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Analysis of Planar Anisotropy of Fibre Systems by Using 2D Fourier Transform

Abstract
This paper describes a simple method of description of the anisotropy of fibre systems using image analysis. The proposed method is based on Fourier transform which is in frequency domains displayed by high values of frequency components corresponding to major structural direction lines in the spatial domain image. The values of frequency components are summarised in directional vectors depending on their certain angle and transformed to a polar diagram and histogram. The polar diagram can be seen as an estimate of the rose of directions.

Key words: *anisotropy, fibre system, Fourier transform, rose of directions.*

Introduction

The article is aims to graphically describe the planar anisotropy of fibre or other planar systems based on image analysis. The method uses spectral techniques with the aid of two-dimensional Fourier transform. The objects are an important part of an image and represent real-world objects. These objects are either randomly placed or they prefer certain directional placement. The objects should be in contrast with the background (gradient of image function on the edges of the object and background). In textile experience, the objects are considered to be fibres, threads, cross – sections of fibres etc., systems containing objects can be webs, fibre layers, woven fabrics, knitted fabrics, nonwoven textiles etc.

The characteristics of planar anisotropy is the angular density of length of thread or fibres $f(\alpha)$, which defines the length of thread or fibres orientated to an angular segment $\alpha \pm \alpha/2$. Function $f(\alpha)$ or rather the polar plot of density $f(\alpha)$ is called the rose of directions. An experimental graphical method for the estimation of $f(\alpha)$ is described in [3]. This method uses the net of angles $\alpha_1 ... \alpha_n$ situated at the top of fibre system being monitored for the construction of the rose of intersections. The rose of directions as an estimate of function $f(\alpha)$ is then obtained from the rose of intersections through the graphical construction of the Steiner compact. The number limit of angles is $n \leq 18$.

The graphical method proposed is based on the spectral method of image analysis. The goal of this method is a fast graphical representation of the directional arrangement of objects (estimation of anisotropy $f(\alpha)$) in the form of rose of directions and histogram.

2D Fourier Transform (2DFT)

The spectral approach is based on two-dimensional (2D) Fourier transform (FT) and is suitable for describing the textured images. The dominating directions (gradient of image function) in the directional textures (spatial domain) correspond to the large magnitude of frequency components distributed along the straight lines in the Fourier spectrum (frequency domain). In contrast, the purely random texture causes, that the frequency components in the power spectrum are approximately isotropic and possess a near circular shape. The Fourier transform is rotation dependent, i.e. rotating the original image by an angle will rotate its corresponding frequency plane by the same angle. The transform of horizontal lines in the spatial domain image appears as vertical lines in the Fourier domain image, i.e. the lines in the spatial domain image and its transformation are orthogonal to each other [5]. Let $f(x,y)$ be the grey level at pixel coordinates (x,y). Let the size of spatial domain image be $M \times N$. For such an image the direct and inverse Fourier transforms are given

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(ux/M + vy/N)} \tag{1}$$

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi(ux/M + vy/N)} \tag{2}$$

where u = 0, 1, 2, ..., N - 1 and, v = 0, 1, 2, ..., M - 1 are frequency variables [4]. If $f(x,y)$ is real, its transform is, in general, complex. $R(u,v)$ and $I(u,v)$ represent the real and imaginary components of $F(u,v)$, the Fourier spectrum is defined as

$$|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)} \tag{3}$$

The power spectrum $P(u,v)$ and the representation of $P(u,v)$ scaled to 8 - bit grey

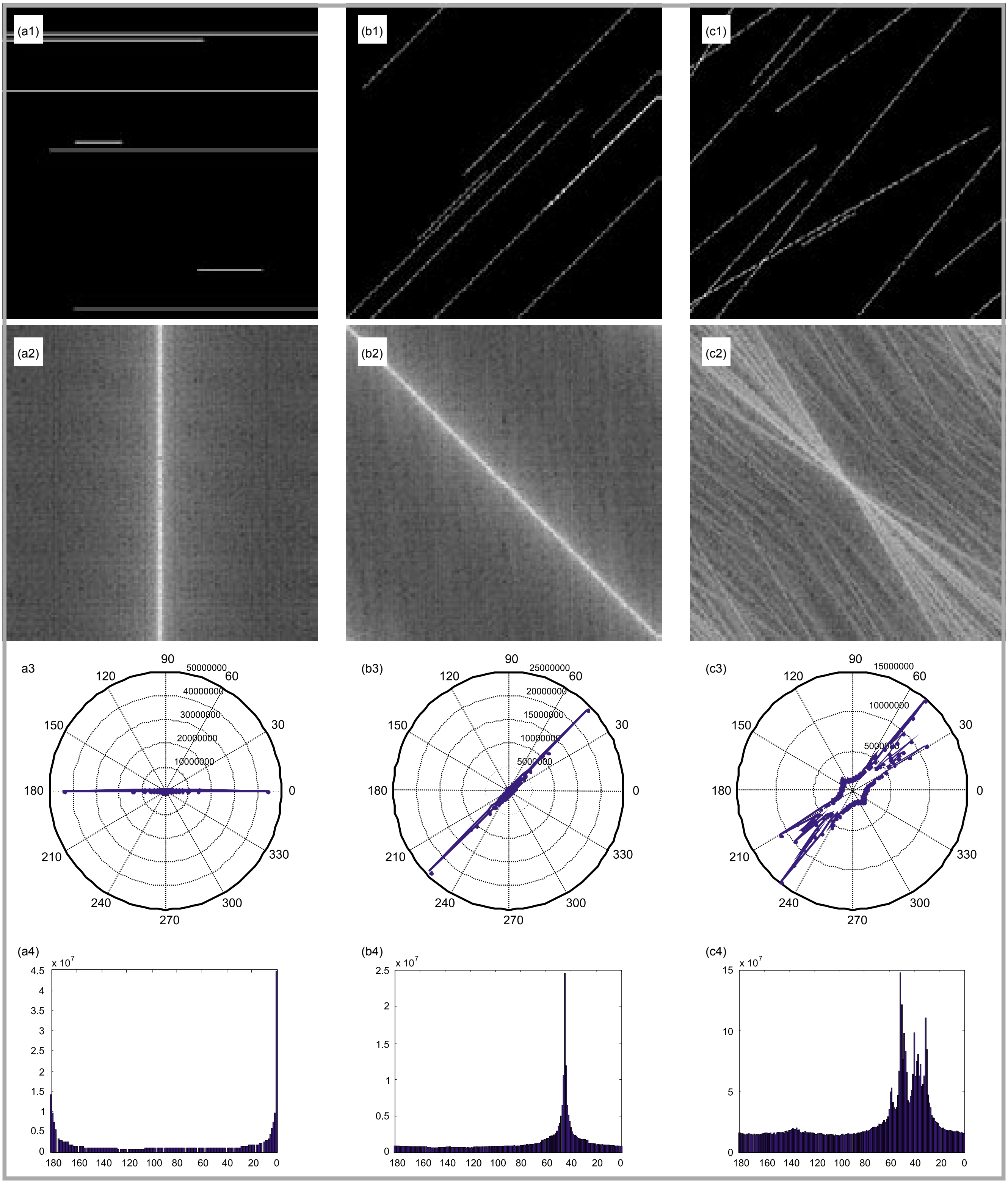

Figure 1. *(a1) - (c1) Binary images of simulated structural lines, (a2) - (c2) power spectrum as an intensity image, (a3) - (c3) polar plot of S_α, (a4) - (c4) histogram of S_α.*

levels is converted

$$P(u,v) = |F(u,v)|^2 \quad (4)$$

$$P(u,v) = \log(1 + |F(u,v)|^2) \quad (5)$$

If *f(x,y)* is real, its Fourier transform is conjugated symmetrically around the origin, that is

$$F(u,v) = F^*(-u,-v) \quad (6)$$

which implies that the Fourier spectrum is also symmetric around the origin

$$|F(u,v)| = |F(-u,-v)| \quad (7)$$

Figures 1 (a1) - (c1) represent binary images of simulated structural lines in the 0° direction, 45° direction, in the interval 30° - 60°, respectively. The length, position and orientation of the lines were randomly generated from uniform distribution. Figures 1 (a2) - (c2) show power spectrums scaled into 256 grey levels.

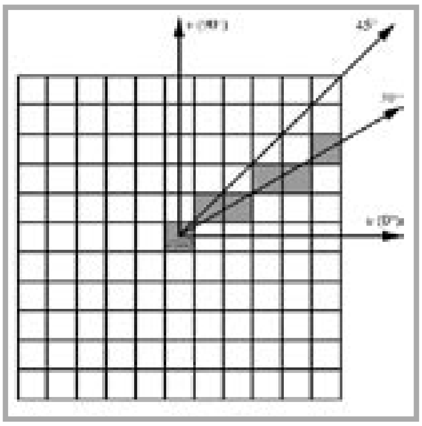

Figure 2. *Coordinates for directional vector dependent on* α *= 30°.*

As can be seen from these figures, information about the direction of major structural lines in the spatial domain is concentrated in the Fourier domain image as the direction of corresponding large magnitude frequency components (represented by white colour).

Assumptions

Let the image matrix be a square matrix of size $M \times M$. Let M be an odd number - it is convenient for the specification of the origin of the Fourier spectrum, and image matrix be scaled to 8 – bit grey levels (monochromatic image). All frequency components from the Fourier frequency spectrum are summarised together in the directional vector of certain angle α. Since the transform of real image function $f(x,y)$ is complex, the absolute magnitudes of frequency components $|F(u,v)|$ are obtained according to relation (3). The sum of frequency components S_α in the directional vector is given by

$$S_\alpha = \sum_{i=1}^{(M+1)/2} |F(u,v)| \qquad (8)$$

where α forms an angle between the directional vector and u axis, $|F(u,v)|$ is a frequency component of the directional vector at the coordinates (u,v) and M is the size of the image.

Computation of directional vector coordinates

As can be seen from equation (7), the Fourier frequency spectrum is symmetric around the origin; it is sufficient to add up the frequency components of directional vectors depending on α in the interval $(0, \pi)$, i.e. to specify that coordinates for the I. and II. quadrant. are symmetric around the ordinate (v axis, $\pi/2$), that is $(u,v) = (-u,v)$, therefore the determination of coordinates for I. quadrant suffices

$$0 \le \alpha \le \frac{\pi}{4} \quad \rightarrow \quad v = u\tan\alpha$$
$$\frac{\pi}{4} < \alpha \le \frac{\pi}{2} \quad \rightarrow \quad u = \frac{v}{\tan\alpha}, \qquad (9)$$

Here u is the abscissa axis or column number, v is the ordinate or row number and coordinates (u,v) are rounded to the closest integer, because the coordinates acquire an integer discrete value. The DC (Direct Current) component is the origin of frequency domain $F(0,0)$, and represents the origin of the system of coordinates. Figure 2 displays an example of coordinates for directional vector in I. quadrant, $\alpha = 30°$.

For an estimation of the rose of directions the magnitude of S_α is plotted onto the polar diagram and consequently into the histogram. The algorithm realising the method proposed was created in MATLAB programming language (Image Processing Toolbox). Input parameters are an image matrix and the output is the visualisation of the direction arrangement of objects in the form of a polar plot of S_α and histogram of S_α, which can be seen as the estimate of the rose of direction.

Figure 3 (a) displays the binary image of simulated structural lines from Figure 1 (c1) and corresponding estimate of the rose of direction achieved by means of the Steiner compact in six directions $\alpha_k = k\pi/6$ for $k = 1, \dots, 5$. The red line in Figure 3 (c) displays the estimate of the rose of direction, also in six directions, and Figure 3 (d) in directions with one-degree step using image analysis with the aid of Fourier transform. Figures 1 (a3) - (c3) display the polar plot of S_α and represent the estimation of function $f(\alpha)$ (rose of directions), and Figure 1 (a4) - (c4) display the histogram of S_α for the binary images from the Figure 1 (a1) - (c1).

Figures 4 (a1) - (c1) show grey level images of nanofibres with a randomly distributed structure, captured by a screnning electron microscope. Figure 4 (a2) - (c.2) represent a corresponding power spectrum, Figure 4 (a3) - (c3) is a polar plot of S_α and Figure 4 (a4) - (c4) is the estimate of the rose of directions by means of the Steiner compact. As can be seen from the polar plot, the image struc-

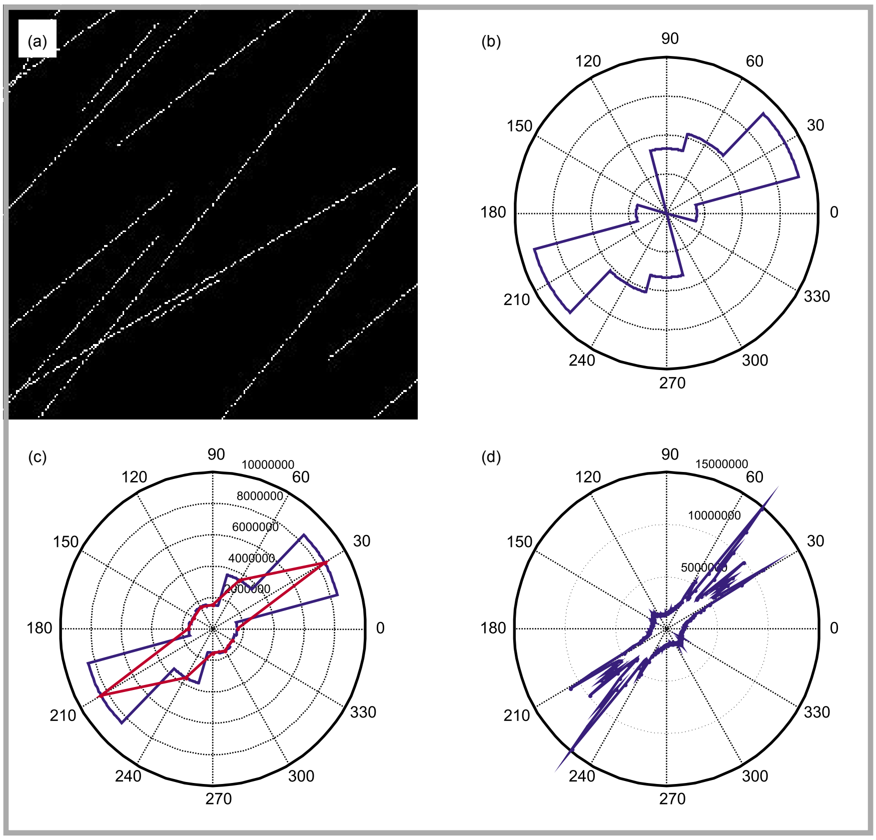

Figure 3. *(a) Simulated fibre system, (b) estimation of the rose of directions by means of Steiner compact, (c) estimation of the rose of directions by using the Fourier transform, plot with 30 degree step, (d) estimation of the rose of directions by using the Fourier transform, plotted with 1 degree step.*

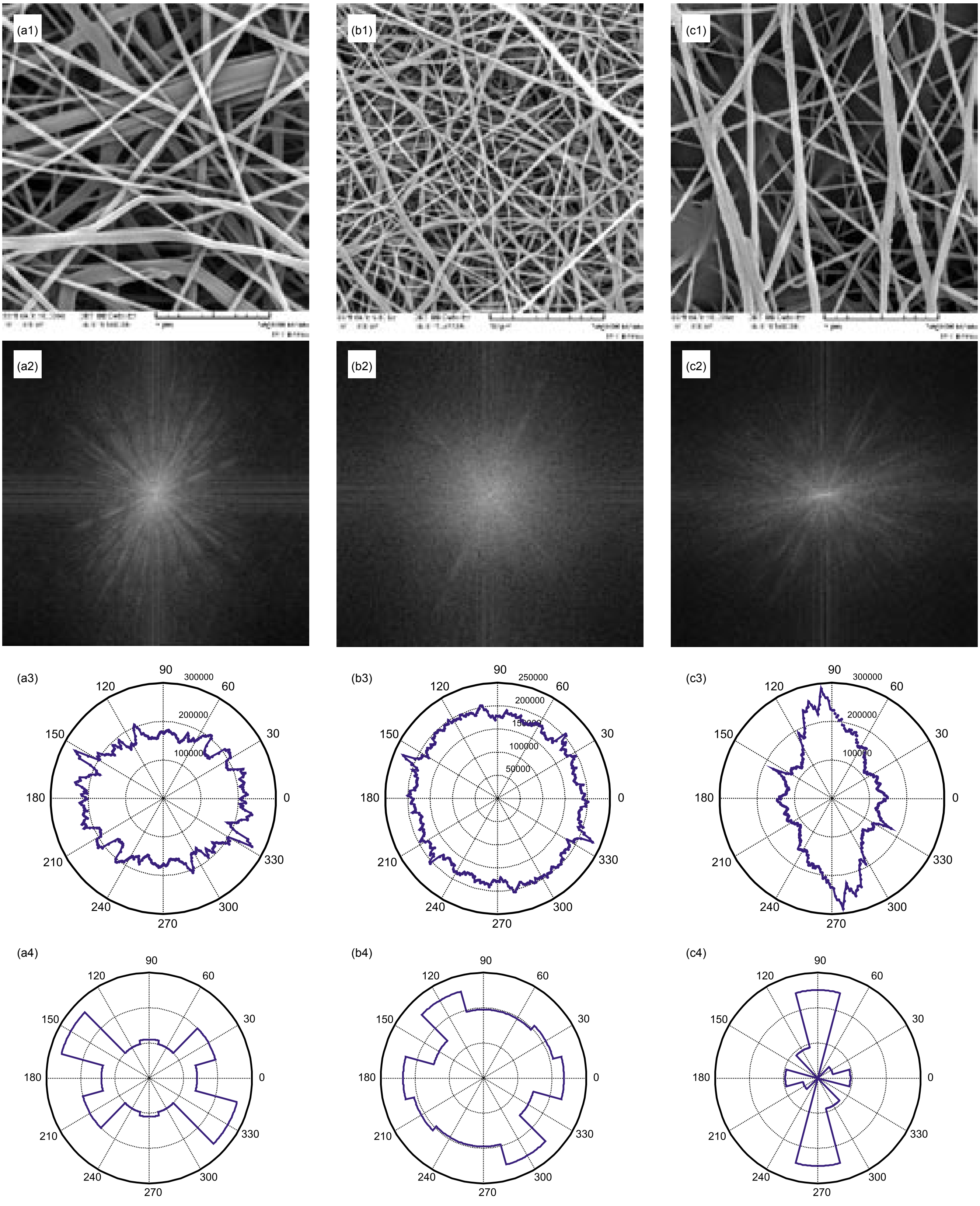

Figure 4. *(a1) - (c1) Textured images, (a2) - (c2) power spectrum as an intensity image, (a3) - (c3) polar plot of* S_α*, (a4) - (c4) estimation of the rose of directions by means of the Steiner compact.*

ture of the nanofibres in Figure 4 (a), and (b) is almost isotropic, but the structure in Figure 4 (c) shows a preference for the directional placement of fibres in a 90°- 120° direction.

Figure 5 (a1) is a grey level image of random Gaussian noise as an example of the isotropic system. The magnitudes of S_α are uniformly distributed along the whole spectrum of angles, which can be seen from the polar plot of S_α in Figure 5 (a2). Figure 5 (b1) displays a system of viscose fibres with preferred directions of orientation between the 0° - 30° and Figure 5 (c.1) is an image of a real fabric in plain weave with a tilted warp set of yarns.

Conclusion

This paper presents a simple graphical method of planar anisotropy analysis for fibre systems. The advantage of this method is its fastness; results are directly available after the acquisition of image and application of algorithm. The visualization of anisotropy is obtained in the form of a polar diagram and histogram. The polar diagram can be seen as an estimate of the rose of directions or function $f(\alpha)$. It is possible to monitor directional vectors with an angular step of 1°. Method can be used for the analysis of anisotropy of other systems, too.

Acknowledgement

This work was supported by project MSMT CR No. 1M06047 and by the Czech Science Foundation under grant No. 106/03/H150.

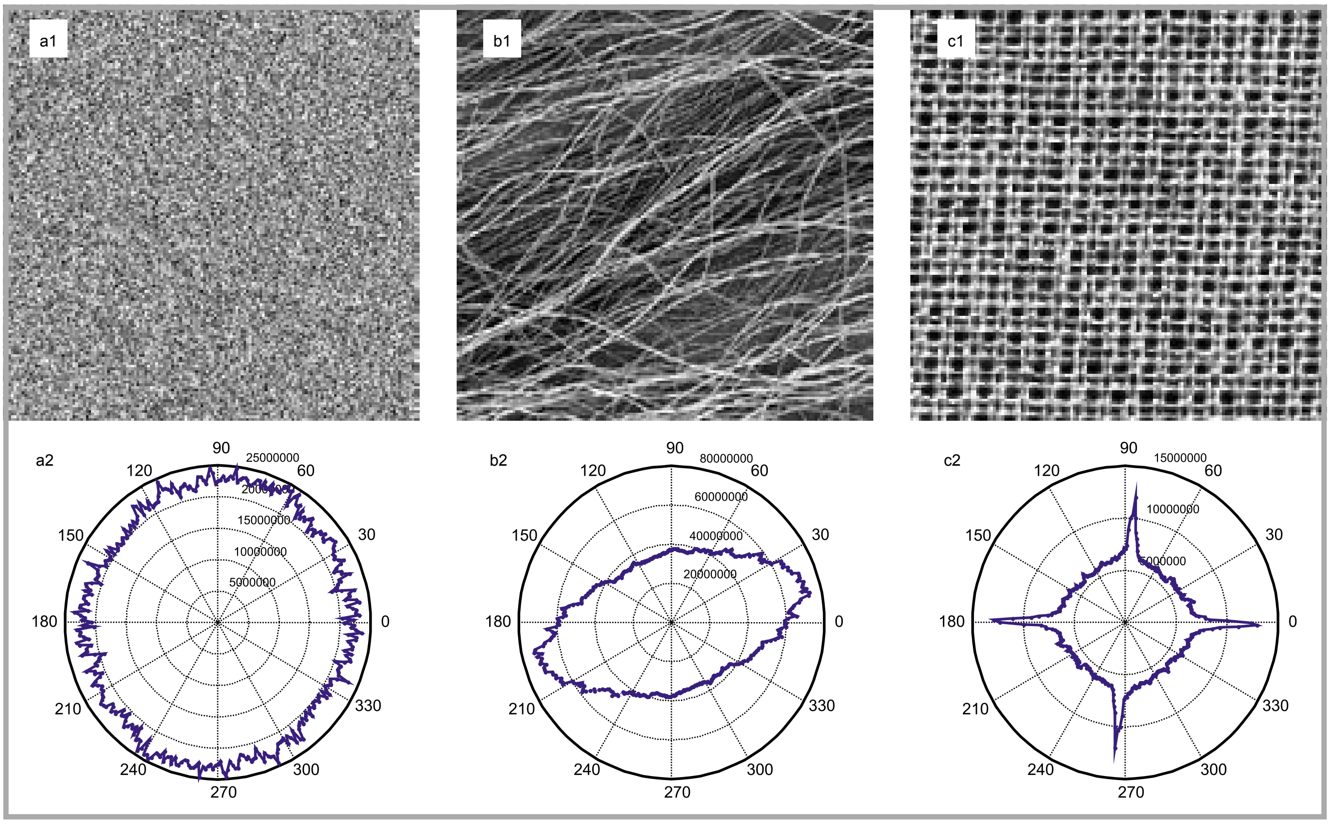

***Figure 5.** (a.1) - (c.1) Textured images, (a.2) - (c.2) polar plot of S_α.*

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Received 15.11.2007 Reviewed 15.01.2008