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# Three-Dimensional Simulation of Warp-knitted Fabric

## Abstract

With detailed research on the structure of warp-knitted fabric, the spatial yarn position of a 3D loop structure was discussed, and the dimension of each parameter was ascertained. After establishing the basic control points of the loop, a 3D model of warp-knitted fabric was created. By calculating the control points of this model in reverse, a more exact control polygon was attained. By utilising the program Visual C++ and the developing tools of Open GL, using Non-Uniform Rational B-Spline surfaces, spatial yarn can be described exactly, and a virtual reality of warp-knitted fabric can be realised on a computer screen.

**Key words:** warp knitting, loop structure, three-dimensional modeling, NURBS, virtual reality.

geometric configuration. The SPEP International Standard regards the NURBS method as the only mathematic method to define the shape of products. The path of yarn in fabric is crooked and variable. To represent this variable effect correctly, the curves or surfaces should be close to the actual shape of the stitches. Modelling a 3D warp-knitted structure using NURBS has more predominant properties than any other curves or surfaces used; therefore NURBS is discussed in this study. Above all, a 3D yarn model of the yarn should be created. Following the definition of the warp-knitted stitch, 3D structures finally come into being by connecting the stitches [2].

## NURBS curves

NURBS curves are defined by the sectional Rational B-Spline polynomial basic function; the formula is:

$$C(u) = \frac{\sum_{i=0}^n \omega_i P_{i,k} N_{i,k}(u)}{\sum_{i=0}^n \omega_i N_{i,k}(u)} \quad u \in [u_{\min}, u_{\max}] \quad (1)$$

Here:  $P_i$  ( $i = 0, 1, \dots, n$ ) is the location vector of the control points of the characterised polygon,  $\omega_i$  is the correlation power factor of  $P_i$ ,  $N_{i,k}(u)$  is the  $k$  degree B-Spline basic function.

Node vector is  $u_i = [u_0, u_1, \dots, u_m]$ , and the number of node is  $m = n + k + 1$  ( $n$  is the number of control points,  $k$  is the degree of the B-Spline basic function).

## NURBS surface

A NURBS surface is a Non-Uniform Rational B-Spline double parameter surface. It maps the region of a 2D space to a 3D one. The definition of parameters in the surface equation is same as in the

curve equation. The equation of the surface is:

$$S(u, v) = \frac{\sum_{i=0}^m \sum_{j=0}^n \omega_{i,j} P_{i,j} N_{i,k}(u) N_{j,l}(v)}{\sum_{i=0}^m \sum_{j=0}^n \omega_{i,j} N_{i,k}(u) N_{j,l}(v)} \quad (2)$$

## Some concepts for the mapping of curves and surfaces

**Value points** are a few measured or calculated data points which describe the geometry shape of a curve or surface.

**Control points** are used to control the shape of the curve. Connecting the control points in turn can form a control polygon; the curve is located in the control polygon's Protrude kits and next to the control polygon. Movement of a control point will affect the shape of part of the curve.

The **Node** is simply a list of non-decreasing number which determines how and where to define the basic function. Some node values may be the same. The nodes that have the same value are called multiplicity nodes. Multiplicity nodes make the curve close to the correlated control points. If the number of node repeats is equal to degree, the curve will pass through the corresponding control points. The node sequence determines where the curve passes through and interpolates between the control points.

The **Power factor** may give power to every control point in order to change the control point's affection to the curve. A control point's affection increases when its power factor is increased, and as a result the affection of the other control point neighbouring it will decrease. Hence the curve will be close to this control point.

CAD technology is widely used in the warp knitting field. But even the latest CAD system in warp knitting can only provide a two-dimensional representation [1]. The lack of a three-dimensional view of fabrics is problematic for designers and limits the further application of CAD technology in the warp knitting field. In this study we analyse the construction of warp-knitted fabric, firstly using the NURBS curve to establish a 3D structure model of warp-knitted fabric and finally leading to a successful realization of the 3D simulation of conventional warp-knitted fabric using Visual C++ programs.

## ■ NURBS curves and surfaces

In the eighties of last century, NURBS (Non-Uniform Rational B-Spline) was one of the most important methods in

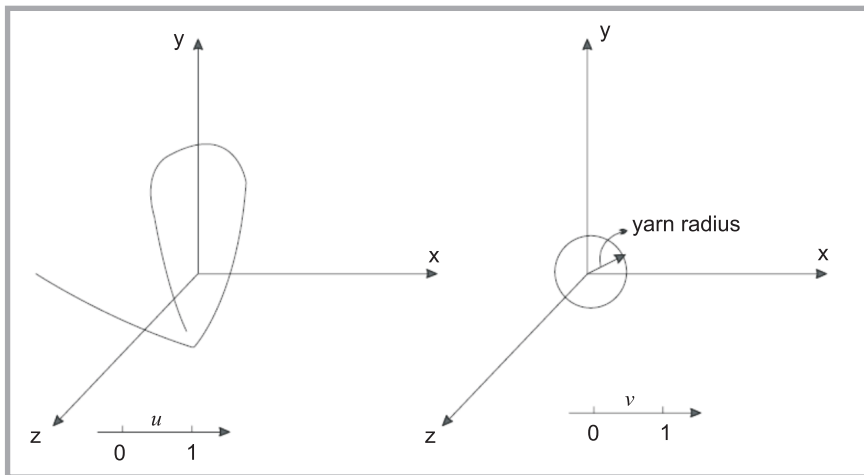


Figure 1. The  $u$  and  $v$  parameters for the solid loop model.

Table 1. Coordinates of the points on the loop; Where:  $d$  is diameter of the yarn,  $b$  is the height of the loop,  $n$  is the length of the underlap (the number of machine gauges that the underlap goes through),  $w$  is the distance between wales.

Value point	Closed loop	Open loop	Inlay
$P_0$	(0.0, 0.0, 0.5d)	(0.0, 0.0, 0.5d)	(0.0, -0.15b, 0.5d)
$P_1$	(0.2b, 0.65b, 1.54d)	(-0.2b, 0.65b, 1.54d)	(-0.15b, 0.0, -0.5)
$P_2$	(0.0, b, 0.5d)	(0.0, b, 0.5d)	(0.0, 0.15b, 0.5d)
$P_3$	(-0.2b, 0.65b, 1.54d)	(0.2b, 0.65b, 1.54d)	(0.5nw, 0.43b, 0)
$P_4$	(-d, 0.15b, 0.5d)	(d, 0.15b, 0.5d)	(nw, 0.87b, 0.5d)
$P_5$	(0.1b, -0.15b, 0.5d)	(0.1b, -0.15b, 0.5d)	
$P_6$	(0.4025nw, 0.35b, -2d)	(0.4025nw, 0.35b, -2d)	
$P_7$	(nw, 0.85b, 0.5d)	(nw, 0.85b, 0.5d)	

The **order** of the curve determines the form of the parametric equations. It is equal to one plus the maximum degree of the variables in the parametric equations. The order of the curve determines the minimum number of control points necessary to define the curve. For example, to create a cubic curve at least four points need to be specified. The order of the curve also affects its behaviour of when a control point is moved. For example, a change in the position of a control point affects three segments of a curve at most. Cubic curves are the most commonly used since they provide enough control for most geometric modelling applications without the drawbacks of higher-order curves. In addition to Cubic curves providing a smooth continuity at break-points as well as other advantages, they are also found to be suitable for accurate representation of warp-knitted stitches.

### Modelling 3D warp-knitted stitches

Warp-knitted fabric is virtually a three-dimensional object. Here we assume that warp-knitted stitches are formed by uniform yarns with a circular cross-section.

The central axis of the yarn is a spatial curve.

#### Solid yarn model

In the description of a yarn path, the yarn is represented as a three-dimensional spatial curve using a cubic NURBS curve. There are quadruple nodes both at the beginning and end of this curve. Hence, the curve passes through the first and last control points. In the solid model, the yarn is simulated as a cylindrical monofilament with a uniform cross-section. The central axis of the yarn refers to the first parameter  $u$  of the NURBS surface, and the cross-section refers to the second parameter  $v$  of the same surface [3] (Figure 1).

#### Creation of a loop model of warp-knitted fabric

##### Determination of the number of value points to define a loop

It is necessary to determine the number of control points to define the shape of a single loop. The number of control points determines the shape of loop to be created. To define a shape, we usually use two points to represent the beginning and

end points, and one point for each turning point of the curves. In this way, ten value points are used to define an open or closed loop, and seven value points are used to define an inlay. The assumption makes the whole warp-knitted stitch integrate, which involves basic a loop and foot. We study the warp-knitted loop structure to define these points. The former G. L. Allison's predigests model, P. Grosberg's first model and P. Grosberg's second model are all 2D models. The 3D modelling of warp-knitted structures put forward by O. Goktepe and S. C. Harlock from the University of Leeds is referenced in the study. The dimension of the loop model is defined by the height  $b$ .  $b$  can be measured by the height of the course, i.e. the number of courses in one centimeter. The loop's shapes of the each lapping structure are different, thus it is necessary to find a general representation for all loops. Therefore, O. Goktepe and S. C. Harlock make the following assumptions [5] (Figure 2):

- 1) **Loop height**,  $b = |AC| = 1.15c$ . The distance between the lowest and highest points of the loop in the wale direction
- 2) **Loop width**,  $|BD| = 0.4b$ . The dimension of the widest part of the loop

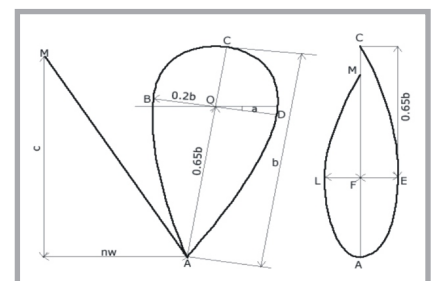


Figure 2. 3D loop model and its dimensions in the fabric plane and fabric profile.

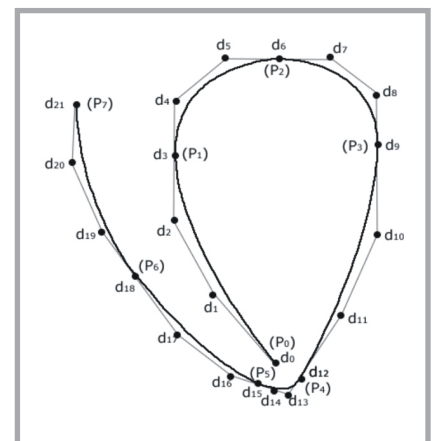


Figure 3. Reconstruction of the control polygon.

- 3)  $|OA| = 0.65b$ . This value shows the distance from the widest section of the loop to the root.
- 4) *Loop bend*,  $|EF| = 2d - (\text{guide bar no.} - 1) \times \text{yarn radius}$ . As shown in the experimental work, back-bar loops bend less than front-bar loops for all the structures produced. The amount of the difference between the back- and front-bar loop bends was approximately equal to the yarn radius. This assumption is based on this observation.
- 5) *Loop-bend distance*,  $|CF| = 0.65b$ . This average value was chosen to define the location of the maximum loop bend (**Figure 3**).
- 6) The *amount of underlap* bend in a fabric cross-section:  $|FL| = (\text{total number of guide bars} - \text{guide bar no.} + 1) \times d$ .

Depending on the above analysis of a warp-knitted structure, 8 points and 5 points can be used to define the knit-locked loops and inlay separately (**Figure 4**).

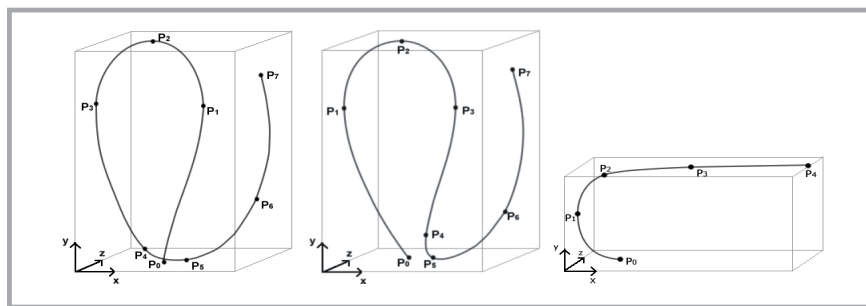
In **Table 1**, we show an example of a fabric knitted using two guide bars. 8 value points of the front guide bar's loop structure and 5 value points of the second guide bar's inlay are defined separately.

#### Calculation of the control point in reverse

For modelling purposes, uniform cubic B-spline curves have a drawback since they do not pass through the points, except the first and last ones. However, for more accuracy and a better control of the curve, it is advisable to have a curve which passes through all the defined points. In this case, it is necessary to establish a series of new control points. The interpolation algorithm is a new NURBS interpolation algorithm, which is brought forward basically owing to the behaviour of the NURBS curve. There is no need for complex calculation of the control point in reverse, and the curve generated has important geometrical behaviours, such as protrusion-preservation and shape-preservation. Furthermore, the modification of the curve has a partial characteristic -  $C^2$  continuous behaviour in the connection of segments of the curve.

We adopt the sequence of value points above:  $P_0, P_1 \dots P_7$ . The correlated power factors are

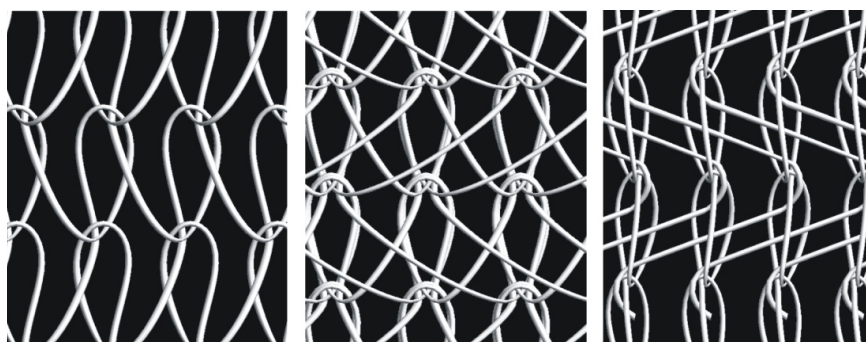
$$\omega_0, \omega_1, \dots, \omega_7 \quad (\omega_i > 0, i = 0, 1, \dots, 7).$$



**Figure 4.** Model of a basic open or closed loop and inlay.

$$\begin{aligned}
 d_{3i} &= P_i & i &= 0, 1, \dots, n \\
 d_{3i+1} &= P_i + \frac{\lambda_i}{\omega_{3i+1} N_{3i+1,3}(u_{3i+2})} T_i & (0 < \lambda_i < \omega_{3i+1} N_{3i+1,3}(u_{3i+2}) t_i, i = 0, 1, \dots, n) \\
 d_{3i+2} &= P_{i+1} + \frac{\lambda_{i+1}}{\omega_{3i+4} N_{3i+4,3}(u_{3i+5})} T_{i+1} & (0 < \lambda_{i+1} < \omega_{3i+4} N_{3i+4,3}(u_{3i+5}) t_{i+1}, i = 0, 1, \dots, n-1)
 \end{aligned} \quad (3)$$

**Equation 3.** The  $d$ -parameters with  $i$  and  $\lambda$  avadritions; Where:  $T_i$  is the tangential vector of point  $P_i$ , thus a new control point sequence  $d_i$  ( $i = 0, 1, \dots, 3n$ ) will be generated.



**Figure 5.** Computer simulated image of a basic warp knitted fabric.

A cubic NURBS shape-preserving interpolation curve is built up using these value points. Control points including the original value points are gained by interpolating two assistant points in the two neighbouring value points [6] (see **Figure 3** and Equation 3).

Due to the fact that both ends of the node vector are quadruplicate nodes, the interpolation points of a cubic NURBS curve are  $d_0(p_0), d_{3n}(P_n)$ . A cubic NURBS curve determined by the control point sequence  $d_i$  interpolates all value points  $P_i(d_{3i})$ . Hence the interpolation curve is regarded as shape-preserving in view of the NURBS curve's variation diminishing behaviour.

In general, at least 8 value points of the loop should be specified exactly to describe the configuration of a basic warp knit loop. We can calculate 14 new control points by the method discussed above and form a control polygon speci-

fied by 22 control points. Hence, a spatial path model of a warp knit loop can be drawn as a cubic NURBS curve. To define a curved surface, the control point and node sequence should be defined by two parameters i.e.  $u$  and  $v$ , which may have a different order and different node sequence.

#### Realization of the simulation of a three-dimensional warp-knitted structure

With the theoretical model of a 3D warp-knitted stitch described above, we used Visual C++ to write the program. Variable configurations of the 3D models can be realised by utilising the Open Graphic Library of Visual C++. Firstly, the individual 3D model of a warp-knitted stitch is generated on a computer. Secondly, this model is used in the warp-knitted fabric design. **Figure 5** shows simulation images generated by the computer program.

By using the model and program, more warp-knitted structures for various basic warp-knitted fabrics can be successfully created.

## ■ Conclusions

The application of the 3D solid modeling concept in the warp knitting field is an even more difficult task than in other fields due to the complexity, flexibility and irregularity of the structures. In our study, starting from finding a three-dimensional model to represent the warp-knitted structure, three-dimensional simulation of basic warp-knitted fabrics made of monofilaments was realized on a computer. The model gives us a suitable simulating effect describing the loop structure of the fabric clearly. Thus, it is a strong basis for further three-dimensional simulation of warp-knitted fabric. Further research work in this area should take the material into account and utilise the irregular cross-section, illumination and texture to realise the simulation of the multifilament and spinning yarn, making the simulation more realistic and intuitionistic; which promote the three-dimensional simulation of more complex warp-knitted structures.



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