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Physical and Mathematical Modelling of the Phenomenon of Fur Knitting Compression

Abstract
While analysing the dependencies between the compression-ability parameter S (the squeezing parameter) and parameters describing fur knittings, i.e. the initial free thickness g_0 *of its pile and the area mass G of the knitting, we have not stated the existence of simple functional relations. This was why we made an attempt to determine the theoretical dependencies between the mechanical and the structural parameters of the fur knittings for the physical model of the fibres' pile which we elaborated and accepted. For the function elaborated,* $S = S_1 (G^2 / g_0^4)^{-a}$*, we obtained a correlation coefficient R of 0.80 by approximating the experimental results.*

Key words: *fur knitting, ability to compression (squeezing), initial free thickness, area mass, model fibres, apparent density, elastic energy of buckled fibre.*

Introduction

This article is a continuation of our earlier publications concerning the properties of fur knitting [1, 2], and includes the results of research into the structural and mechanical parameters of these knittings, as well as an interpretation of the model developed.

A scheme of a fur-knitting segment under different loads, together with the dependency of compression vs. load, and the physical fibre model, is presented in Figure 1.

The compression-ability parameter S (squeezing parameter), according to the literature [4] for fur knittings, has been defined as

$$S = \frac{g_o - g_1}{g_o} \cdot 100\% \qquad (1)$$

where:
g_0 – the initial free thickness under the load of N_1 = 1.0 cN/cm², and
g_1 – the final thickness under the load of N_f = 70.0 cN/cm²

For the purpose of our considerations, twelve different fur knittings, each of four nominal densities, were selected. The results of the measurements of the initial free thickness g_0, the area mass G, and the elaborated values of the compression-ability parameter S, differentiated by the four nominal knitting thicknesses, are presented in Table 1. The dependencies between the initial free thickness g_0, the area mass G, the apparent density after compression d_1, the initial apparent density and the compression-ability parameter S are presented in Figures 2, 3, and 4.

The dependencies between the individual knitting's thickness g_0 and the parameter S, as well as between the area mass and the parameter S, have low correlations.

The dependency between the parameter S and the apparent density after compres-

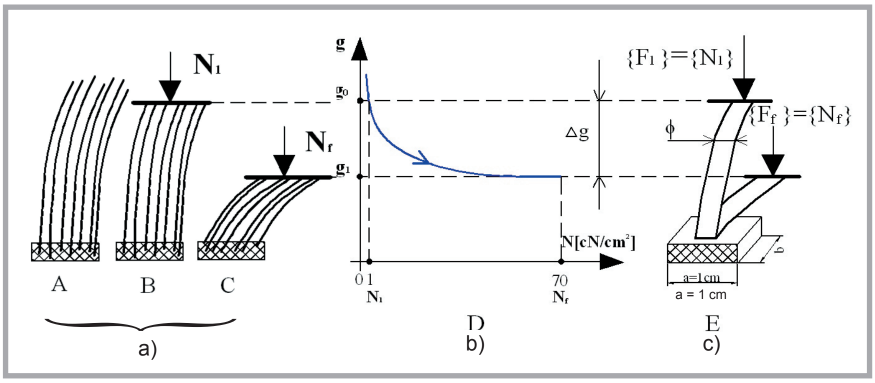

Figure 1. *Scheme of a fur-knitting segment under different loading; a) the process of compressing a fibre bundle, A – free state, B – state under an initial load of* $N_1 = 1.0$ *cN/cm², C – state under a final load of* $N_f = 70$ *cN/cm², b) compression dependency, i.e. pile thickness vs. load), and c) the physical model of the fibre related to the dependency.*

Table 1. *Structural parameters of 48 fur knitting variants with nominal thickness of* g_n *= 10, 20, 30, and 40 mm.*

No.	Prod. No.	Initial thickness g_o, mm Nominal thickness g_n=10, 20, 30, 40 mm				G, g/m²				S, %			
		10	20	30	40	10	20	30	40	10	20	30	40
1	3891	6.53	8.96	9.56	9.15	306.7	366.7	374.7	372.8	63	62	61	63
2	3892	7.81	11.28	12.76	13.15	483.5	545.3	549.3	544.0	50	57	62	61
3	3893	7.30	10.92	10.24	10.04	425.9	526.9	517.9	519.5	51	56	55	53
4	3894	4.05	5.28	5.48	5.60	199.7	224.8	224.3	205.3	66	61	67	70
5	3920	5.27	6.40	5.92	6.80	213.1	258.1	266.9	266.7	64	70	63	74
6	3923	7.61	10.60	10.92	10.35	383.2	470.4	483.2	437.3	62	63	58	59
7	3931	8.54	11.44	11.48	11.80	425.6	526.4	530.1	522.4	59	56	56	56
8	3941	7.13	9.56	10.04	8.95	355.2	400.5	403.7	408.8	65	63	65	61
9	3950	9.84	14.22	13.12	14.15	481.1	636.0	629.1	633.6	59	63	57	63
10	3956	8.84	13.05	14.88	16.80	474.4	593.9	605.1	605.6	56	59	66	66
11	3965	9.70	16.02	17.16	18.60	576.3	705.1	708.5	703.2	51	61	66	66
12	3975	9.93	16.90	18.32	18.70	666.9	797.1	838.9	842.4	51	63	66	63

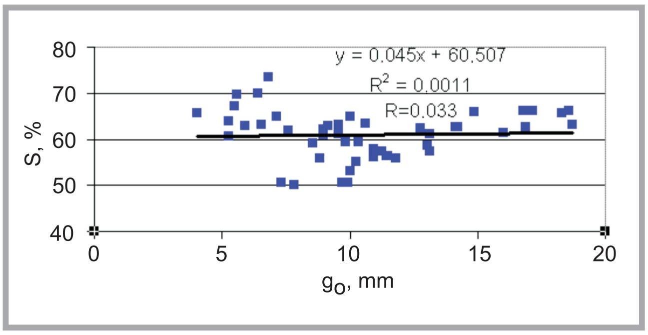

Figure 2. *Dependency between the compression-ability parameter S and the initial free thickness g_0 for 48 different knittings.*

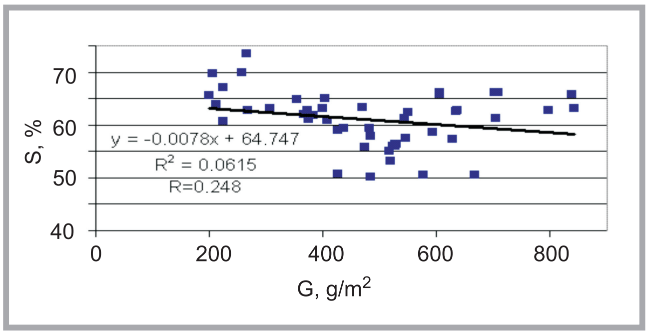

Figure 3. *Dependency between the compression-ability parameter S and the area mass G for 48 different knittings.*

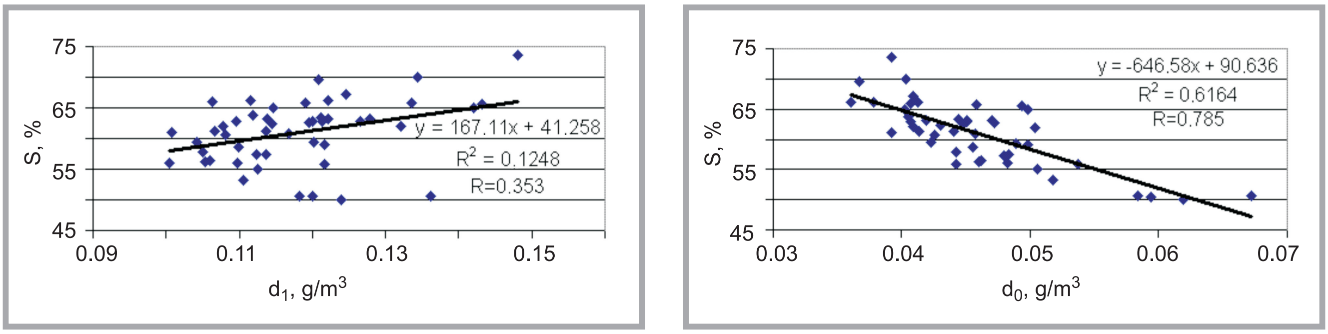

Figure 4. *Dependency between a) the compression-ability parameter S and the final apparent density (after compression), and b) the compression-ability parameter S and the initial apparent density for 48 different knittings.*

sion d_1, where

$$d_1 = G \cdot 10^{-3}/g_1,\ \mathrm{g/cm^3} \qquad (2)$$

has a correlation coefficient of $R = 0.353$ (Figure 4.a), whereas the dependency between the parameter S and the apparent initial density d_0, where

$$d_0 = G \cdot 10^{-3}/g_0,\ \mathrm{g/cm^3} \qquad (3)$$

has a correlation coefficient of $R = 0.785$ (Figure 4b).

As is visible from the results presented above, designing such an important quality parameter as compression ability only on the basis of its direct dependencies on the initial thickness g_0 and the area mass G is impossible. These dependencies can by determined indirectly from the relation $S = f(d_0)$, as is illustrated by Figure 4.b and Equation (3). These relations have a factual character, and do not explain the phenomena which occurs. Considering this situation, a physical and mathematical model of the phenomenon of fur knitting compression (squeezing) has been elaborated.

Assumptions for the physical model presented in Figure 1.c

1. A fibre bundle from the pile with a mass m_w and area p = 1 cm^2 is represented by one elastic fibre with diameter ϕ, fastened at the base in the structure of the binding stitches.
2. The numeric value of the force {F} which compresses the model fibre vertically is related to the numerical value of the load {N} acting on the fibre pile {F = N}.
3. The deformations occurring during compressing (buckling) the model fibre are related to the deformations of the knitting; this is illustrated by comparing the Figures 1.b. and 1.c.

Comparison of the physical model with the fur knitting

The fibre's elastic energy V during buckling is described by equation (4):

$$V = F_{cr} \cdot \Delta g \qquad (4)$$

where:

F_{cr} – the critical force for which the state of balance takes place at the curvilinear shape of the rod and free deflection of the rod's end [5], and

Δg – the absolute deformation (deflection) of the fibre.

Considering how the model fibre is fastened, i.e. by one end to the base, the equation describing the critical force F_{cr} is related to the classical form of the Euler equation, which is valid if the buckling length is twice the fibre's length:

$$F_{cr} = \frac{\pi^2}{4} \cdot \frac{EI_z}{g_o^2} \qquad (5)$$

where:

E – Young's modulus,

I_Z – the smallest main middle inertia model of the rod's cross-section,

g_0 – the length (height) of the fibre (rod).

The mass, in g/cm^2, of the fibre bundle from a 1 cm^2 knitting with the area mass G in g/m^2 equals:

$$m_b = 10^{-4} \cdot \mathrm{G},\ \mathrm{g/cm^2} \qquad (6)$$

whereas for the model fibre, the mass can be calculated from equation (7):

$$m_b = \frac{g_o \cdot \pi \cdot \phi^2}{4} d_m \qquad (7)$$

where:

d_m, g/cm^3 – the apparent density of the model fibre.

By comparing equations (6) and (7), we can determine the diameter of the model fibre:

$$\phi^2 = \frac{4 \cdot 10^{-4} \cdot G}{\pi \cdot g_o \cdot d_m},\ \mathrm{cm^2} \qquad (8)$$

The inertia modulus I of the model fibre with diameter ϕ is described by:

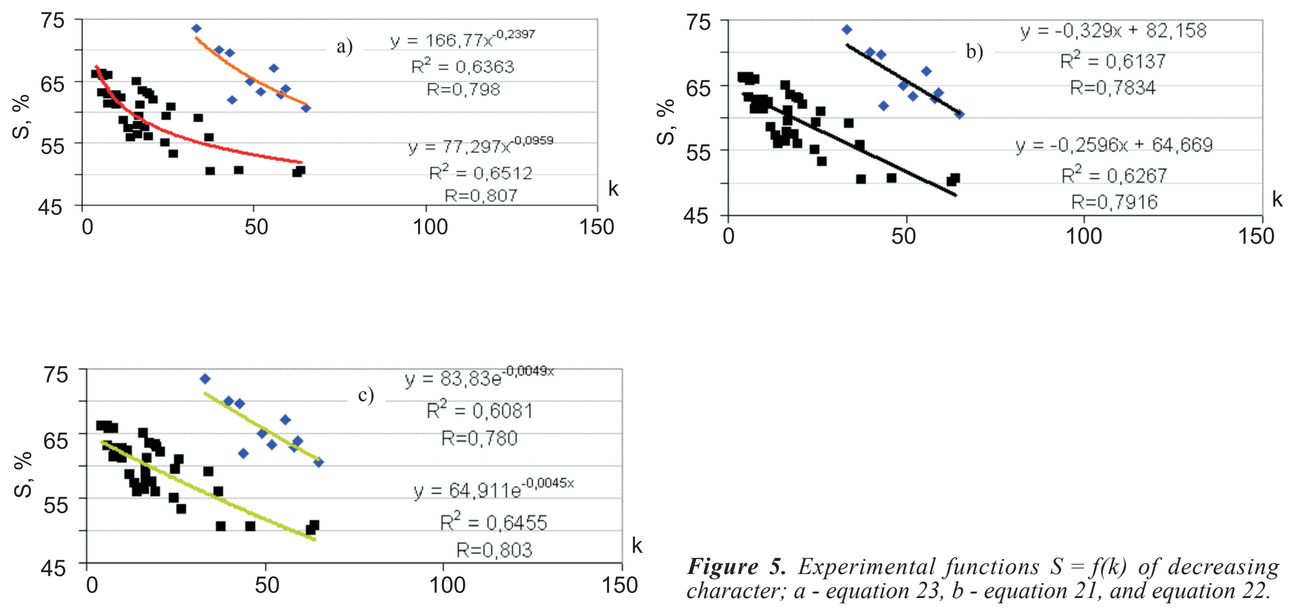

Figure 5. *Experimental functions S = f(k) of decreasing character; a - equation 23, b - equation 21, and equation 22.*

$$I_z = \frac{\pi \cdot \phi^4}{64} \tag{9}$$

After substituting equation (8) into (9), and then (9) into (5), we obtain the equation describing the critical force in the form:

$$F_{cr} = \frac{\pi^2 \cdot 10^{-8} \cdot E \cdot G^2}{16 \cdot d_m^2 \cdot g_o^4} \tag{10}$$

By setting apart the distinctive parameter:

$$k = \frac{G^2}{g_o^4} \tag{11}$$

from equation (10), we obtain equation (12) in the form of:

$$Fcr = C \cdot k \tag{12}$$

where:

$$C = \frac{\pi^2 \cdot 10^{-8} \cdot E}{16 \cdot d_m^2} = const \tag{13}$$

Introducing the compression-ability parameter S in the form of equation (14) instead of equation (1):

$$S = \frac{\Delta g}{g_o} \cdot 100\% \tag{14}$$

and the relative elasticity V_m of the model fibre strip

$$V_m = \frac{V}{g_o} \tag{15}$$

then considering equation (4), we obtain for parameter Z:

$$S = \frac{V_m}{F_{kr}} \cdot 100\% \tag{16}$$

After substituting the critical force F_{cr} from equation (12) into equation (16), we have:

$$S = \frac{V_m}{C \cdot k} \cdot 100\% \tag{17}$$

and by further substituting

$$S_1 = \frac{V_m \cdot 100\%}{C} \tag{18}$$

we obtain

$$S = \frac{S_1}{k} \tag{19}$$

After generalising this equation to the form

$$S = S_1 \cdot k^{-a} \tag{20}$$

we finally obtain a theoretical function which can be used to approximate the experimental results.

Comparison of the experimental results with the model analysis

The experimental functions $S = f(k)$ presented in Figures 5.a, b and c have a decreasing character, similar to equation (20). The function shown in Figure 5.b was presented by a linear function of the type

$$S = S_2 - b \cdot k \tag{21}$$

whereas the function from Figure 5.c by the exponential function:

$$S = S_3 \cdot e^{-c \cdot k} \tag{22}$$

and that from Figure 5.a by

$$S = S_1 \cdot k^{-a} \tag{23}$$

the only one which is similar to equation (20).

The values of all the correlation coefficients are close to R=0.8. It is clearly apparent that the approximation of the experimental results and the level of the correlation coefficient prove the correctness of the physical and mathematical model elaborated.

From the population of $N = 48$ variants of knittings, the following two groups could be distinguished on the basis of the function $S = f(k)$:

- 38 items with a value of the average apparent density of $\overline{d_o} = 0.0432$, g/cm^3, and
- 10 items with this value of $\overline{d_o} = 0.0461$, g/cm^3.

The above mentioned densities d_0 are related to fur knittings with the initial thickness g_0.

An independent complex problem would be to search for theoretical relations, firstly between the apparent density of fibres & yarns and the densities of fur knittings (d_0 and d_1), and next for dependencies with the density d_m of the model, and furthermore with the compression-ability parameter S.

Summary

1. Simple functional dependencies between the compression-ability parameter S and the initial thickness g_0 under

a load of $N = 1.0$ cN/cm^2, as well as the area mass (in g/m^2), could not be stated.

2. An attempt was made to determine the theoretical dependencies, which involve the mechanical and structural parameters of fur knittings for a physical model of the textile pile's fibre during the process of compression (squeezing).
3. The theoretical function we obtained, of the shape

$$S = S_1 \cdot k^{-a}$$

where:

$$k = \frac{G^2}{g_o^4}$$

enabled us to obtain a correlation coefficient within the range of $R_k = 0.80$ by approximating the experimental results. This is a basis for the statement that the preliminary assumptions of the model of the phenomenon of compression fur knittings, and the mathematical analysis based on them, present a physical and mathematical analogy.

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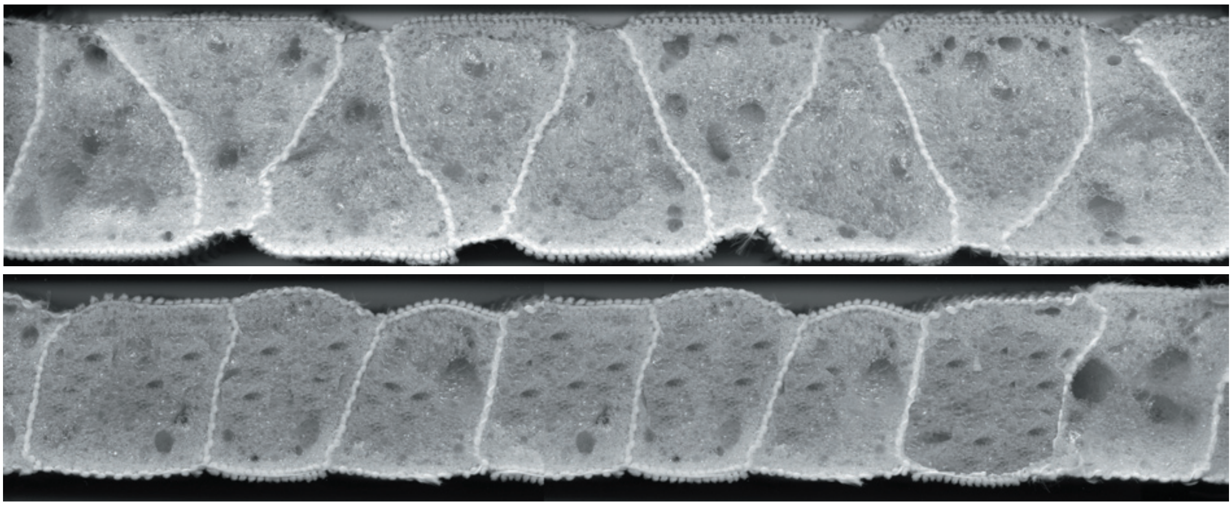
The research group from the **Institute of Architecture of Textiles, the Technical University of Lodz**, consisting of **Izabela Frontczak-Wasiak, Ph.D., Aleksander Koryciński, Ph.D., Henryk Suszek Ph.D., conducted by Marek Snycerski Ph.D. D.Sc., prof. of TUŁ,** was awarded a **Gold Medal at the Brussels-Eureka 2006** World Exhibition of Innovation, Research and New Technology for

'Multilayer textile product and its technology'

Spacer textile products have hitherto been manufactured as 3-layer products, with the internal layer in the form of threads connecting the two external layers. The new invention consists of a new class of multilayer products, with the internal layer in the form of woven ribs coherent with the external layers. To meet various needs, the ribs can be located at different angles in relation to the external layers: perpendicularly or obliquely, creating trapezoid, triangular, and rhomboidal cross-sections, with links between the next ribs.



Cross-section photos of examples of heat-insulating composites based on multilayer spacer fabrics with internal ribs.

The specific quality of the new product is its cellular structure, which shows a coherency of the fabric structure in all the partitions. The product consists of three layers. Two external layers are parallel woven structures; the internal layer in the form of ribs is woven simultaneously with both external layers to create a uniform product which is all of a piece. In order to obtain different product constructions, appropriate manufacturing methods were developed. Four patent applications for the product's construction and technology have been registered.

Manufacturing tests were carried out in laboratory and industrial conditions. The results obtained indicate that it is possible to produce the multilayer spacer fabric with integral ribs on rapier looms.

The multilayer spacer textile products with integral ribs manufactured on rapier looms so far do not have any constructive or functional textile equivalent. They can only be compared with technical goods manufactured by methods using corrugated boards or plastic finned panels. Compared with products of similar construction known so far, the multilayer spacer textile products with integral ribs are characterised by higher tenacity per unit mass, coherency of structure, functionality and technical applicability.

The technology presented can be of technical, economical and ecological benefit for composite construction reinforced by fabrics. Depending on the raw material used, including polyester, carbon and glass fibres, this can result in new constructional materials, characterised by higher strength, rigidity and lower unit mass, which would be resistant to atmospheric conditions and chemical corrosion. After filling the multilayer textile product with polyurethane foam, for instance, it is possible to obtain excellent heat-insulating and soundproof products.

Coating all the product's layers with a sealing material of controlled gas and liquid permeability will result in obtaining a sealing material for chemical tanks. In the case of underground reservoirs, applying such products equipped with leakage sensors placed into the inter-rib spaces would allow early detection and warning of possible leakages, facilitate their location and protect the environment form ecological disaster.

The main advantage of the multilayer spacer textile product with integral ribs is its high usability. Depending on the needs, the same product can be used to manufacture a wide variety of goods of different properties and applications.

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