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Periodic and Chaotic Motion in Sirofil Yarn Spinning

Abstract

A dynamic model is established for two-strand yarn processing. The oscillating frequencies of Sirofil spinning in vertical and horizontal directions are approximately obtained. The stability and chaotic characters of Sirofil spinning are analysed analytically, and an optimal condition for the spinning system is suggested.

Key words: *two-strand spinning, Sirospun, Sirofil, chaotic motion.*

■ **Introduction**

Sirofil has been widely used to produce composite yams with a low twist factor in a single operation during yarn spinning. Sirofil yarns are stronger and have larger extensibility and rupture energy than corresponding normal ring spun yarns or air-

vortex spinning [1, 2]. More importantly, Sirofil yarns have a low initial modulus, and hence the yarns are softer than normal ring spun yarns [3]. Furthermore, the strands are texturised to improve the bulk of the resultant yarns, which have been demonstrated to possess more desirable properties, for example the weaveability of fabric formed withSirofil yarns (See Figure 1) is significantly better than its counterparts.

In our previous study [4 - 7], the linear and nonlinear dynamical characters of Sirospun were studied using the homotopy perturbation method [8, 9] or the variational method [8, 10]. In this paper, we will develop a dynamical model for Sirofil spinning. The characteristics of two-strand spun yarns mainly depends on how the two strands are combined and mixed. The different oscillating frequencies of the convergent point in both vertical and horizontal directions have been proven to be the dominant factors [6].

■ **Dynamical model**

Figure 1 illustrates two-strand yarn spinning. We first assume the system is in a stable condition. Using our quasistatic model [4], the convergent point can be determined with ease. Due to some perturbations, the convergent point (equilibrium position, *O* in Figure 2) moves to an instantaneous new position (*O'*). The distances *x* and *y* are measured from the equilibrium position.

Let the ends of the two strands above the convergent point be fixed at a distance $L_1 + L_2$ apart, and the equilibrium position be *H* below. The equations of the motion in *x*- and *y*-directions are

$$\begin{cases} M\dfrac{d^2x}{dt^2} + F_1\cos\alpha - F_2\cos\beta = 0 \\ M\dfrac{d^2y}{dt^2} + F_1\sin\alpha + F_2\sin\beta - F = 0 \end{cases} \quad (1)$$

Here *M* is the total mass of a fixed control volume *ABC,D* illustrated in Figure 3; the control volume is chosen in such a way that the mass centre coincides with the convergent point (*O*) of the two strands.

If *x* and *y* are much smaller than $\sqrt{L_1^2 + H^2}$ and $\sqrt{L_2^2 + H^2}$, and we then apply the binomial theorem to expand the square-root terms, we have approximately (equation 1.a)
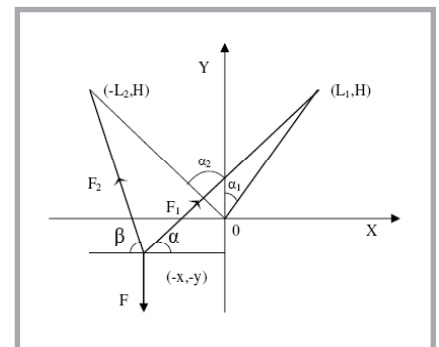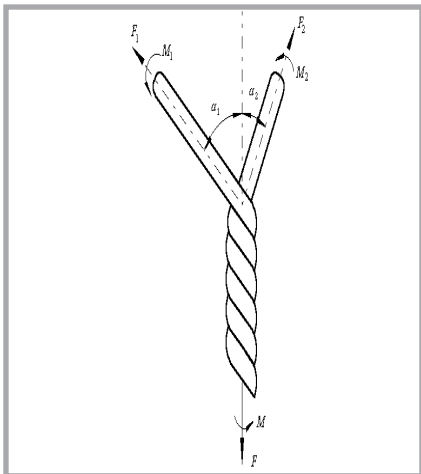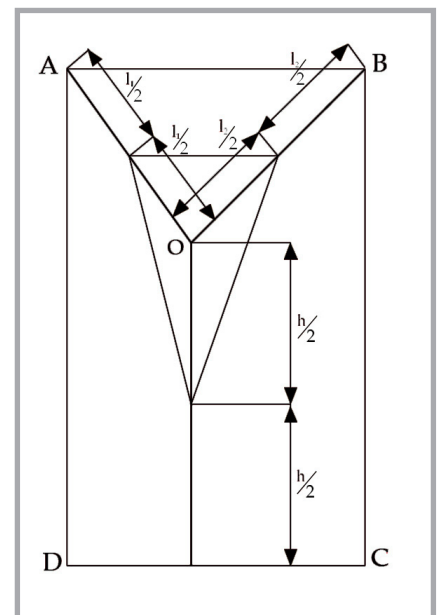

Figure 2. *The dynamical illustration of two-strand spun??*

Figure 3. *The control volume, the mass centre at the convergent point.*

Figure 1. *Two-Strand Yarn Spinning (Sirofil Spinning).*

Figure 4. Closed trajectories of the convergent point under different conditions;
a) $p_1 : p_2 = 2 : 1$,
b) $p_1 : p_2 = 3 : 1$,
c) $p_1 : p_2 = 4 : 1$,
d) $p_1 : p_2 = 5 : 1$,
e) $p_1 : p_2 = 6 : 1$,
f) $p_1 : p_2 = 4 : 3$,
g) $p_1 : p_2 = 3 : 2$.

Figure 5. Chaotic motion of the system;
a) $p_1 : p_2 = \pi : 1$, b) $p_1 : p_2 = 2 \pi : 1$,
c) $p_1 : p_2 = 5 \pi : 4$, d) $p_1 : p_2 = 4 : \pi..$
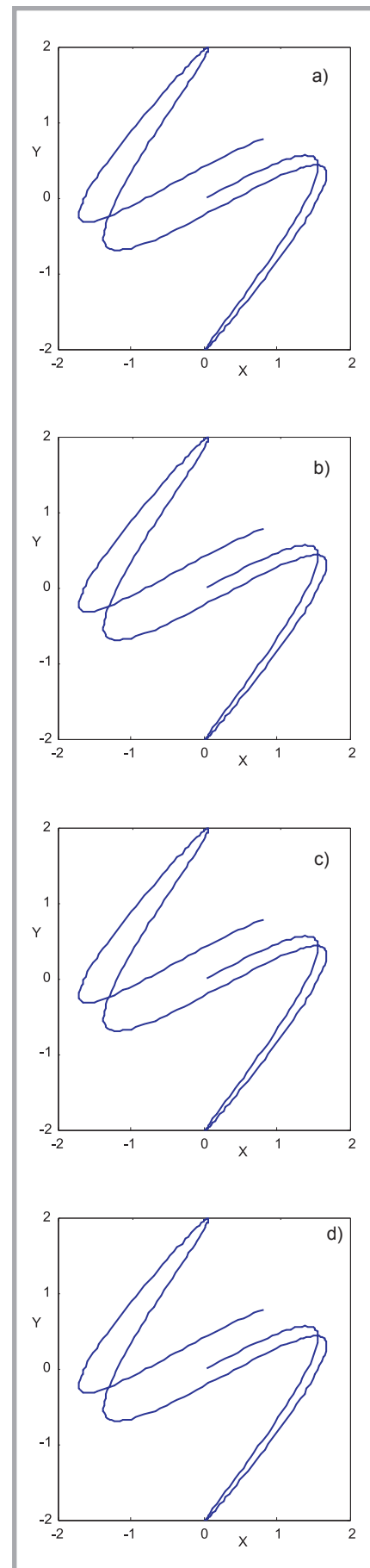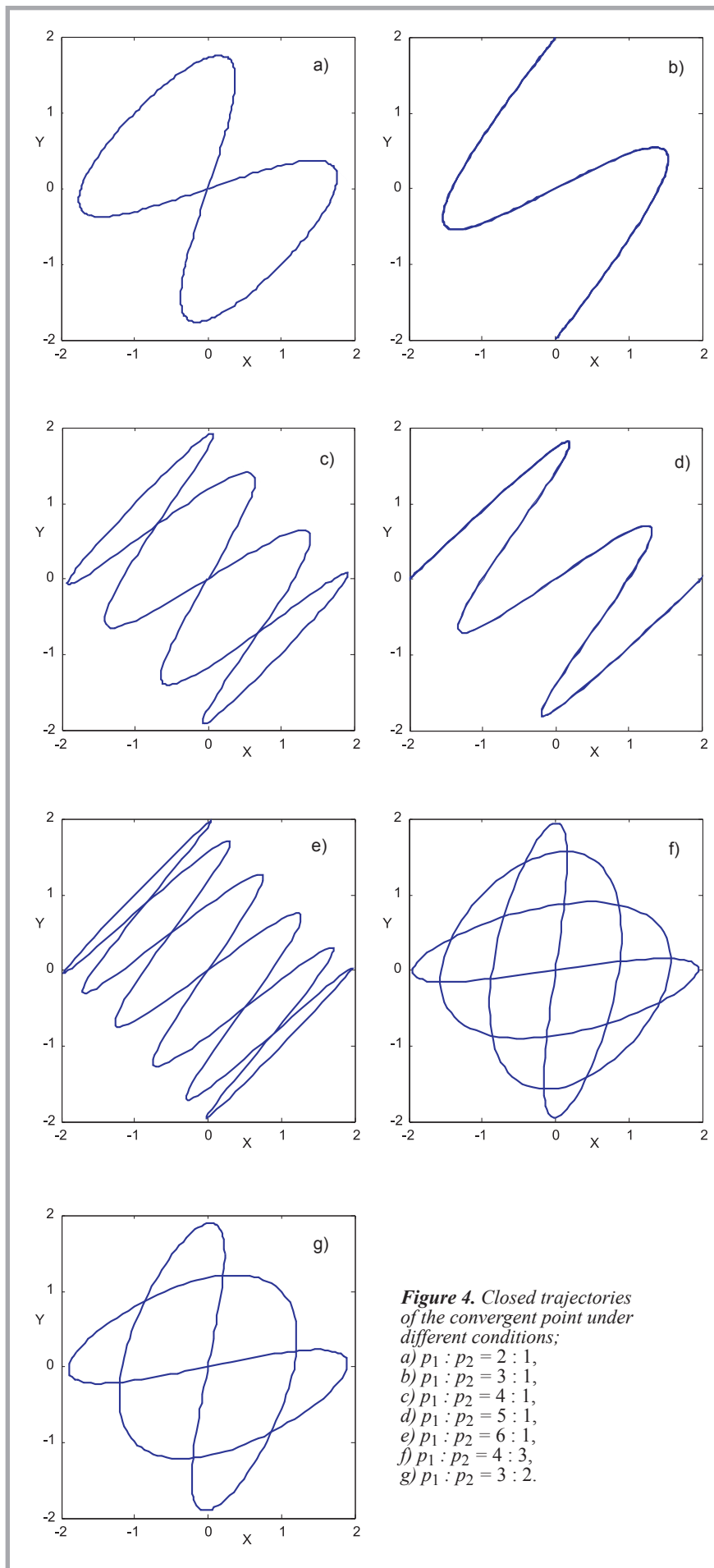
Equation (1) becomes

$$\begin{cases} \ddot{x} + \omega_1^2 x + by = 0 \\ \ddot{y} + \omega_2^2 x + bx = 0 \end{cases} \quad (2)$$

where

$$\omega_1^2 = \frac{F_1 \cos^3 \alpha_1 + F_2 \cos^3 \alpha_2}{MH},$$

$$b = \frac{F_2 \sin \alpha_2 \cos^2 \alpha_2 - F_1 \sin \alpha_1 \cos^2 \alpha_1}{MH},$$

$$\omega_2^2 = \frac{F_1 \sin^2 \alpha_1 \cos \alpha_1 + F_2 \sin^2 \alpha_2 \cos \alpha_2}{MH}$$

In the case of $b = 0$, i.e., $\alpha_1 = \alpha_2$ and $F_1 = F_2$, the system turns out to be Sirospun spinning, which was discussed in [5, 6].

Solving Equations (2), we can obtain the following solutions:

$$\begin{aligned} x &= a \sin p_1 t + b \sin p_2 t \\ y &= c \sin p_1 t + d \sin p_2 t \end{aligned} \quad (3)$$

where

$$p_1 = \sqrt{\frac{\omega_1^2 + \omega_2^2 + \sqrt{\left(\omega_1^2 - \omega_2^2\right)^2 + 4b^2}}{2}}$$

$$p_2 = \sqrt{\frac{\omega_1^2 + \omega_2^2 - \sqrt{\left(\omega_1^2 - \omega_2^2\right)^2 + 4b^2}}{2}} \quad (4)$$

$a, b, c,$ and d are constants which can be determined by the initial conditions. The trajectories of the convergent point are illustrated in Figure 4.

From Figure 4 we know that we have closed trajectories when p_1/p_2 is an integer or can lead to a ratio of two integer numbers, i.e., $p_1/p_2 = n/m$, where m and n are integer numbers. Closed trajectory means that the convergent point, O, moves periodically.

Chaotic motion occurs when ω_1/ω_2 or p_1/p_2 are not rational numbers as illustrated in Figure 5.

$$\begin{cases} \cos\alpha = \dfrac{L_1 + x}{\sqrt{(L_1 + x)^2 + (H + y)^2}} = \cos\alpha_1 - \dfrac{\sin\alpha_1 \cos^2\alpha_1}{H}x + \dfrac{\sin^2\alpha_1 \cos\alpha_1}{H}y \\ \cos\beta = \dfrac{L_2 - x}{\sqrt{(L_2 - x)^2 + (H + y)^2}} = \sin\alpha_1 + \dfrac{\cos^3\alpha_1}{H}x - \dfrac{\sin\alpha_1 \cos^2\alpha_1}{H}y \end{cases}$$

and $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ *1.a*

$$\begin{cases} \sin\alpha = \dfrac{H + y}{\sqrt{(L_1 + x)^2 + (H + y)^2}} = \sin\alpha_2 - \dfrac{x}{H}\cos^3\alpha_2 - \dfrac{y}{H}\sin\alpha_2 \cos^2\alpha_2 \\ \sin\beta = \dfrac{H + y}{\sqrt{(L_2 - x)^2 + (H + y)^2}} = \cos\alpha_2 + \dfrac{\sin\alpha_2 \cos^2\alpha_2}{H}x + \dfrac{\sin^2\alpha_2 \cos\alpha_2}{H}y \end{cases}$$

■ Conclusion

To conclude, we obtained conditions for stability ($b=0$ for Sirospun spinning), periodic and chaotic motion in the two-strand spun process. The results show that by suitably choosing certain geometry parameters and forces, the spinning process can be optimally matched. Chaotic motion should be avoided in practical applications as chaotic motion leads to yarns easily breaking.

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