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Thermal Properties of Functionally Graded Fibre Material

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Abstract
In this paper the problem of modelling graded materials in the form of a fibre composite with varying fibre diameter is considered. The aim of modelling was to determine the micro and macroscopic thermal properties of this type of material, in which the average thermal conductivity in relation to fibre saturation changes was calculated at any point of fibre FGM, and then the effective thermal conductivity of a whole layer of the material was determined. To do that, a unit cell of the material of given structure was isolated and the one-dimensional heat flux passing through it was considered. As an effect of the investigation, the procedure of effective thermal conductivities calculation was presented and illustrated with a numerical example. Additionally the discrete and continuous approach to the effective thermal conductivities calculations were analysed and compared.

Key words: *functionally graded material, fiber composites, graded materials modeling, effective thermal conductivity.*

Introduction

Heat transfer problems are widely considered in many areas of engineering. For example, the human need for thermal comfort leads to constructing proper building materials [1, 2] and parts of clothing [3, 4]; knowledge of the behaviour of structures under an applied thermal load can lead to identification of their material or structural features [5, 6]; the manufacturing process can require strictly designed thermal conditions to perform a technological process correctly [7, 8]; the operating conditions of machine parts can force engineers to create a material structure highly resistant to thermal stresses [9].

In many areas of engineering, thermal stresses play an important role. In some engineering structures like simple and complicated elements of machines, car engines, turbine blades, aerospace structures and energy conversion systems, which work at high and non-uniform temperature fields, the main importance is to design thermal resistant structures. The necessity of designing material for a longer lifetime implies the application of new technologically advanced materials. In addition to well-known and commonly used classic materials of uniform mechanical and thermal properties, functionally graded materials are also utilised.

Functionally graded materials (FGM) are a new class of composite materials where the composition of components generates continuous and smooth gradation of properties of a composite. Multiphase composites with a continuously varying volume of fractions are characterised by smoothly changing mechanical and thermal properties. The concept of functionally graded materials was proposed in the early 1980s by materials scientists in Japan as a means of preparing thermal barrier materials [10].

The new generation of functionally graded fibre materials has dynamic, effective thermal properties and the volume fraction of the materials changes gradiently. The non-homogeneous, variable microstructures of these materials cause continuously graded macroscopic properties, such as the thermal conductivity, specific heat, mass density and elastic modulus. These materials have been developed as super-resistant materials in order to decrease thermal stresses and increase the effect of protection from heat [11, 12].

According to manufacturing techniques, FGM may exhibit orthotropic or anisotropic material properties due to practical engineering requirements. Apart from their main advantage of good heat resistance, they are characterised by very low density, resulting in lightweight structures with very good thermo-mechanical properties and small operating costs [13, 14].

There are different kinds of fabrication processes for producing functionally graded materials. Each of the manufacturing processes of gradient materials is adapted to the type, shape and size of material, as well as to the value of the gradient of material variations or the microstructure of gradient components [15]. Functionally graded materials are always manufactured by mixing two different material phases, for example metal and ceramic or fibres and resin. The concept is to make a composite material by varying the microstructure from one material to another with a specific gradient. This enables the material to have varying, designed properties. The reasonably low thermal stresses allow us to create high thermal resistance materials which can work in changing environmental conditions.

The most commonly used gradient materials made of fibre composites are those in which fibres have a constant diameter, but their saturation can be different. Graded materials can be also created from regularly arranged fibres of variable diameter.

Glass and carbon fibres are the most popular classes of filling fibres. Glass fibres are commonly used due to their low cost and excellent properties. Carbon fibres are one of the most important classes of filling fibres because of their properties, such as high stiffness, high tensile strength, low weight, high temperature tolerance and low thermal expansion. These features make them very popular in many engineering applications. However, carbon fibres are relatively expensive in comparison to filling fibres, such as glass or plastic fibres [16, 17]. By controlling the density of the arrangement of fibres or their diameter, it is possible to influence the durability of some structural parts. An example of such usage of gradient material can be a beam structure made of composite material filled with fibres of constant diameter but with different saturation of the matrix or beam filled with fibres of variable diameter. In this case the reinforcing fibres can be more densely arranged in parts more distant from the neutral axis. Proper arrangement of filling fibres enables better usage of their components, which means a more expensive and usually more resistant fraction can have greater saturation at points of higher stresses. Generally, this type of construction material can be used as structural elements of buildings, such as beams, plates or shell elements In addition to the mechanical properties of fibre FGM, its thermal properties can also be important when considering thermal comfort inside buildings.

In this paper the effective thermal conductivity of a layer of given thickness built from fibre FGM in relation to varying fibre saturation was determined. To do that, the average thermal conductivity at an arbitrary point of the structure was considered. In the investigations it was assumed that long parallelly-placed fibres of variable diameter are arranged in a regular hexagonal microstructure. With respect to the real, discrete structure of fibre FGM (built with many layers of composite material), the thermal behaviour of a discrete material structure was compared with that of the gradient material considered as a continuum.

Problem formulation

Let us consider two-dimensional steady state heat transfer for a body built with orthotropic fibre FGM material. To describe the behaviour of the body at each point, we can use typical relations described by a heat equation and Fourier's law [18]:

$$div\mathbf{q} + f = 0,\ q = -\boldsymbol{\lambda}_e \nabla T \text{ in } \Omega \qquad (1)$$

Where, $\mathbf{q}$ and f denote the heat flux intensity and heat source, respectively, $\boldsymbol{\lambda}_e$ - the matrix of average thermal conductivity coefficients (following from properties of the matrix and fibre materials), and ∇T denotes the gradient of the temperature field. The main problem for FGM is the knowledge of elements of the matrix $\boldsymbol{\lambda}_e$ at each point of the body domain. In the Cartesian coordinate system for orthotropic axes of the material, we have to know elements λ_{ex} and λ_{ey} of the main diagonal of the $\boldsymbol{\lambda}_e$ matrix. Additionally to be able to solve the problem, we have to know proper boundary conditions characteristic for environmental conditions of the structure analysed describing, for example, the temperature, heat flux intensity or convection in proper parts of the boundary (***Figure 1***). The conditions mentioned can be written in the form:

$$T = T^0 \text{ on } \Gamma_T,$$
$$q_n = \mathbf{qn} = q_n^0 \text{ on } \Gamma_q, \qquad (2)$$
$$q_n = h(T - T_\infty) \text{ on } \Gamma_c$$

where, T^0 & q_n^0 denote prescribed values of the temperature and heat flux, h – the convection coefficient, T_∞ - the environmental temperature, and $\mathbf{n}$ is the normal unit vector of the boundary line at a chosen point.

In many practical applications we should know the effective thermal conductivity for an element of given thickness t in relation to fibre saturation changes, which characterises the total thermal resistance of a layer built from fibre FGM. In this case we are interested, in fact, in one-dimensional heat flux (as shown in ***Figure 2***) of prescribed temperature T_0 and effective temperature T_t, for instance. Knowledge of this coefficient gives us the possibility to consider the thermal properties of fibre FGM as constant in relation to fibre saturation changes, and leads to the simplification of typical practical calculations.

Average thermal conductivities for fibre FGM

Fibre FGM is a mixture of matrix and fibres. To determine the average thermal conductivity of this type of material we should know how to determine the thermal resistance of composite material when heat flux passes through their components arranged in parallel or in a serial manner, and how to take into account the shape of intrusions. The fibre arrangement, cross section shapes and fibre saturation in the matrix play a significant role for thermal properties of the whole material.

Parallel and serial arrangement of components

The goal now is to determine the effective thermal conductivities for one-dimensional heat flow for composites built with many layers of different, thermally isotropic materials. The problem is well known and was solved many years ago, but its awareness is really important for further investigations carried out in this paper.

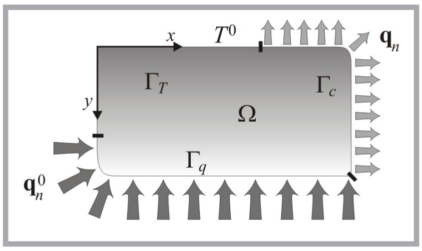


***Figure 1**. FGM structure subjected to service loading.*

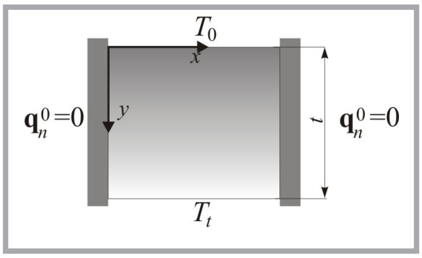


***Figure 2.** One-dimensional heat flow through FGM structure.*

Let us consider steady state heat transfer through a multilayer structure of unit length, built with n layers of different thicknesses t_i and thermal conductivities λ_i, as shown in ***Figure 3.a*** (see page 70).

First we consider the one-dimensional heat flow perpendicular to the layer direction, ***Figure 3.b***. Taking into account that the flux conducted through layers is the same in each layer of the discrete structure and should be the same in homogenised material, we can write:

$$\lambda_1 \frac{\Delta T_{01}}{t_1} = \lambda_2 \frac{\Delta T_{12}}{t_2} = \ldots = \\ = \lambda_n \frac{\Delta T_{n-1\,n}}{t_n} = \lambda_{sy} \frac{\Delta T_{0\,n}}{\sum_{i=1}^{n} t_i} \qquad (3)$$

where, ΔT_{ij} denotes the temperature jump between *i-th* and *j-th* surfaces, and λ_{sx} - the effective thermal conductivity of the composite in the y direction. Noting that

$$\sum_{i=1}^{n} \Delta T_{i-1\,i} = \Delta T_{0\,n} \qquad (4)$$

we can rearrange relationships (3) to obtain the effective thermal conductivity and write it in the form:

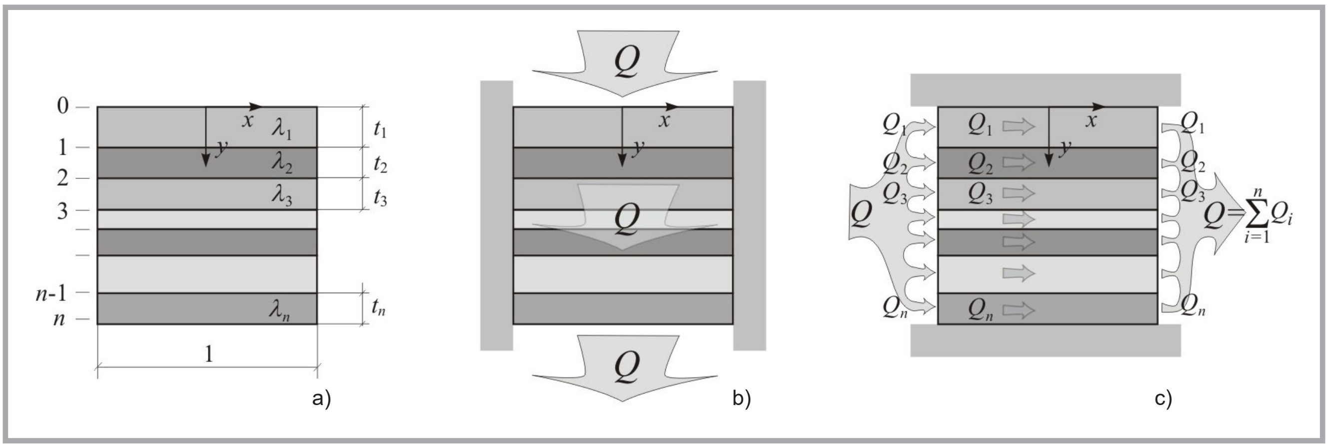


Figure 3. a) sandwich structure, b) heat flow through elements connected in series, c) heat flow through elements connected in parallel.

$$\lambda_{sy} = \frac{\sum_{i=1}^{n} t_i}{\sum_{i=1}^{n} \frac{t_i}{\lambda_i}} \quad (5)$$

In the case of one-dimensional heat flow parallel to the layers (***Figure 3.c***), the total flux passing through the structure is equal to the sum of fluxes passing through all layers, and should be equal to the flux passing through the homogenised material. Consequently assuming an additionally constant temperature at each boundary, we can write:

$$\Delta T \sum_{i=1}^{n} \lambda_i t_i = \Delta T \lambda_{sx} \sum_{i=1}^{n} t_i \quad (6)$$

to finally obtain the effective thermal conductivity in the y direction in the form:

$$\lambda_{sx} = \frac{\sum_{i=1}^{n} \lambda_i t_i}{\sum_{i=1}^{n} t_i} \quad (7)$$

Average thermal conductivities at an arbitrary point of fibre FGM

Let us take into account a repeatable structure of functionally graded material filled with long fibres of changing diameter and arranged in a hexagonal structure, as shown in ***Figure 4***. The main goal is to determine the average thermal conductivity coefficient of FGM in the y direction. To do that, we have to define its value at each point of the structure.

To determine the average thermal conductivity coefficient at an arbitrary point, we isolate the repeatable unit cell, shown in the ***Figure 4,*** and we connect it to the local coordinate system $\xi - \eta$. With respect to symmetricity of the cell, we consider only one-forth of the cell (***Figure 4***).

Because of the different possible diameters of filling fibres, we can observe two different arrangements of the element considered. In the first case (***Figure 5.a***) we can observe the overlapping of fibres ($r < r_1 + r_2$), and in the second case (***Figure 5.b***) a matrix material layer exists between fibres ($r > r_1 + r_2$).

In the first case, similar to what was done in [19], the average thermal conductivity coefficient at the y (cf. ***Figure 4***) coordinate point can be calculated as a parallel connection of three parts of the cell: *mf*, *fmf* & *fm* (***Figure 5.a***). Using the proper notation in (7) characteristic for our unit cell, we can write:

$$\lambda_{y1}(y) = \frac{\lambda_{mf1}(r - r_2) + \lambda_{fmf}(r_1 + r_2 - r) + \lambda_{fm1}(r - r_1)}{r} \quad (8)$$

where, λ_{mf1}, λ_{fmf}, λ_{fm1} denote the effective thermal conductivity of *mf*, *fmf* & *fm* parts, respectively.

All effective thermal conductivities mentioned in ***Equation 8*** can be calculated in a similar way. Let us consider the procedure of calculation of the λ_{fmf} coefficient. To calculate it, we can write the total heat balance for the infinitesimal section $d\xi$, shown in ***Figure 6,*** or simply use ***Equation 5*** applying the proper notation. Adapting ***Equation 5,*** we can write:

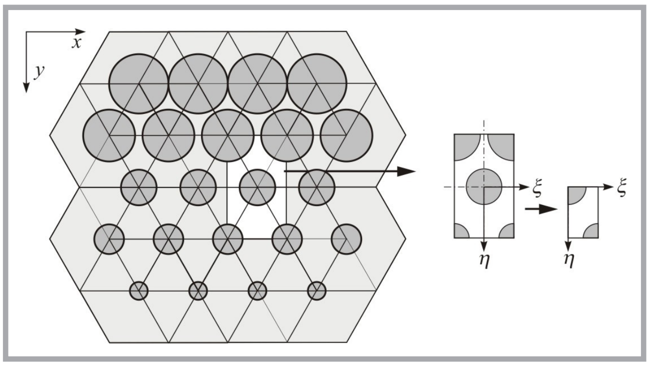


Figure 4. Hexagonal repeatable arrangement of fibers in FGM and unite cell.

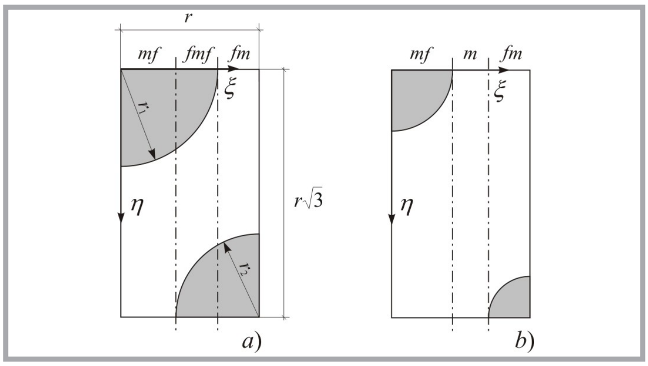


Figure 5. The one forth of the unit cell; a) with overlapping fibers, $r < r_1 + r_2$, b) with not overlapping fibers, $r > r_1 + r_2$.

$$\lambda_{fmf\,\xi}(\xi) = \frac{AD}{\frac{1}{\lambda_f}\left(CD + AB\right) + \frac{1}{\lambda_m} BC} \quad (9)$$

In this way we obtain the substituting thermal conductivity coefficient for element $d\xi$ as a function of the proper lengths of the element considered as well the thermal conductivities of matrix λ_m and fibre materials λ_f. Noting that $CD = \sqrt{r_1^2 - \xi^2}$, $BA = \sqrt{r_2^2 - (r - \xi)^2}$ and $AD = CD + BC + AB = r\sqrt{3}$, relation (9) can be rewritten in the form:

$$\lambda_{fmf\,\xi}(\xi) = \frac{r\sqrt{3}}{\left(\frac{1}{\lambda_f} - \frac{1}{\lambda_m}\right)\left(\sqrt{r_1^2 - \xi^2} + \sqrt{r_2^2 - (r - \xi)^2}\right) + \frac{r\sqrt{3}}{\lambda_m}} \quad (10)$$

Knowing that the total heat flux passing through the part of the cell considered is the same in real and homogenised material, we can write ***Equation 11***:

$$\int_{r-r_2}^{r_1} \lambda_{fmf\,\xi} \frac{\Delta T}{AD} d\xi = \lambda_{fmf} \frac{\Delta T}{AD}\left(r_1 + r_2 - r\right), \quad (11)$$

where, ΔT is the jump in temperature between points A and D of the unit cell, which is assumed to be constant along the ξ axis in the cell considered. Consequently we can write:

$$\lambda_{fmf} = \frac{1}{r_1 + r_2 - r} \int_{r-r_2}^{r_1} \lambda_{fmf\,\xi} d\xi \quad (12)$$

The missing coefficients in (8) can be obtained in a similar way. Consequently substituting the thermal conductivity in the infinitesimal section $d\xi$ for parts *mf* and *fm* can be expressed as follows:

$$\lambda_{mf\,\xi}(\xi) = \frac{r\sqrt{3}}{\left(\frac{1}{\lambda_f} - \frac{1}{\lambda_m}\right)\left(\sqrt{r_1^2 - \xi^2}\right) + \frac{r\sqrt{3}}{\lambda_m}}$$
$$\lambda_{fm\,\xi}(\xi) = \frac{r\sqrt{3}}{\left(\frac{1}{\lambda_f} - \frac{1}{\lambda_m}\right)\left(\sqrt{r_2^2 - (r - \xi)^2}\right) + \frac{r\sqrt{3}}{\lambda_m}} \quad (13)$$

and effective thermal conductivities required have the form:

$$\lambda_{mf1} = \frac{1}{r - r_2} \int_0^{r-r_2} \lambda_{mf\,\xi} d\xi,$$
$$\lambda_{fm1} = \frac{1}{r - r_1} \int_{r_1}^{r} \lambda_{mf\,\xi} d\xi \quad (14)$$

Generally the integrals in ***Equation 12*** and ***Equation 14*** have to be calculated in a numerical manner.

In the second case (***Figure 5.b***), the average thermal conductivity coefficient can be calculated similarly and can be written in the form:

$$\lambda_{y2}(y) = \frac{\lambda_{mf2} r_1 + \lambda_m (r - r_1 - r_2) + \lambda_{fm2} r_2}{r} \quad (15)$$

Where, λ_{mf2} and λ_{fm2} are determined as follows:

$$\lambda_{mf2} = \frac{1}{r_1} \int_0^{r_1} \lambda_{mf\,\xi} d\xi,$$
$$\lambda_{fm2} = \frac{1}{r_2} \int_{r-r_2}^{r} \lambda_{mf\,\xi} d\xi \quad (16)$$

The average thermal conductivity coefficient for the whole cell (at an arbitrary point of FGM for a y coordinate) is easily determined from ***Equation 5*** in the serial connection of the upper and lower one-fourth of cell parts (cf. ***Figure 4***):

$$\lambda_{ay}(y) = \frac{\lambda_{yi} + \lambda_{yj}}{\lambda_{yi}\lambda_{yj}} \quad (17)$$

where, i and j are equal to 1 or 2 according to the character of the upper and lower one-fourth of the cell.

If necessary, we can obtain the average thermal conductivity λ_{ax} for an arbitrary y coordinate in the same manner. It is worth noting that with respect to the fibre material arrangement, coefficients λ_{ax} and λ_{ay} for the chosen y are constant along the x direction.

Effective thermal conductivity for a layer of fibre FGM

Having determined the average thermal conductivity coefficients at each point of the material, we can determine the effective thermal conductivity coefficient of the whole layer. Let us assume that the layer of the material has thickness t (as shown in ***Figure 2***) and we know function $\lambda_{ey}(y)$. According to Fourier's law, the intensity of heat flux in the y direction is equal to:

$$q_y = \lambda_{ay}(y) \frac{dT}{dy} \quad (18)$$

Separating the variables and integrating both sides with limits $0 \le y \le t$, we obtain:

$$q_y = \frac{T_t - T_0}{\int_0^t \lambda_{ay}^{-1} dy}, \quad (19)$$

where, T_0 and T_t denote the temperatures on the upper and lower surface of the layer, respectively. Comparing the heat flux passing in the y direction through section dx in real and homogenised material, we can write:

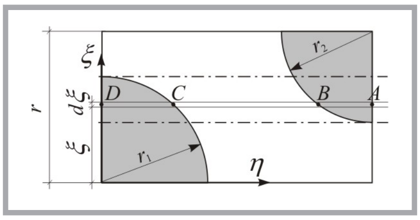


***Figure 6**. The one-forth of the unit cell with singled infinitesimal section dy.*

$$\frac{T_t - T_0}{\int_0^t \lambda_{ay}^{-1} dy} dx = \lambda_{ey} \frac{T_t - T_0}{t} dx, \quad (20)$$

where, λ_{ey} denotes the effective thermal conductivity in the y direction of the FGM layer analysed, which can be easily determined as:

$$\lambda_{ey} = \frac{t}{\int_0^t \lambda_{ay}^{-1} dy} \quad (21)$$

The expression ***Equation 21*** can be used when we treat the material structure as continuous. The FGM material structure is in fact discrete. Consequently instead of ***Equation 21***, knowing the real structure of FGM connected with the manufacturing process (diameters of fibres, the number of fibre layers, etc.), we can also use ***Equation 5*** to determine the effective thermal conductivity required.

Numerical examples

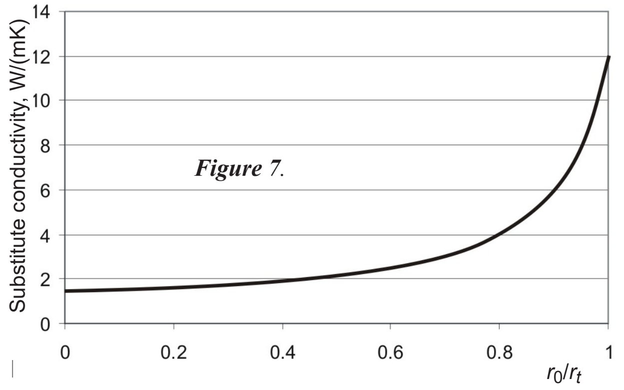
The main goal of this section is to illustrate the approach to effective thermal conductivity calculations proposed and next to compare the discrete and continuous approach for their calculation.

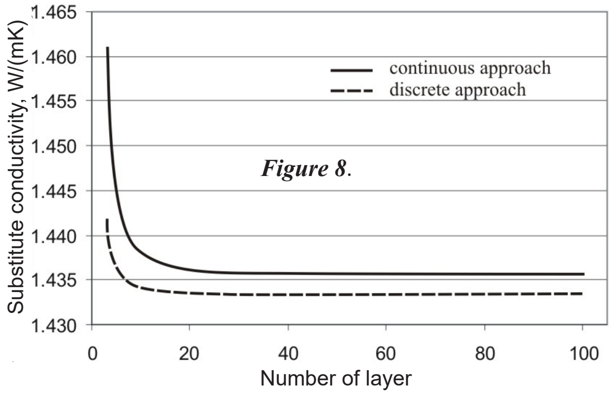
Numerical example of calculation of the effective thermal conductivity for a layer of FGM

To show the ability of the method of effective thermal conductivity calculation proposed (according to ***Equation 21***), a numerical example was realised. It was assumed that the radius of fibres is known at any point of the material and can by described by the function $R(y)$. In our case this function was assumed in the linear form:

$$R(y) = \frac{r_t - r_0}{t} y + r_0 \quad (22)$$

where, r_t and r_0 denote the radius of fibre at points $y = t$ and $y = 0$ - which means on external boundaries of lamina of thickness t. Calculations were performed for changing ratio r_0/r_t assuming the maximal possible size $r_t = r$ (cf. ***Figure 6***). In particular, it was assumed: r = 0.5 mm,





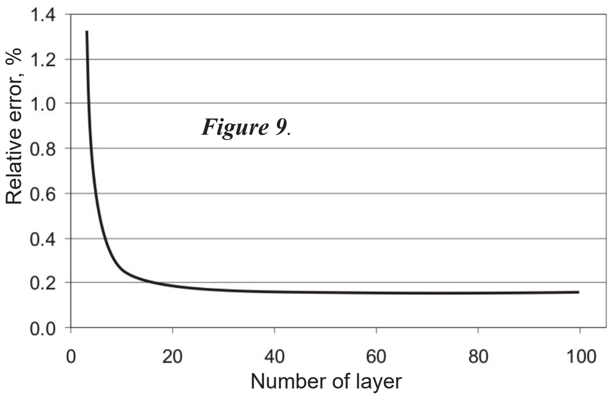


***Figure 7**. The effective thermal conductivity vs. changes of fibers' radius along lamina thickness.*

***Figure 8.** The effective thermal conductivities for fiber FGM.*

***Figure 9.** The relative error of calculation of the effective thermal conductivity obtained as a result of discrete approach vs. continuous approach.*

number of layers 20, λ_m = 1 W/(mK), and λ_f = 25 W/(mK). A plot of the effective thermal conductivity obtained is shown in ***Figure 7***.

From geometrical investigations of the unit cell proposed, we can see that the maximal saturation of fibre material in the matrix (for r_0/r_t = 1 and $r_t = r$) is equal to $\pi/(2\sqrt{3}) \approx 0.9069$, which allows to create a composite with effective thermal conductivity approximately two times lower than for pure fibre material.

Comparison of discrete and continuous approaches to calculation of effective thermal conductivity coefficients

Even if we know the real discrete structure of a fibre FGM, we treat it as continuous material. The question is how good this approximation is and when we can treat a discrete material as continuous. The question is similar to that when we treat fibre composites as uniform homogenised material forgetting about their inner structure.

In the case of fibre FGM considered, the effective thermal conductivity coefficient can be obtained using the discrete (cf. ***Equation 5***) or continuous approach (cf. ***Equation 21***). The question now is how many layers are needed to treat a discrete structure as a continuous one.

To examine this, we can do numerical tests using ***Equation 5*** or ***Equation 21***, where the effective thermal conductivity coefficient at an arbitrary point is calculated in the same manner using ***Equation 17***.

In the case of the discrete approach, we have to calculate the effective thermal conductivity coefficient in each layer according to ***Equation 5***.

In the case of the continuous approach, we have to calculate, in a numerical manner, the integral appearing in ***Equation 21***. In this case we forget about the real material structure, but we are interested in the thermal conductivity coefficient at some points according to the method of numerical calculations chosen – in our case Gauss points. The structure of the unit cell (the diameters of fibres) results from the assumed function $R(y)$ of fibre diameter variability, and can be described at any point and not only in real fibre positions. This approach creates virtual (not existing) fibres which are used in the calculation process.

Numerical tests were carried out for lamina composed of 3 to 100 layers. Each layer had a constant thickness resulting from the assumed dimension r = 0.5 mm. Additionally it was assumed in ***Equation 22*** that $r_t = r$ and r_0 = 0. This assumption causes a change in fibre saturation in the matrix from the maximum to minimum possible value. During numerical integration in ***Equation 21*** the number of Gauss points was assumed to be equal to 2, which means that the integral in ***Equation 21*** was calculated as a result of calculation λ_y at two points, and function course $\lambda_y(y)$ was approximated with a third order polynomial. Plots of the effective thermal conductivities versus the number of layers for the discrete and continuous approaches are shown in ***Figure 8***. The relative difference between the coefficients obtained using both approaches is depicted in ***Figure 9***.

The continuous approach gives a really small relative error in comparison to the discrete one. For a number of layers equal to 3 the error is smaller than 1.5%, and for a material built from more than 15 layers it is always smaller than 0.2%. The greater the number of Gauss points used in numerical calculations, the better the accuracy that can be achieved;

however, it seems not to be necessary with respect to satisfactory precision of calculations for a number of Gauss points equal to 2.

Conclusion remark

In the paper, a simple way of average thermal conductivity calculation of FGM filled with long parallely-arranged fibres was presented. Next the effective thermal conductivity for a layer of fibre FGM was determined in the gradient direction of fibre diameter changes. Numerical calculations were carried out for a material of linear variability of fibre diameter, assuming different velocity of fibre radius changes across the thickness of the material sample.

Additionally it was shown that in the case of thermal properties, lamina consisting of three or more layers can be treated as continuous with good accuracy. Consequently there is no need to consider all layers of lamina because we can homogenise them in one structure with the prescribed function defining the variability of the fibre diameter, and then treat it as continuous.

The effectiveness of the proposed method of calculation of effective thermal conductivities of fibre FGM is defined by the number of elementary mathematical operations which have to be done to calculate proper integrals. Assuming numerical integration using five Gauss points to calculate the average thermal conductivity at a given point and two Gauss points to calculate the effective thermal coefficient of the whole layer of fibre FGM, we need a little more then one hundred elementary mathematical operations. In the real world of numerical calculations, the time of execution of this number of calculations is negligibly small.

To verify the accuracy of this approach we need practical tests. In the case of insufficient compatibility of practical test results with numerical calculations, the method proposed should be revised taking into account the stochastic approach to fibre arrangements in the unit cell.

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