensity coefficient K. According to this concept, crack propagation is governed by the variability of local stresses in the crack tip, whereas coefficient K describes the effect of load action and the stress field in the area around (neighborhood of) the tip. Depending on the crack model considered (**Figure 6**), one should distinguish adequate stress intensity coefficients: crack opening – K_I, crack under longitudinal shear – K_{II} and for a crack relevant to tangent shear – K_{III}. In general, the stress intensity coefficient K is expressed by **Equation (5)**:

$$K = \sigma \cdot \sqrt{\alpha \cdot \pi \cdot l} \quad (5)$$

where: σ external stress at the crack tip, α – parameter depending on the shape of the specimen and crack geometrical form.

In the present paper, we utilise the formula for determination of the range of the stress intensity coefficient ΔK as a function of the range of the relevant stress, i.e.: $\Delta\sigma = \sigma_F > \Delta\sigma_{F\ lim}$ and the ultimate length of the crack l:

$$\Delta K = \Delta\sigma \cdot \sqrt{\pi \cdot a} = \sigma_F \cdot \sqrt{\pi \cdot a} \quad (6)$$

Coefficients C and m – which appear in formula (4) – are material-based constants determined via experimental investigations.

The parameters of the model were determined based on the results of investigations of geared wheel-specimens made of 1.6523 (20 HNM) steel. Similar (equivalent) materials considered in different countries are listed in **Table 1** for comparison.

Technical specifications i.e. geometrical parameters of the geared wheels. are listed in **Table 2**. Within the framework of the experiments, a pair of idler gear wheels of a FIOMAX spinner were utilised. The adequate tooth numbers were as follows: $z_1 = 39$ and $z_2 = 64$ [11], respectively. The wheels were subjected to heat treatment, carburization, nitration and hardening (reaching hardness: 58 ± 2 HRC). Investigations were performed ona hydro-pulsating test machine, at a maximal pressure force equal to 150 kN. The experimental tooth loading applied had a constant-amplitude, sinusoidal course of 30 Hz frequency [38].

The following measurements were performed: measurements of fatigue crack propagation for 12 teeth of the wheel-specimen, And the loading acting in the direction normal to the pitch diameter, which caused a bending stress equal to 1500 MPa. Measurement of the fatigue crack length were performed by means of a laboratory microscope, after every 5×10^4 cycles from the moment of crack detection.

In **Figure 12**, curves of growth of the fatigue crack depending on the number of loading cycles are shown.

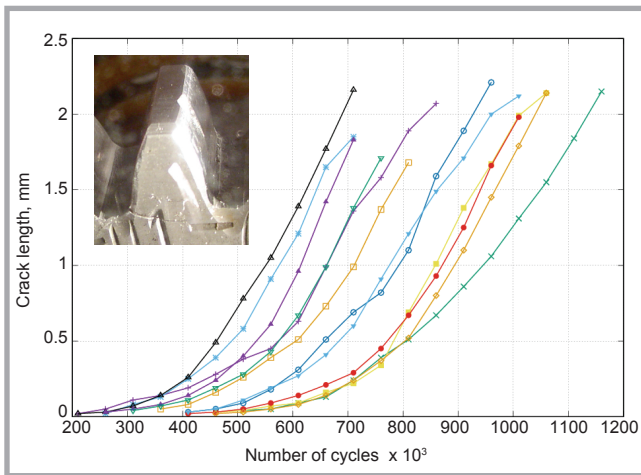Based on an analysis of the data shown in **Figure 12**, one can state that it is possible

Figure 12. *Curves representing growth of fatigue cracks at the tooth foot for 12 geared wheels loaded at moment M = 750 MPa, manufactured of 1.6523 (20HNM) steel.*
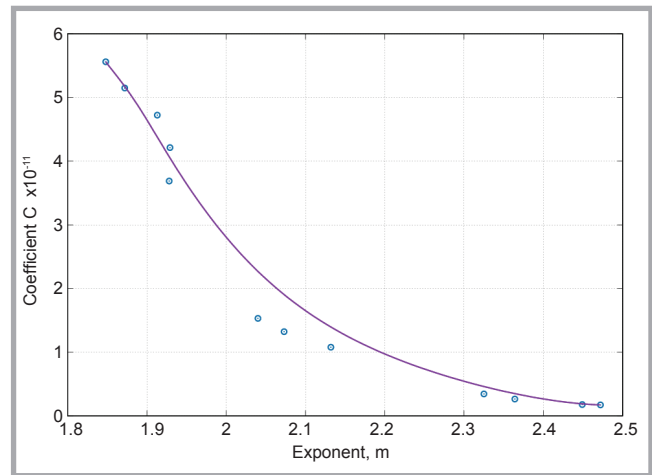

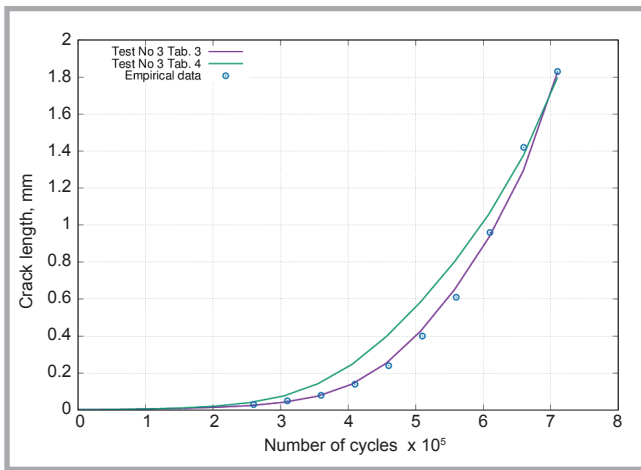**Figure 13.** *Approximation function of C and m summarized in Table 3.*

Figure 14. *Comparison of fracture curves obtained based on the solution of Equations (8) and (4) with empirical data relevant to test no 3.*
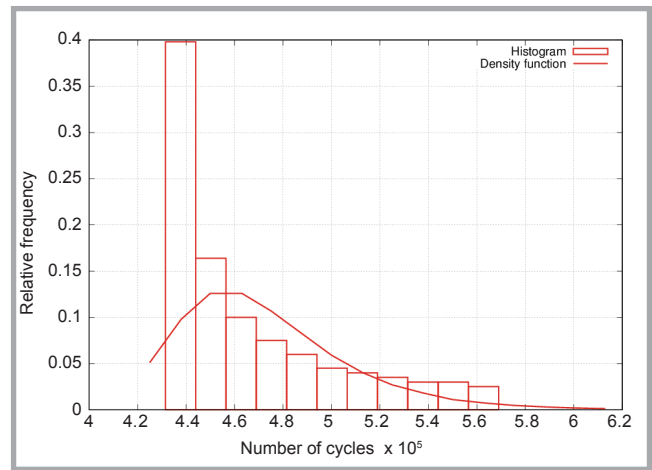

Figure 15. *Bar chart (histogram) of times of reaching the critical value by the fatigue crack for 1.6523 (20HNM) steel and stresses at the tooth foot $\sigma_F = 1500$ MPa in the drive system of a Fiomax 2000 spinner.*

to determine the working time of a wheel (of the drive system) as a function of crack growth (near tooth foot) depending on the loading excited. For a crack length greater than 2 mm, accelerated fatigue crack growth was observed.

Determination of constants in Paris-Erdogan equation

In the second phase of investigations, constants C and m for the Paris model (4) were established based on experimental results. These constants were derived utilising the modified method of definite integral [2]. The results obtained are presented in *Table 3*.

In *Figure 13*, a chart of the relationship of constant C against exponent m is shown. It was made utilising the experimental data gathered in *Table 3*.

Based on an analysis of the chart, one can conclude that constants C and m depend on one another and the adequate relationship can be expressed by means of a particular function. For the data presented in *Figure 13*, the approximation relationship between C and m is proposed in the following form:

$$C(m) = a_0 \cdot m^{a_1} + a_2. \qquad (7)$$

Where, coefficients $a_0 = 7.49 \times 10^{-8}$, $a_1 = -11.435$, & $a_2 = -1.675 \times 10^{-12}$ have been determined by means of the method of minimization of the average square error between the data given in *Table 3* and those calculated utilizing the approximation function (7). Inserting *Equation (7)* into *(4)*, we obtain the following Equation:

$$\frac{dl}{dN} = (a_0 \cdot m^{a_1} + a_2) \cdot (\Delta K(l))^m. \quad (8)$$

Furthermore utilizing *Equation (6)* for approximation of the empirical data, we obtained the constants of the model – gathered in *Table 4*.

In *Figure 14*, there are exemplary solutions of *Equations (8)* and *(4)* related to the data collected during test no 3, prepared according to the items shown in *Tables 3* and *4*, respectively. Based on an analysis of the charts, we can draw the conclusion that the application of *Equation (8)* gives some errors in comparison to empirical data describing the growth of the fatigue crack in time. If the points of chart 14 are placed on the approximation curve, then we obtain the compatibility of the crack growth chart with the empirical data. A characteristic phenomenon is the incompatibility of data determined by means of *Equation (8)* within the middle range of the process of crack growth,

whereas in the final range the empirical and model-based data overlap each other. In industrial practice, an engineer is interested in the time when the crack reaches the ultimate size, after which final tooth breakage takes place. Thus one could utilize *Equation (8)* due to the compatibility of results with empirical data in the upper range of the crack growth chart (curve of fracture).

Prediction of the number of fatigue cycles acting on the geared wheel of the spinner drive system until reaching the crack critical length

Assuming that the probability distribution value m in *Equation (8)* is uniform (even) within the range $m_{min} \leq m_i \leq m_{max}$, the time (number of cycles) until the crack reaches the ultimate value can be determined using the transformed relationship (8):

$$N_i = \int_{l_{min}}^{l_{kr}} \frac{1}{(a_2 + a_0 . m_i^1) \cdot \Delta K(l)^{m_i}} dl. \quad (9)$$

where: m_i – value of exponent m established within the range $m_{min} \leq m_i \leq m_{max}$, l_{kr} – critical crack length, l_{min} – minimal crack length.

In *Figure 15* (see page 141), we present a histogram of the distribution of time N_i of reaching the critical fatigue crack length by the tooth foot /material 1.6523 (20HNM) steel/. A bar chart was prepared taking into account 200 values of parameter m within the interval $2.14 \leq m \leq 2.395$ for stresses near the tooth foot equal to $\sigma_F = 1374$ *MPa*, criti-

cal crack length $l_{kr} = 2,4$ *mm* and minimal crack length $l_{min} = 0,05$ *mm*.

Based on an analysis of the data shown in the chart, one can state that the longer the operating time of the geared wheel, the lower its fatigue life. The highest fatigue life was achieved in the following interval $(4.3 – 4.5)$ x 10^5 cycles. The scatter is connected, as usual, with the propagation of defects or a particular damage mechanism.

In practice, it means that the recommended change of the geared wheel should be made before the performance of 3 x 10^6 cycles (ISO 6336-3:2006, Calculation of load capacity of spur and helical gears – Part 3: Calculation of tooth bending strength).

Due to the wear of the most frequently co-operating geared wheels, their continuous monitoring (SHM) is recommended. Taking into account the quality of the products as well as the manufacture profile relating to the assortment and thickness of yarns, the change of the most utilised geared wheels in the stretching apparatus has to be made at least once per two years.

Utilizing the commercial package Statistica, it was stated that the most suitable distribution which approximates the bar chart (histogram) is asymptotic Gumbel's distribution, having the distribution density function given in the standard [3, 5]:

$$f(y) = \alpha \cdot exp\left[-\alpha \cdot (y - u) - e^{-\alpha \cdot (y-u)}\right]. \quad (10)$$

Central moments of the distribution (mean value and variance) are expressed by *Equation (11)*:

$$m_y = u + \frac{\delta}{\alpha} \; ; \quad \sigma_y^2 = \frac{\pi^2}{6 \cdot \alpha^2}, \quad (11)$$

where: δ – Euler constant, equal to $\delta \approx 0.577$.

The distribution parameters (8) for 1.6523 (20 HNM) steel were as follows: $m_y = 4.594 \cdot 10^5$, $\sigma_y = 3.557 \cdot 10^4$, $u = 4.434 \cdot 10^4$, $\alpha = 3.605 \cdot 10^{-5}$.

Reliability i.e. the probability that after the passage of N_i cycles, a particular crack reaches the critical length l_{kr} – is expressed by the following formula:

$$R = \int_0^{y_1} f(y) dy \quad (12)$$

Verification of the numerical model

During machine operation, diagnostics symptoms were registered which indicated an initiation of the damage process of the gear teeth in the drive system of the Fiomax 200 spinner. Based on eye inspection of toothing, a crack near the tooth foot of a length equal to approx. 0.5 mm was detected. The time of gear operation until reaching the critical crack length $l_{kr} = 2,14$ *mm* was evaluated. A fatigue crack was spotted on the pinion of the first stage of the reducer, working at a constant rotational velocity $n = 1460$ obr/min. The pinion was made of 1.6523 (20 HNM) steel, hardened up to 60 ± 2 HRC. Geometrical parameters of the gear tooth are given in *Table 2*.

The probability of damage occurrence can be determined based on *Equation (9)*, assuming that we know the density distribution function $f(y)$, whose parameters u and α are calculated utilizing *Equation (8)*, if moments m_y and σ_y^2, determined based on adequate statistics, are known. The number of possible cycles N_i until reaching the critical fatigue crack length is determined utilizing *Equation (4)* after some transformations, see *Equation (13)*.

The values of stress intensity coefficients were chosen based on rules given in [30], where *Equations (14)* and *(15)*. Where additionally:

$\rho_{fP} = 0,25\ m_n$, $G = \frac{\rho_{fP}}{m_n} - \frac{h_{fP}}{m_n} + x,$

$h_{fP} = 1,25\ m_n,$

ϑ – angle, calculated by means of the equation: $\vartheta = \frac{2 \cdot G}{z_n} \cdot tg(\vartheta) - H,$

$A_1 = 0.625$; $A_2 = -2.766$; $A_3 = 10.17$; $A_4 = -16.178$; $A_5 = 10.293$;
$2.14 \leq m_i \leq 2.395$ for $i = 1 \cdots 200$.

$$N_i = \int_{0,05}^{2,4} \frac{1}{(7.49 \cdot 10^{-8} \cdot m_i^{-11,435} - 1.675 \cdot 10^{-11}) \cdot \Delta K(l)^{m_i}} dl. \quad (13)$$

$$\Delta K(l) = \sigma \cdot \sqrt{\pi \cdot l} \cdot \left[A_1 + A_2 \cdot \frac{l}{b} + A_3 \cdot \left(\frac{l}{b}\right)^2 + A_4 \cdot \left(\frac{l}{b}\right)^3 + A_5 \cdot \left(\frac{l}{b}\right)^4\right], \quad (14)$$

$$b = \left[z_n \cdot sin\left(\frac{\pi}{3} - \vartheta\right) + \sqrt{3} \cdot \left(\frac{G}{cos(\vartheta)} - \frac{\rho_{fP}}{m_n}\right)\right] \cdot m_n = 4.99. \quad (15)$$

$$f(y) = 2,898 \cdot 10^{-5} \cdot exp\left[-2,898 \cdot 10^{-5} \cdot (y - 5,022 \cdot 10^5) - \right.$$
$$\left. - e^{-2,898 \cdot 10^{-5} \cdot (y - 5,022 \cdot 10^5)}\right]. \quad (16)$$

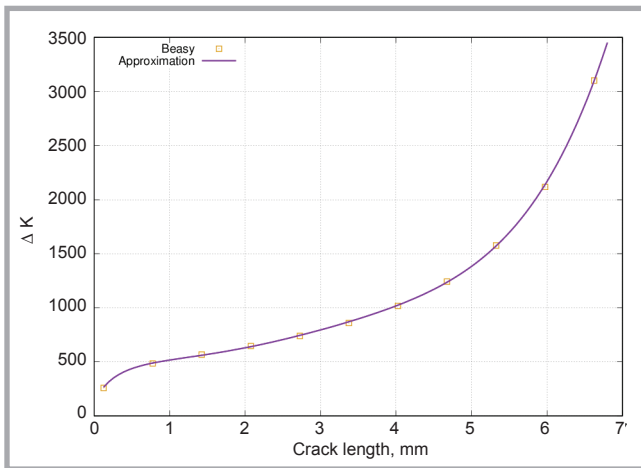*Equation (13), (14), (15)* and *(16)*.

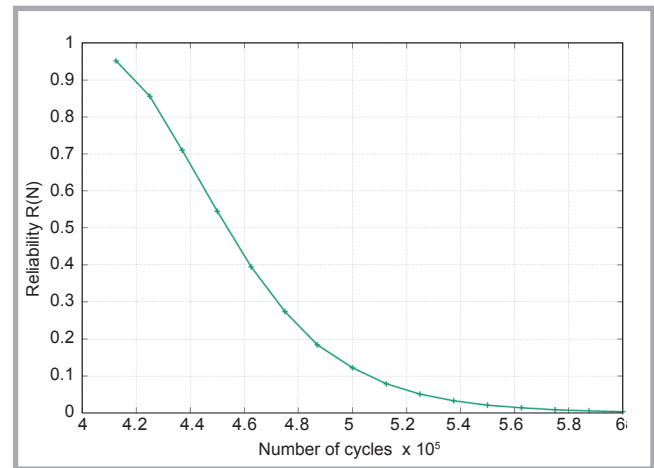**Figure 16.** *Approximation of the stress intensity coefficient determined by means of the Beasy system.*

Figure 17. *Reliability function for toothing operation against number of cycles, for stress equal to σ = 1316 MPa and value x = 0.005 mm.*

The constants are determined based on the approximation of the stress intensity coefficient obtained (in turn) via simulation of the propagation of the fatigue crack in the BEASY package for the stress near the tooth foot equal to 1374 MPa.

The BEASY package [21-22] is an engineering calculation system utilizing the boundary element method for the solution of several different problems e.g. related to elastic theory, problems of elastic contact as well as cracking. The modules connected with fatigue and fracture calculations are based on the procedures designed by the international research team Flagro – NASA. Specialized files i.e. NASMFM and NASMFC are added to this package, which contain a considerable database of strength properties of versatile materials.

In *Figure 16*, charts of changes in the stress intensity coefficient and its approximation function are shown.

Central moments for statistics N_i are equal to $m_y = 5{,}221 \cdot 10^5$ & $\sigma_y = 4{,}436 \cdot 10^4$, which, in turn, entered into relationship (9) allow for the determination of distribution parameters $u = 5{,}022 \cdot 10^5$ & $\alpha = 2{,}898 \cdot 10^{-5}$.

The distribution density function of time (i.e. number of cycles) until reaching the critical length by the crack analyzed, determined based on relationship (10), has the following form, see *Equation (16).*

In *Figure 17*, a chart of the course of the reliability function against the number of working cycles for a gear having a broken tooth (breakage near tooth foot) is presented.

Assuming an allowable value of the reliability function at a probability level equal to 0.95, we can determine the time of operation of a gear with a propagating fatigue crack until reaching the critical value (not exceeding $\theta = 1.45$ hour), simultaneously assuming a level of loading not exceeding the allowable stresses $\sigma = 1316$ MPa (near tooth foot) for 1.6523 (20HNM) steel, according to the standard [3].

In the case where nucleation of the prospective fatigue crack has been caused by a sudden stopping of the production process, where additionally the transient stresses have exceeded the elasticity limit of the material and induced a fatigue crack above the threshold level, then the bending stress near the tooth foot can be assumed as 1.3 times lower. The minimal computational safety coefficient assumed for calculation of the fatigue strength near the tooth foot under bending conditions according to the standard [4] can be assumed depending on the allowable stresses. The operating time of a gear in which the process of crack propagation takes place elongates up to $\theta = 2.5$ hours, in the case of an assumption of reliability not less than $R = 0.95$.

■ Summary

The modified probabilistic model of fatigue crack propagation at the tooth foot of a geared wheel proposed allows prediction of the working time of a gear in which an initiated fatigue crack propagates. One of the essential factors permitting effective utilization of the model is knowledge (proper evaluation) of the stresses at the tooth foot (root) in the case of initiation of a fatigue crack. Based on the guidelines formulated in the standard [4], in cases where the geared wheel has been subjected to more than the ultimate number of cycles $N_g = 3 \cdot 10^6$ (for carburized alloy and hardened steel), the stresses at the tooth foot can be considered as having values under those allowable (ultimate) in the standard [3]. In industrial practice, some cases occur frequently i.e. a tooth crack takes place when the particular geared wheel was subjected to the ultimate number of cycles. Determination of the stress value at the tooth foot for the moment of crack initiation is the major problem in the model proposed, since based on this knowledge, one can predict the time until the crack reaches the ultimate length.

The Uneveness of working of the spinner drive system has a direct influence on yarn ends down. The stretching apparatus of the ring spinner consists of several mutually co-operating gears. Even a trivial defect of a single element e.g. the geared wheel, bearing or shaft disturb or preclude the performance of the manufacturing process.

In industrial practice, the problem discussed is solved by installation of diagnostic gauges on a particular machine which allow the monitoring of parameters of co-operating elements of the drive system. Additionally these solutions can be supported by expert systems enabling an almost on-line performance of machine repairs. Due to the high costs of adaptation of particular

procedures, it seems that the most useful is the application of probabilistic methods for evaluation of the fatigue lives of particular elements of drive systems. These activities could be planned and performed even at the design phase. In the case of machines which are currently operated, one can assure that during the repairs of particular parts, the spare parts fulfil requirements according to fatigue life.

The probabilistic models proposed enabled determination of the operating time of a particular machine until the planned repair. Knowledge of the fatigue life of key elements of a drive system reduces costs connected with the damage risk of a particular machine as well as non-planned break-downs. The method proposed can be also useful for other types of ring spinners.

References

1. Sobczyk K, Billie F and Spencer F. *Stochastic models of material fatigue*. WNT (Scientific and Technical Publishing), Warsaw, 1996.
2. Ostregaard D F and Hillberry B M. *Characterization of variability in fatigue crack propagation data. In Probabilistic Method for Design and Maintenance of Structures*. SSP-798. Philadelphia, American Society for Testing and Materials, 1983, p. 97-115.
3. DIN 3990: 1989. Calculation of load capacity of cylindrical gears. Deutsches Institut Für Normung E.V. (German National Standard).
4. ISO 6636/1: 1986. Calculation of load capacity of spur and helical gears. Part 1: Basic principles, introduction and general in influence factors.
5. Gumbel E J. Statistic of Extremes. Columbia University Press, New York, 1958.
6. Ortiz K and Kiremidjian A S. Stochastic Modeling of Fatigue Crack Growth. *Engineering Fracture Mechanics* 1986; 29(3): 657-676.
7. Ditlevsen O. Random Fatigue Crack Growth – a First Passage Problem. *Engineering Fracture Mechanics* 1986; 23(2): 467-477.
8. Madsen H O, Krenk S, Lind N C. *Methods of Structural Safety. Prentice-Hall*, Englewood Cliffs, New Jersey, 1986.
9. Gross D and Sellig T. *Fracture mechanics with an introduction to micromechanics*, Springer, 2011, ISBN 978-3-642-19239-5.
10. Schijve J. *Fatigue of structures and materials*. Springer 2009. ISBN:978-1-4020-6807-2.
11. FIOMAX 2000 – Technical specification, 1999.
12. Paris P and Erdogan F. A critical analysis of crack propagation laws. *Trans. ASME* 1963, 85: 528-534.
13. Drewniak J and Rysiński J. Model of fatigue damage accumulation. *Key Engineering Materials* 2014; 598, 51-56.
14. Drewniak J and Rysiński J. Evaluation of fatigue life of cylindrical geared wheels. *Solid State Phenomena* 2013; 199: 93-98.
15. Drewniak J, Rysiński J and Praszkiewicz M. Analysis of fatigue life and dynamics of gear train by boundary element method. *Mechanik* 2013; (12): 1-8.
16. Tomaszewski J and Rysiński J. Diagnostics of gears and compressors by means of advanced automatic system. *Acta Mechanica et Automatica* 2015; (9), 19-22.
17. Rysiński J and Wróbel I. Diagnostics of machine parts by means of reverse engineering procedures. *Advances in Mechanical Engineering* 2015; (7)1-9.
18. Tomaszewski J and Rysiński J. Diagnostics of elements of a drive system performed by means of the WiViD device. *Pomiary, Automatyka, Kontrola* 2014; (8): 629-632.
19. Drewniak J and Rysiński J. Kinetics of fatigue crack propagation at the tooth root of a cylindrical gear wheel. *Solid State Phenomena* 2015; 220-221, 333-338.
20. Drewniak J and Rysiński J. Model of fatigue damage accumulation. *Key Engineering Materials*; 2014, 51-56.
21. Drewniak J and Rysiński J. Evaluation of fatigue life of cylindrical geared wheels. *Solid State Phenomena* 2013; (199): 93-98.
22. Drewniak J and Rysiński J. Fatigue life and reliability of power engineering machines and their elements. *Energetyka* 2013; (107): 12, 9-16.
23. Rysiński J and Sidzina M. Insitu diagnostic investigations of gear parameters with use of industrial automation installations and http protocol. *Pomiary, Automatyka, Kontrola* 2012; (58) 11: 950-952.
24. Rysiński J and Wróbel I. Modern methods for assessment of machine part surface. *Pomiary, Automatyka, Kontrola* 2011; (57) 2: 1595-1597.
25. Kwiecień A, Rysiński J and Sidzina M. *Application of distributed system in control and diagnostic toothed gears, Computer networks: proceedings* / eds. Andrzej Kwiecień, Piotr Gaj, Piotr Stera (Communications in Computer and Information Science ; Vol. 39, Berlin, Heidelberg, Springer-Verlag, 2009.
26. Mirota K, Tomaszewski J and Rysiński J. Diagnostics of Scoring in Gears, Solid State Phenomena, 2009, vol.144, (Mechatronic Systems and Materials II: selected, peer reviewed papers from the 3rd International Conference: Mechatronic Systems and Materials (MSM 2007), Kaunas, Lithuania, 27-29 September, 2007 / ed. Inga Skiedraite, Jolanta Baskutiene, Arunas Lipnickas. – Stafa-Zurich : Trans Tech Publications Ltd., 2009. ISBN 3-908451-60-4. – ISBN 978-3-908451-60-0), pp. 118-123.
27. Tomaszewski J and Rysiński J. Measurements of heat parameters in cylindrical gears. *Pomiary, Automatyka, Kontrola* 2007; (53)12: 35-37.
28. Mirota K and Rysiński J. *Zatarcie kół zębatych określane metodą temperatury błyskowej, Zacieranie przekładni zębatych*, praca zbiorowa pod red. Jerzego Tomaszewskiego, Józefa Drewniaka, 2007, Gliwice: Centrum Mechanizacji Górnictwa KOMAG.
29. Tomaszewski J and Rysiński J. Diagnostics model of crack of gears *Czasopismo Techniczne. Mechanika* 2006; (1) M, 367-377.
30. Dietrich M. (red): Podstawy Konstrukcji Maszyn tom 1, 1995, WNT Warszawa
31. Drobina R and Machnio M. Evaluation of properties of spliced joints of ends of wool worsted yarns, in Polish. *Przegląd Włókienniczy* 2000; 54, 10, 3-7.
32. Drobina R, Włochowicz A, Machnio M, Drobina E and Lewandowski S. Fatigue Curves Elaborated for Selected Worsted Wool Yarn. *Fibres and Textiles in Eastern Europe* 2007; 15, 5-6: 54-58.
33. Drobina R, Włochowicz A and Machnio M S. Fatigue curves elaborated for selected worsted wool yarns. *Fibres and Textiles in Eastern Europe* 2007; 15, 5-6: 64-65.
34. Drobina R. *Probabilistic model of the fatigue durability of cotton yarns smooth and fancy ones*, 2012. Trial scientific No. p. 40, Scientific Publishing House University in Bielsko-Biala.
35. Drobina R and Drobina R. Assessment of the Fatigue Durability of Standard Smooth and Fancy Flame Cotton Yarns Using a Statistical Model. Fibres and Textiles In Eastern Europe 2013; 21, 2(98): 61-67.
36. Drewniak J. *Probabilistyczny model obliczeniowy trwałości zmęczeniowej elementów i zespołów maszyn*, 1992, Wydawnictwo Politechniki Łódzkiej Filii w Bielsku-Białej.
37. Drobina R. Probabilistic Model of Fatigue Strength. *Fibres and Textiles in Eastern Europe* 2013; 21; 3(99): 61-67.
38. Włochowicz A, Kukla S and Drobina R. Static and Fatigue Strength of Linear Textile Products. *Fibres and Textiles in Eastern Europe* 2016; 24, 3(117): 8-16.
39. Drewniak J and Rysiński J. Derived equations for damage accumulation in geared wheels based on modified laws of crack propagation. Scientific Journal of Silesian University of Technology. Series Transport. 2016, 91, 19-31. ISSN: 0209-3324. DOI: 10.20858/sjsutst. 2016.91.2.
40. Więcek D. Implementation of artificial intelligence in estimating prime costs of producing machine elements. *Advances in Manufacturing Science and Technology* 2013; 37, 1: 43-53.
41. Szala J and Zawiślak S. Application of computer simulation method for determination a distribution type of construction parts fatigue life. *Archive of Mechanical Engineering* 1990; 37.3, 145-167.