Ryszard Korycki

Optimisation of Thermal Conditions in a Composite Wet Diving Suit

Technical University of Lodz, Department of Technical Mechanics and Informatics
Zeromskiego 116,
90-924 Lodz, Poland
ryszard.korycki@p.lodz.pl

Abstract
The main goal was to optimise the thermal conditions within a diving suit and improve the user's comfort. The wet diving suit consisted of (i) an external neoprene layer with air bubbles as additional insulation, and (ii) internal textile clothing to improve the insulation effect. Both layers were connected as a composite because the friction coefficient is very high. The core temperature should be constant and monitored during activity because a change in it is a sign of hypothermia. The internal textile layer contained textronic systems to transfer the medical parameters selected. The state variable is the temperature, whereas the design variables are the coordinates of the crucial points within the layers. Heat transfer is described by a state equation and set of boundary and initial conditions. The first-order sensitivities of an arbitrary behavioural functional are formulated and implemented into the optimality conditions of the problem. A simple numerical example of the optimisation of the thermal conditions is considered.

Key words: *composite diving suit, optimization, thermal conditions.*

Nomenclature

A, B, C, D – *constants acc. Liang and Qu [2],*
A – *matrix of thermal conduction coefficients,*
b – *vector of design parameters,*
b_j^1; b_j^2 – *vertex coordinates A_j, j = 1,2…m, of the Bezier polygon,*
C – *constraint functional, in this case the global cost of the structure,*
C_0 – *constant value of the constraint functional imposed (structural cost),*
c – *volumetric heat capacity,*
F – *objective functional,*
F' – *Lagrange functional (the auxiliary function),*
$g_p = \frac{DG}{Db}$ – *global (material) derivative of g with respect to design parameter b_p,*
$g^p = \frac{\partial g}{\partial b_p}$ – *partial (local) derivative of g with respect to design parameter b_p,*
H – *mean curvature of external boundary Γ,*
h – *surface film conductance,*
K_f – *volumetric strain modulus of the material within the filling,*
K_m – *volumetric strain modulus of the textile material,*
n – *unit vector normal to the external boundary Γ, directed outwards to the domain Ω bounded by this boundary,*
P – *number of design parameters during the sensitivity analysis,*
q – *vector of heat flux density,*
$\mathbf{q}^*$ – *vector of initial heat flux density,*
$q_n = \mathbf{n} \cdot \mathbf{q}$ – *heat flux density normal to the external boundary,*
T – *temperature,*
T_a; T_b – *temperatures of the upper and lower surface of composite acc. Liang and Qu [2],*
T^0 – *prescribed value of temperature,*
T_{0L} – *assumed level of temperature,*
T_∞ – *temperature of surrounding water,*
t – *real time in primary and additional structures,*
u – *unit cost of structure,*
V_f – *volume of free spaces between the textile material,*
V_m – *volume of textile material,*
$\mathbf{v}^p(\mathbf{x}, \mathbf{b}, t)$ – *transformation velocity field associated with design parameter b_p,*
$v_n^p = \mathbf{n} \cdot \mathbf{v}^p$ – *transformation velocity normal to the external boundary Γ,*
$\mathbf{v_f}$ – *vector of flow velocity,*
x – *vector of points' coordinates,*
Γ – *external boundary of structure,*
γ – *boundary integrand of the objective functional,*
ε – *effective porosity of textile material,*
ξ – *slack variable of the Lagrange functional for inequality problems,*
ξ_m; ξ_f – *coefficients for the textile material and material within the free spaces,*
λ_s – *substitute heat transfer coefficient,*
λ_m; λ_f – *heat transfer coefficients of the textile material and material within free spaces,*
Σ – *discontinuity line between adjacent parts of the piecewise smooth boundary Γ,*
σ – *Stefan-Boltzmann constant,*
ρ – *density of fibres,*
τ – *transformed time in the adjoint structure,*
χ – *Lagrange multiplier,*
Ψ – *domain integrand of the objective functional,*
Ω – *domain of the structure,*
∇ – *gradient operator,*
$\binom{m}{j} = \frac{m!}{j!(m-j)!}$ – *Newton symbol.*

Problem definition. Hypothermia

Protection against heat loss and, consequently, hypothermia is the key role of a diving suit. Hypothermia is caused by severe environmental exposure [12 - 15]. The key to establishing a diagnosis of hypothermia is the rapid determination of the true core temperature, i.e. the temperature of the human body measured by direct contact. At a given temperature, specific physical examination findings vary among patients. However, an examination does provide a frame of reference for dividing presenting symptoms into mild, moderate, and severe hypothermic signs [13]. Typical mild hypothermia for most people is shivering vigorously (34 °C – 35 °C), which may develop into altered judgment, amnesia and dysarthria (below 34 °C), the respiratory rate may increase, and ataxia and apathy may be experienced (33 °C). During moderate hypothermia, oxygen consumption decreases, with most people being in a stupor (32 °C); the body loses its ability to generate heat by shivering (31 °C), and finally people may become brain dead (28 °C – 30 °C). During severe hypothermia, the body becomes markedly susceptible to ventricular fibrillation (28 °C), with 83% of people becoming comatose (below 27 °C).

The most popular and cheapest protection against heat loss is a wet diving suit, which consists of an external neoprene layer with air bubbles as additional insulation, as well as internal textile clothing to improve the heat insulation of the body. These suits are typically used where the water temperature is 10 °C – 25 °C [1, 5]. Although water can enter the suit, it prevents excessive heat loss because little of the water warmed inside the suit escapes from it. A correct shape ensures optimal thermal conditions, which are critical for warmth. A suit that is too loose will al-

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low too much water to circulate over the diver's skin. A suit that is too tight is very uncomfortable and can impair circulation at the neck, a very dangerous condition which can cause blackouts. For this reason, many divers choose to have wetsuits custom-tailored to optimise the thermal conditions.

Both layers are connected in a similar fashion to the composite because the friction coefficient between the neoprene and internal textile clothing is very high. The core temperature should be constant during work and permanently monitored because a change therein can be a sign of hypothermia. The design variables are geometric coordinates of the characteristic points describing the current shape of the layers. The heat transfer is defined by a state equation and a set of boundary and initial conditions. The textile layer contains textronic systems to transfer the medical parameters selected.

Less popular and much more expensive is the dry diving suit. These suits can be used in a wide range of water temperatures depending on the structure. A dry suit is a composite made of different materials, where water cannot enter the structure. Thus, the body is protected by the material. Additional protection can be the thin layer of air between the suit and the skin.

The main idea of the paper was to optimise the thermal conditions of a wet diving suit. The problem also concerns the user's comfort but not the optimal adaptation of the diving suit to the body's shape. Furthermore it is necessary to discuss the heat transport model for non-isotropic composite structures of specific conditions and next solve the problem. The solution is sensitivity oriented, containing the sensitivities of the state fields as well as the sensitivity expressions within the structure. The material derivative concept as well as the direct and adjoint approaches to sensitivity analysis are considered, cf. Dems, Mróz [2], Dems, Korycki, Rousselet [3], Korycki [7 - 9]. The sensitivity oriented optimisation of the thermal conditions within a composite diving suit was not found in the literature analysed. The problem is very common for divers, as well as the different special textile structures, smart clothing, etc. The microclimate formed by a diving suit is totally different from that within typical clothing, cf. [16].

Physical model

A wet diving suit consist of two different layers. The external layer is of neoprene foam which can result from (i) the chemical reaction with the special substance during the hardening process or (ii) the injection of nitrogen inwards into the foam. The internal textile clothing has a typical compact, inhomogeneous structure which can be, for example, nylon fabrics or synthetic fur fabrics. Water can enter the suit, and the textile structure consists of textile elements as well as the water within the free spaces. Thus we have to homogenise the wet suit to create a physical model. The homogeneous structure has the same conditions of heat transfer within the whole domain, which considerably simplifies the solution procedure.

There are a few efficient homogenisation methods. Golański, Terada, Kikuchi [1] introduced the classic *rule of mixture* to determine the substitute heat transfer coefficient in the form

$$\lambda_s = \lambda_m \xi_m + \lambda_f \xi_f\ ; \quad \xi_m = \frac{V_m}{V_m + V_f}\ ;$$
$$\xi_f = \frac{V_f}{V_m + V_f}. \qquad (1)$$

Turner's model is developed according to the hydrostatic analogy, with the substitute coefficient equal to

$$\lambda_s = \frac{\lambda_m \xi_m K_m + \lambda_f \xi_f K_f}{\xi_m K_m + \xi_f K_f}. \qquad (2)$$

Medical parameters can be transmitted from the fabrics by special printed elements on the textile surface or by special metallic elements. The textronic system has a negligible volume/mass in relation to the whole textile clothing, and the structure can be homogenised by means of the above methods.

The neoprene layer can be homogenised by means of specially developed procedures. Liang and Qu [2] determined the substitute coefficient of a material subjected to the different temperatures on the parallel external surfaces. The model introduced includes the radiation of the free spaces filled with gas or liquid, located symmetrically and repeatable in the material. The authors discussed the two shapes of the spaces: the cylinder (2D problem) and the sphere (3D problem). The substitute heat transfer coefficient has the form of ***Equation 3*** (see page 108).

The problem of calculating the time comes from the scale. A macro scale implies a 3D problem, whereas a micro scale can be treated as a 2D problem. Let us next assume, for simplicity, that the surrounding fluid is subjected to laminar flow, which ensures the other mechanism of heat transfer is a turbulent one. The heat balance allows to formulate a state equation describing heat transfer within the suit.

Mathematical model

The state variable is the temperature T. The transient heat transport is described within each layer by a second-order correlation with respect to the state variable and a first-order with respect to time. The equation is applied, in general, by Korycki [9] but can be simplified because a typical diving suit does not contain internal heat sources. The problem is accompanied by a set of boundary and initial conditions. The structure contacts the body; the boundary is also the portion Γ_T subjected to the first-kind condition. An optimal microclimate is secured by the prescribed temperature T^0 of the water layer between the skin and diving suit. Heat is transported unidirectionally from the skin to the surroundings. The heat flux density on the side surfaces Γ_q can be consequently neglected, $q_n = 0$, and the structure is subjected to the second-kind condition. The external boundary portion Γ_C is subjected to the third kind boundary condition, i.e. convectional heat flux. The fourth-kind boundary conditions are defined for the common surfaces of the internal boundary, i.e. between the neoprene layer and internal clothing. The heat flux density normal to this boundary portion Γ_N has the same value. Heat can be additionally radiated from the external boundary portion Γ_d to the water layers surrounding the diving suit. Some questions concerning radiation are introduced, for example, by Bialecki [2]. Li [10] discussed the parameters describing the combined conduction and radiation. The initial condition describes the temperature distribution within the structure.

The diving suit contacts the skin through a thin water layer entering the suit. The boundary Γ_T of the internal clothing is also subjected to the constant temperature of the water layer, $T^{0(1)} = 35$ °C, which is the minimal temperature of a normal human metabolism. Heat is transported unidirectionally from the skin to the surroundings, with the heat flux density being negligible on the side surfaces Γ_q of the textile clothing and neoprene. The external part of the neoprene layer is subjected to thermal convection and thermal radiation. We assume the surrounding temperature T_∞ and surface film conductance h. The problem has the form of ***Equation 4***.

The problem can be simplified considerably for steady heat transfer. The time derivative of the temperature with respect to time is negligible, and the problem has the form of ***Equation 5***.

We can analyse the heat balance, i.e. the heat densities within a unit volume of the surrounding fluid caused by different thermal phenomena. The state parameter is still the temperature T. The energy source is the 3D fluid convection as well as the 3D heat conduction within the unit domain. Heat is lost by convection, conduction and the accumulation within the material. Consequently the energy correlation for the fluid contains an additional term which describes the fluid convection and depends on the flow velocity $\mathbf{v}_f$ and fluid temperature T, cf. Zarzycki [17], as is presented in ***Equation 6***.

We integrate the above equations numerically and determine the temperature distribution between the external boundary of the diving suit and the surroundings with respect to the flow velocity.

Sensitivity oriented optimisation

Let us consider an arbitrary behavioral functional associated with the transient heat transfer problem, described within the structure as is presented in ***Equation 7***, where Ψ and γ are continuous and differentiable functions of their arguments. According to the material derivative concept, the first-order sensitivity is assumed as the material derivative of the functional with respect to the design parameter and analysed by means of the direct and adjoint approaches.

Let us first analyse the direct approach. The unknown sensitivities of the state fields are obtained by means of the additional structure associated with each design parameter. This approach is useful for calculating the sensitivities of the entire response field with respect to a few design variables. The number of problems is equal to the number of design parameters, and we additionally solve the primary problem. The additional structure has the same shape, with the thermal properties as primary, and is characterised by the correlations following from the differentiation of primary equations with respect to design parameters. The state equation and set of conditions are characterised, acc. [9], with respect to the vanishing heat source $f = 0$ see ***Equation 8***.

The first-order sensitivity expression can be simplified to the form of [9] ***Equation 9***.

The adjoint approach for calculating the first-order sensitivity requires the solution of the one adjoint and the primary heat transfer problem. The adjoint and primary structures have the same shape as well as thermal and radiation properties. The adjoint method is most convenient for estimating first-order sensitivities with respect to a few objective functionals. The equations of the adjoint heat conduction problem are the heat conduction equation and the boundary and initial conditions, cf. Korycki [9] see ***Equation 10***.

$$\lambda_s = \frac{1}{A} - \frac{1}{3A^2CD^2(T_b - T_a)}\ln\frac{T_b + D}{T_b + D} + \frac{1}{6A^2CD^2(T_b - T_a)}\ln\frac{T_b^2 - DT_b + D^2}{T_a^2 - DT_a + D^2} - \frac{1}{\sqrt{3}A^2CD^2(T_b - T_a)}\left(\operatorname{arctg}\frac{2T_b - D}{\sqrt{3}D} - \operatorname{arctg}\frac{2T_a - D}{\sqrt{3}D}\right). \quad (3)$$

$$\begin{cases} -\operatorname{div}\mathbf{q} = c\dfrac{dT}{dt} \\ \mathbf{q} = \mathbf{A}\cdot\nabla T + \mathbf{q}^* \end{cases} \mathbf{x}\in\Omega; \quad \begin{array}{l} T(\mathbf{x},t) = T^0(\mathbf{x},t) \ \ \mathbf{x}\in\Gamma_T; \ \ q_n(\mathbf{x},t) = 0 \ \ \mathbf{x}\in\Gamma_q; \\ q_{nc}(\mathbf{x},t) = h[T(\mathbf{x},t) - T_\infty(\mathbf{x},t)] \ \ \mathbf{x}\in\Gamma_C; \ \ q_n^{(1)}(\mathbf{x},t) = q_n^{(2)}(\mathbf{x},t) \ \ \mathbf{x}\in\Gamma_N; \\ q_n^r(\mathbf{x},t) = \sigma\, T(\mathbf{x},t)^4 \ \ \mathbf{x}\in\Gamma_d; \quad T(\mathbf{x},0) = T_0 \ ; \ \ \mathbf{x}\in(\Omega\cup\Gamma). \end{array} \quad (4)$$

$$\begin{cases} -\operatorname{div}\mathbf{q} = 0 \\ \mathbf{q} = \mathbf{A}\cdot\nabla T + \mathbf{q}^* \end{cases} \mathbf{x}\in\Omega; \quad \begin{array}{l} T(\mathbf{x}) = T^0(\mathbf{x}) \ \ \mathbf{x}\in\Gamma_T; \ \ q_n(\mathbf{x}) = 0 \ \ \mathbf{x}\in\Gamma_q; \\ q_{nc}(\mathbf{x}) = h[T(\mathbf{x}) - T_\infty(\mathbf{x})] \ \ \mathbf{x}\in\Gamma_C; \ \ q_n^{(1)}(\mathbf{x}) = q_n^{(2)}(\mathbf{x}) \ \ \mathbf{x}\in\Gamma_N; \\ q_n^r(\mathbf{x}) = \sigma\, T(\mathbf{x})^4 \ \ \mathbf{x}\in\Gamma_d. \end{array} \quad (5)$$

$$\begin{cases} -\operatorname{div}(\rho c\, \mathbf{v}_f T) + \operatorname{div}\mathbf{q} = \rho c\dfrac{dT}{dt} \\ \mathbf{q} = \mathbf{A}\cdot\nabla T + \mathbf{q}^* \end{cases} \mathbf{x}\in\Omega_{fluid}; \quad \begin{array}{l} T(\mathbf{x},t) = T^0(\mathbf{x},t) \ \ \mathbf{x}\in\Gamma_T; \ \ q_n(\mathbf{x},t) = 0 \ \ \mathbf{x}\in\Gamma_q; \\ q_{nc}(\mathbf{x},t) = h[T(\mathbf{x},t) - T_\infty(\mathbf{x},t)] \ \ \mathbf{x}\in\Gamma_C; \quad T(\mathbf{x},0) = T_0 \ \ \mathbf{x}\in(\Omega\cup\Gamma). \end{array} \quad (6)$$

$$F = \int_0^{t_f}\left[\int_\Omega \Psi(T,\nabla T,\mathbf{q},\dot{T})\,d\Omega + \int_\Gamma \gamma(T,q_n,T_\infty)\,d\Gamma\right]dt; \ \ \dot{T} = \frac{dT}{dt}; \quad (7)$$

$$\begin{cases} -\operatorname{div}\mathbf{q}^p = c\dfrac{dT^p}{dt} \\ \mathbf{q}^p = \mathbf{A}\cdot\nabla T^p + \mathbf{q}^{*p} \end{cases} \mathbf{x}\in\Omega; \quad \begin{array}{l} T^p(\mathbf{x},t) = T^{0p} = T_p^0 - \nabla T^0\cdot\mathbf{v}^p \quad \mathbf{x}\in\Gamma_T; \\ q_n^p(\mathbf{x},t) = \left(q_n^0\right)_p + \mathbf{q}_\Gamma^0\cdot\nabla_\Gamma v_n^p - \nabla_\Gamma q_n^0\cdot\mathbf{v}_\Gamma^p - q_{n,n}^0 v_n^p \quad \mathbf{x}\in\Gamma_q; \ \ i = 1,2; \\ q_{nc}^p(\mathbf{x},t) = h\left(T^p - T_\infty^p\right) + \mathbf{q}_\Gamma\cdot\nabla_\Gamma v_n^p \quad \mathbf{x}\in\Gamma_c; \\ q_n^{p(1)}(\mathbf{x},t) = q_n^{p(2)}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_N \\ q_n^{rp} = 4\sigma T^3\left(T^p + \nabla T\cdot\mathbf{v}^p\right) - \nabla q_n^r\cdot\mathbf{v}^p + \mathbf{q}^r\cdot\nabla_\Gamma v_n^p \quad \mathbf{x}\in\Gamma_d; \\ T_0^p(\mathbf{x},0) = T_{0p} - \nabla T_0\cdot\mathbf{v}^p \ \ \mathbf{x}\in(\Omega\cup\Gamma). \end{array} \quad (8)$$

Equations 3, 4, 5, 6, 7 and 8.

The time transformation is introduced and the final time $t = t_f$ at the primary and additional problem is equivalent to the starting time at the adjoint problem $\tau = 0$. The first-order sensitivity expression has now the form [9] of ***Equation 11***.

The problem is sensitivity oriented, i.e. we introduce the first-order sensitivity expressions in the optimisation correlation. The optimisation problem is defined as the minimisation of the objective functional with the imposed constraint of the structural cost *C*. Assuming a homogeneous structure in technical problems, the structural cost is proportional to the area of domain *Ω*. Introducing the Lagrange functional $F' = F + \chi\left(C - C_0 + \xi^2\right)$ cf. [5] and its stationarity correlations, we formulate the optimality conditions

$$\begin{cases} F_p = -\chi \int\limits_{\Omega} u\, v_n^p\, d\Gamma \\ \int\limits_{\Omega} u\, d\Omega - C_0 + \xi^2 = 0. \end{cases} \quad (12)$$

The functional most applied is the measure of the heat flux density in the form:

$$F = \int_0^{t_f} \left[\int_{\Gamma} q_n d\Gamma \right] dt \;\; ; \;\; \Gamma \in \Gamma_{external} \quad (13)$$

The minimisation of the above functional corresponds to the design of a diving suit of optimal insulation.

The functional can be a global measure of the maximum local temperature in the domain. We determine the optimal heat conditions by minimising the distribution of the state variable within the structure:

$$F = \int_0^{t_f} \left[\int_{\Omega} \left(\frac{T}{T_{0L}} \right)^n d\Omega \right] dt \;\; ; \;\; n \to \infty \quad (14)$$
$$\Gamma \in \Gamma_{external}$$

Numerical example

Let us now optimise the wet diving suit within a water environment. Of course, the diving suit should ensure optimal insulation, cf. ***Figure 1***. The primary structure can be characterised by ***Equation 4***. The diving suit contacts the skin through the thin water layer entering the suit. The boundary Γ_T of the textile has the temperature of the water layer $T^{0(1)} = 34$ °C, from $t = 0$ to $t = 10$ s. The temperature increases in conjunction with the value $T^{0(1)} = 36$ °C till the end of optimisation. The external

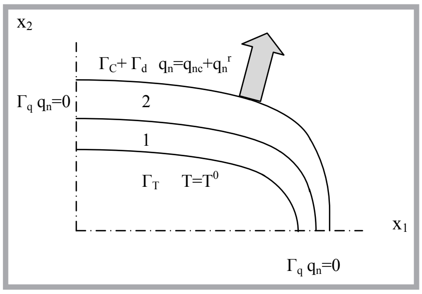

Figure 1. *Shape optimization of the segment of the wet diving suit, boundary conditions, 1 – textile clothing, 2 – neoprene.*

part of the neoprene layer is subjected to thermal convection as well as thermal radiation to the surrounding temperature $T_\infty = 0$ °C, with the surface film conductance $h = 5$. Additionally let us introduce the negligible initial heat flux density $\mathbf{q}^* = 0$. The primary problem is defined with respect to the above assumptions as presented in ***Equation 15***.

In this case the objective functional, ***Equation 13***, should be minimised, and the integrand is $\gamma = q_n$ on the external boundary portion Γ_C and Γ_d. The additional structures are defined by ***Equation 8***. Introducing the

$$F_P = \left[\int_{\Omega} \Psi,_{\dot{T}} T^p d\Omega \right]_0^{t_f} + \int_0^{t_f} \left\{ \int_{\Omega} \left[\left(\Psi,_T - \frac{d(\Psi,_{\dot{T}})}{dt} \right) T^p + \nabla_{\nabla T}\Psi \cdot \nabla T^p + \nabla_{\mathbf{q}}\Psi \cdot \mathbf{q}^p \right] d\Omega + \int_{\Gamma_T} \left[\gamma,_T \left(T_p^0 - \nabla_\Gamma T^0 \cdot \mathbf{v}_\Gamma^p - T,_n^0 v_n^p \right) + \right. \right.$$
$$\left. + \gamma,_{q_n} \left(q_n^p - \mathbf{q}_\Gamma \cdot \nabla_\Gamma v_n^p \right) \right] d\Gamma_T + \int_{\Gamma_q} \left[\gamma,_T T^p + \gamma,_{q_n} \left(q_{np}^0 - \nabla_\Gamma q_n^0 \cdot \mathbf{v}_\Gamma^p - q_n^0,_n \cdot v_n^p \right) \right] d\Gamma_q + \int_{\Gamma_c} \left[\gamma,_T T^p + \gamma,_{q_n} h\left(T^p - T_\infty^p \right) \right] d\Gamma_c + \quad (9)$$
$$\left. + \int_{\Gamma_d} \gamma,_T T^p d\Gamma_d + \int_{\Gamma} \left(\Psi + \gamma,_n - 2H\gamma \right) v_n^p \, d\Gamma + \int_{\Gamma} \gamma,_{T_\infty} T_\infty^p \, d\Gamma + \int_{\Sigma} \left] \gamma \mathbf{v}^p \cdot \boldsymbol{\upsilon} \right[\right\} dt; \qquad p = 1, 2, \ldots P.$$

$$T^a(\mathbf{x},\tau) = T^{0a}(\mathbf{x},\tau) \quad \mathbf{x} \in \Gamma_T \;\; ; \;\; q_n^a(\mathbf{x},\tau) = \mathbf{n} \cdot \mathbf{q}^a = q_n^{0a}(\mathbf{x},\tau) \quad \mathbf{x} \in \Gamma_q;$$
$$q_n^a(\mathbf{x},\tau) = \mathbf{n} \cdot \mathbf{q}^a = h\left[T^a(\mathbf{x},\tau) - T_\infty^a(\mathbf{x},\tau) \right] \mathbf{x} \in \Gamma_c \qquad \mathbf{n} \cdot \mathbf{q}^{ar} = q_n^{ar}(\mathbf{x},\tau) \quad \mathbf{x} \in \Gamma_d$$
$$T^a(\mathbf{x},\tau = 0) = T_0^a(\mathbf{x},\tau = 0) \quad \mathbf{x} \in (\Omega \cup \Gamma); \; i = 1,2; \qquad T^a(\mathbf{x},\tau = 0) = \frac{1}{c} \Psi,_{\dot{T}}(\mathbf{x}, t = t_f) \quad \mathbf{x} \in (\Omega \cup \Gamma);$$
$$\begin{cases} -\operatorname{div} \mathbf{q}^a = c \dfrac{dT^a}{d\tau} \\ \mathbf{q}^a = \mathbf{A} \cdot \nabla T^a + \mathbf{q}^{*a} \end{cases} \quad \mathbf{x} \in \Omega; \qquad \mathbf{q}^{*a}(\mathbf{x},\tau) = \nabla_{\nabla T}\Psi(\mathbf{x},t) + \nabla_{\mathbf{q}}\Psi(\mathbf{x},t) \cdot \mathbf{A}(\mathbf{x}) \quad \mathbf{x} \in \Omega; \qquad T^{0a}(\mathbf{x},\tau) = \gamma,_{q_n}(\mathbf{x},t) \quad \mathbf{x} \in \Gamma_T \quad (10)$$
$$q_n^{0a}(\mathbf{x},\tau) = -\gamma,_T(\mathbf{x},t) \quad \mathbf{x} \in \Gamma_q; \qquad T_\infty^a(\mathbf{x},\tau) = \frac{1}{h}\gamma,_T(\mathbf{x},t) + \gamma,_{q_n}(\mathbf{x},t) \quad \mathbf{x} \in \Gamma_c$$
$$q_n^{ar}(\mathbf{x},\tau) = \sigma\left[T^a(\mathbf{x},\tau) \right]^4; \;\; T^a(\mathbf{x},\tau) = \left[\frac{-\gamma,_T(\mathbf{x},t)}{\sigma} \right]^{0,25} \quad \mathbf{x} \in \Gamma_d$$

$$F_p = -\left[\int_{\Omega} \left(\Psi,_{\dot{T}} - cT^a \right)\left(T_p - \nabla T \cdot \mathbf{v}^p \right) d\Omega \right]_{t=0} + \int_0^{t_f} \left\{ \int_{\Omega} \left[\left(\nabla_q \Psi + \nabla T^a \right) \cdot \mathbf{q}^{*p} \right] d\Omega + \int_{\Gamma_T} \left[\left(\gamma,_T + q_n^a \right)\left(T_p^0 - \nabla_\Gamma T^0 \cdot \mathbf{v}_\Gamma^p - T,_n^0 v_n^p \right) - \gamma,_{q_n} \mathbf{q}_\Gamma \cdot \nabla_\Gamma v_n^p \right] d\Gamma_T + \right.$$
$$+ \int_{\Gamma_q} \left[\left(\gamma,_{q_n} - T^a \right)\left(q_{np}^0 - \nabla_\Gamma q_n^0 \cdot \mathbf{v}_\Gamma^p - q_n^0,_n v_n^p \right) - T^a \mathbf{q}_\Gamma^0 \cdot \nabla_\Gamma v_n^p \right] d\Gamma_q + \int_{\Gamma_c} \left[T^a h T_\infty^p - T^a \mathbf{q}_\Gamma \cdot \nabla_\Gamma v_n^p - \gamma,_{q_n} h T_\infty^p \right] d\Gamma_c - \int_{\Gamma_d} T^a q_n^{rp} \, d\Gamma_d + \quad (11)$$
$$\left. + \int_{\Gamma} \left(\Psi + \gamma,_n - 2H\gamma \right) v_n^p \, d\Gamma + \int_{\Gamma} \gamma,_{T_\infty} T_\infty^p \, d\Gamma + \int_{\Sigma} \left] \gamma \mathbf{v}^p \cdot \boldsymbol{\upsilon} \right[\right\} dt.$$

Equations 9, 10 and 11.

conditions shown in ***Figure 1***, with the material derivatives of temperature $T^0_P = T_{0P}$ known in advance, we simplify the equation to the form presented ***Equation 16***.

According to ***Equation 9***, the sensitivity expression has the form of ***Equation 17***.

The adjoint approach can be defined by ***Equation 18***, cf. ***Equation 10***.

The sensitivity expression is consequently described by the simplified ***Equation 11*** presented as ***Equation 19***.

Practically speaking, we assume that the fabrics of the internal clothing have isotropic thermal properties, which means that the matrix of heat conduction coefficients now has only one component – the heat conduction coefficient determined by the homogenisation method $\mathbf{A} = \lambda$. The material parameters of the clothing are $\mathbf{A}^{(1)} = 0{,}030$ W/(mK), $c^{(1)} = 1610 \cdot 10^3$ J/(m^{3}K). This layer was homogenised by means of the *rule of mixture*. The external neoprene layer has the parameters $\mathbf{A}^{(2)} = 0{,}050$ W/(mK) and $c^{(2)} = 2500 \cdot 10^3$ J/(m^{3}K). The neoprene is homogenised by means of the special method with air bubbles, according to Liang and Qu [11].

The optimisation is determined for the 5 steps within the time period $t_{init} = 0$; $t_{final} = 40$ s; $\Delta t = 10$ s. The design parameters are the 24 coordinates of the Bezier polygon points shown in ***Figure 2***. The Bezier curve is determined by means of these polygon points according to the correlation

$$\begin{vmatrix} x_1 \\ x_2 \end{vmatrix} = \sum_{j=0}^{m} \begin{vmatrix} b_j^1 \\ b_j^2 \end{vmatrix} \binom{m}{j} t^j (1-t)^{m-j}, \qquad (20)$$

where t is the real parameter from the range $0 \le t \le 1$; Of course, the first and last point of the Bezier polygon are situated on the curve and the boundary is tangent to the polygon within these points. The number of design parameters is equal to 24, and consequently there are coordinates b_j^1 and b_j^2, cf. ***Figure 2***.

$$\begin{aligned} & -\mathbf{A} \cdot \mathrm{div}(\nabla T) = c\frac{dT}{dt} \quad \mathbf{x} \in \Omega; \\ & T(\mathbf{x},t) = T^0(\mathbf{x},t) \quad \mathbf{x} \in \Gamma_T; \quad q_n(\mathbf{x},t) = 0 \quad \mathbf{x} \in \Gamma_q; \\ & q_{nc}(\mathbf{x},t) = h[T(\mathbf{x},t) - T_\infty(\mathbf{x},t)] \quad \mathbf{x} \in \Gamma_C; \quad q_n^{(1)}(\mathbf{x},t) = q_n^{(2)}(\mathbf{x},t) \quad \mathbf{x} \in \Gamma_N; \\ & q_n^r(\mathbf{x},t) = \sigma\, T(\mathbf{x},t)^4 \quad \mathbf{x} \in \Gamma_d; \quad T(\mathbf{x},0) = T_0 \;; \quad \mathbf{x} \in (\Omega \cup \Gamma). \end{aligned} \qquad (15)$$

$$\begin{aligned} & -\mathbf{A} \cdot \mathrm{div}(\nabla T^p) = c\frac{dT^p}{dt} \quad \mathbf{x} \in \Omega; \\ & q_n^p(\mathbf{x},t) = 0 \quad \mathbf{x} \in \Gamma_q; \quad T^p(\mathbf{x},t) = -\nabla T^0 \cdot \mathbf{v}^p \quad \mathbf{x} \in \Gamma_T; \\ & q_n^p(\mathbf{x},t) = h(T^p - T_\infty^p) + \mathbf{q}_\Gamma \cdot \nabla_\Gamma v_n^p \quad \mathbf{x} \in \Gamma_c; \\ & q_n^{rp} = 4\sigma T^3 (T^p + \nabla T \cdot \mathbf{v}^p) - \nabla q_n^r \cdot \mathbf{v}^p + \mathbf{q}^r \cdot \nabla_\Gamma v_n^p \quad \mathbf{x} \in \Gamma_d; \\ & T_0^p(\mathbf{x},0) = -\nabla T_0 \cdot \mathbf{v}^p \quad \mathbf{x} \in (\Omega \cup \Gamma); \quad i = 1,2. \end{aligned} \qquad (16)$$

$$F_p = \int_0^{t_f} \left\{ \int_{\Gamma_c} h(T^p - T_\infty^p) d\Gamma_c + (q_{n,n} - 2Hq_n) v_n^p \, d\Gamma_C \right\} dt; \qquad p = 1, 2, \ldots P. \qquad (17)$$

$$\begin{aligned} & -\mathbf{A} \cdot \mathrm{div}(\nabla T^a) = c\frac{dT^a}{d\tau} \quad \mathbf{x} \in \Omega; \\ & T^a(\mathbf{x},\tau) = T^{0a}(\mathbf{x},\tau) \quad \mathbf{x} \in \Gamma_T \;; \quad q_n^a(\mathbf{x},\tau) = \mathbf{n} \cdot \mathbf{q}^a = q_n^{0a}(\mathbf{x},\tau) \quad \mathbf{x} \in \Gamma_q; \\ & q_n^a(\mathbf{x},\tau) = \mathbf{n} \cdot \mathbf{q}^a = h[T^a(\mathbf{x},\tau) - T_\infty^a(\mathbf{x},\tau)] \, \mathbf{x} \in \Gamma_c; \quad \mathbf{n} \cdot \mathbf{q}^{ar} = q_n^{ar}(\mathbf{x},\tau) \quad \mathbf{x} \in \Gamma_d; \\ & T^a(\mathbf{x},\tau=0) = T_0^a(\mathbf{x},\tau=0) \quad \mathbf{x} \in (\Omega \cup \Gamma); \quad T^a(\mathbf{x},\tau=0) = 0 \quad \mathbf{x} \in (\Omega \cup \Gamma); \\ & \mathbf{q}^{*a}(\mathbf{x},\tau) = 0 \quad \mathbf{x} \in \Omega; \quad T^{0a}(\mathbf{x},\tau) = 0 \quad \mathbf{x} \in \Gamma_T; \quad q_n^{0a}(\mathbf{x},\tau) = 0 \quad \mathbf{x} \in \Gamma_q; \\ & T_\infty^a(\mathbf{x},\tau) = 1 \quad \mathbf{x} \in \Gamma_c; \quad q_n^{ar}(\mathbf{x},\tau) = \sigma[T^a(\mathbf{x},\tau)]^4; \; T^a(\mathbf{x},\tau) = \left[\frac{-\gamma_{,T}(\mathbf{x},t)}{\sigma}\right]^{0,25} \quad \mathbf{x} \in \Gamma_d. \end{aligned} \qquad (18)$$

$$\begin{aligned} F_p = & \left[-\int_\Omega c T^a (\nabla T \cdot \mathbf{v}^p) d\Omega \right]_{t=0} + \int_0^{t_f} \left\{ -\int_{\Gamma_T} \left[q_n^a \left(\nabla_\Gamma T^0 \cdot \mathbf{v}_\Gamma^p + T_{,n}^0 v_n^p \right) \right] d\Gamma_T + \right. \\ & + \int_{\Gamma_q} \left[T^a \left(\nabla_\Gamma q_n^0 \cdot \mathbf{v}_\Gamma^p + q_{n,n}^0 v_n^p \right) - T^a \mathbf{q}_\Gamma^0 \cdot \nabla_\Gamma v_n^p \right] d\Gamma_q + \\ & \left. + \int_{\Gamma_c} \left[T^a h T_\infty^p - T^a \mathbf{q}_\Gamma \cdot \nabla_\Gamma v_n^p - h T_\infty^p \right] d\Gamma_c - \int_{\Gamma_d} T^a q_n^{rp} \, d\Gamma_d + \int_{\Gamma_C} (q_{n,n} - 2Hq_n) v_n^p d\Gamma_C \right\} dt. \end{aligned} \qquad (19)$$

Equations 15, 16, 17, 18 and 19.

We have to introduce additional constraints on the material shape and thickness. Only a quarter of the whole cross-section is optimised due to the symmetry of the problem. All the points of the Bezier polygon on the external and internal boundaries of the neoprene layer can change the location no greater than 10% of the initial value. The same points on the external boundary of the clothing contacting the skin can change maximally 5% only in the direction outside the thorax. Thus, water can perhaps enter the suit more dynamically. The additional constraint is the material thickness of the front side of thorax (the upper part in ***Figure 3***), which is 20% greater in relation to the side part of the suit (the right-hand side in ***Figure 3***).

The analysis step allows to introduce a Finite Element Net made of 600 nodes. The synthesis step is performed by means of the external penalty function. The optimal shape is obtained in 9 iterations, and the optimal functional is equal to 81,79% of the initial value.

Conclusions

Heat transport and optimal thermal conditions are the main factors of the user's comfort and safety in extreme environmental conditions, for example while diving. The transient problems are complicated and should be solved approximately by means of different numerical methods. The problem can be considerably simplified for steady heat transport in a diving suit, especially for some particular cases. Boundary conditions are formulated by means of the real physical phenomena within the composite structure of the diving wet suit.

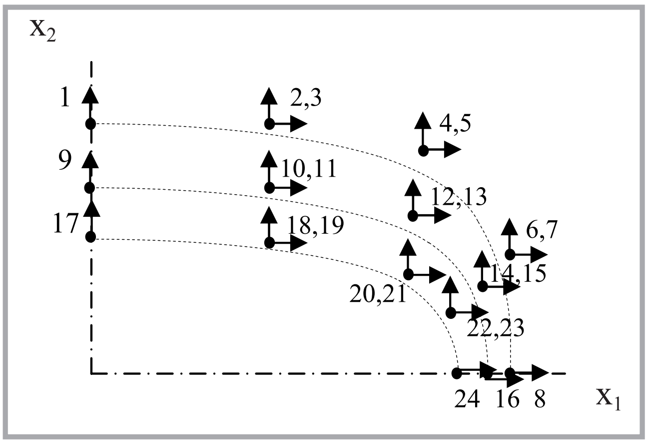

Figure 2. Shape optimization of the segment of the wet diving suit, design parameters, 1 – 24 coordinates f the Bezier polygon points.

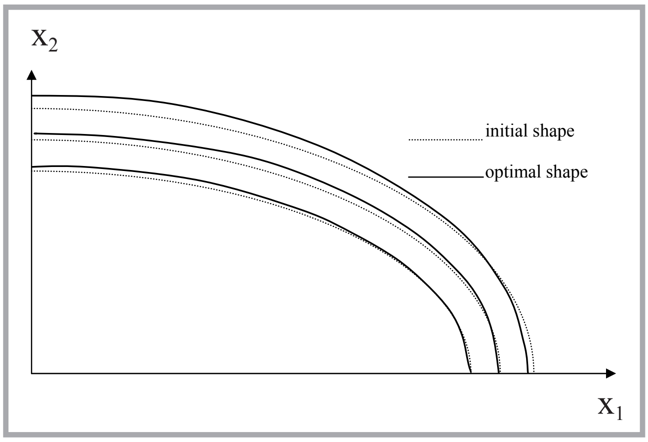

Figure 3. Shape optimization of the segment of the wet diving suit, initial and optimal shapes.

The results presented show that the method discussed can be a promising tool to optimise the thermal conditions within a composite diving suit. The boundary was modelled by means of the Bezier curve, which makes the analysis similar to the real shape of the thorax and other parts of the human body. Different constraints can be introduced to improve the insulation effect (for example different material thicknesses, maximal changes in the position of the Bezier points etc.). The shape obtained has the minimal increased material thickness of the front, decreased thickness of the sides and more space between the skin and clothing. Thus for the diver the thin layer of water entering the suit can now be additional insulation of constant temperature during the optimisation time. Of course, a complete wet diving suit of different boundary conditions needs modelling by means of 3D elements. The optimal thermal conditions need much more calculation time. However, basic conclusions can be formulated by means of simple 2D optimisation. The additional advantage is the little calculation time in relation to the other methods.

It is evident that the optimisation results obtained should be practically verified, which is beyond the scope of the publication presented and will be introduced in the next paper. The main difficulty is always the balance between the computational effort required to solve the problem and the complexity of the modelling. Basic 2D or simple 3D models as well as the applied FE Net are relatively uncomplicated, therefore the effective calculation time is low. The more complicated shapes need the advanced space FE Net, in which the calculation time grows significantly. Moreover, the results can be verified for existing wet diving suits and the temperature measured within selected points of the structure.

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References

1. US Navy Diving Manual, 6th revision. United States: US Naval Sea Systems Command. 2006. http://www.supsalv.org/00c3_publications.asp?destPage=00c3&pageID=3.9. Retrieved 2008-04-24.Bialecki R.A., Solving the heat radiation problems using the boundary element method, Computational Mechanics Publications, Southampton and Boston, 1993.
2. Dems K., Mróz Z., Shape sensitivity in mixed Dirichlet-Neumann boundary-value problems and associated class of path-independent integrals, Eur. J. Mech. A/Solids, 14 (2), 169-203, 1995.
3. Dems K., Korycki R., Rousselet B., Application of first – and second-order sensitivities in domain optimization for steady conduction problem, J.Therm. Stress. 20, 697-728, 1997.
4. Fulton H.T., Welham W., Dwyer J.V., Dobbins R.F. (1952). "Preliminary Report on Protection Against Cold Water". US Naval Experimental Diving Unit Technical Report NEDU-RR-5-52. http://archive.rubicon-foundation.org/3387. Retrieved 2008-04-25.
5. Golański D., Terada K., Kikuchi N., Macro and micro scale modeling of thermal residual stresses in metal matrix composite surface layers by the homogenization method, Computational Mechanics, 19, 188-202, 1997.
6. Korycki R., Identification and optimal design of the shape of the heat conducting textile products and structures (in Polish), Scientific Bulletin of Lodz Technical University, 929, 2003.
7. Korycki R., Sensitivity analysis and shape optimisation for transient heat conduction with radiation, Int. J. Heat and Mass Transfer 49, 33-38, 2009.
8. Korycki R., Shape optimization in oppositely directed coupled diffusion within composite structures. Structural and Multidisciplinary Optimization, 39, 283-296, 2009.
9. Li H.Y., Estimation of thermal parameters in combined conduction and radiation, Journal of Heat and Mass Transfer, Vol.42, 3, 565-572, 1999.
10. Liang X.-G., Qu W., Effective thermal conductivity of gas-solid composite materials and the temperature difference effect at high temperature, Int. J. Heat Mass Transfer, vol.42, nr 11, 1885-1893, 1999.
11. Long W.B. 3rd, Edlich R.F., Winters K.L., Britt L.D., Cold injuries. J Long Term Eff Med Implants. 15(1), 67-78, 2005.
12. McCullough L., Arora S., Diagnosis and treatment of hypothermia. Am Fam Physician. 70(12), 2325-32, 2004.
13. Polderman K.H., Mechanisms of action, physiological effects, and complications of hypothermia. Crit Care Med., 37(7 Suppl), 186-202, 2009.
14. Sessler D.I., Thermoregulatory defense mechanisms. Crit Care Med. 37(7 Suppl), 203-10, 2009.
15. Więźlak W., Kobza W., Zieliński W., Słowikowska-Szymańska Z., Modeling of the microclimate formed by a single-layer clothing material pack, Part 1, Fibres & Textiles in Eastern Europe, 2, 49-53, 1996.
16. Zarzycki R., Heat exchange and mass transfer in the environmental engineering (in Polish), WNT, Warsaw, 2005.

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