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Distribution of Bending Stiffness of Orthotropic Fibrous Material Based on an Example of Corrugated Board

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Abstract

Commonly known methods for calculating the bending stiffness of corrugated board allow to calculate values related to the machine and cross direction of corrugated board but do not allow to calculate bending stiffness in any intermediate direction. The article presents a method for calculating bending stiffness in any direction in the plane of corrugated board, on the basis of the bending stiffness values in the machine and cross direction. Comparing the results of measurements of bending stiffness in selected directions with those of their calculations, the method proposed was verified and its practical usefulness confirmed.

Key words: *bending stiffness, corrugated board, fibrous material.*

Introduction

Despite the fact that fibrous materials show the characteristics of viscoelastic bodies, in many cases they are treated as elastic bodies. This is possible when elastic deformations dominate in the total deformation of the fibrous material, for example at low stress levels or during the short times of their interaction, when permanent deformations have negligibly small values. In practice, when testing papers and solid and corrugated boards, their properties are determined, such as Young's modulus and bending stiffness.

Machine-made fibrous materials in the form of a solid paper board show features of orthotropic bodies. This is due to the orientation of fibres, whose longitudinal axes have an orientation close to the direction of manufacture, referred to as the machine direction. The consequence of the fibre structure orientation is the diversification of mechanical properties in the plane of the web. In machine-made papers, the tensile and compressive strength of the web as well as Young's modulus in the machine direction are usually about twice as high as in the cross direction perpendicular to it; their distribution in the plane has been analysed in various research works [1]. The features mentioned are also characterised by laminates made from paper and solid board, of which the most complicated construction is that of corrugated board. It consists of flat and undulating layers, and for that reason it is not always easy to combine its properties with those of materials of individual laminate layers.

In the cardboard plane, two main directions of orthotropy, machine MD and transverse CD, can be distinguished, in which there are extreme values of mechanical properties of its component layers as well as the entire laminate.

One of the basic strength indicators used to assess the suitability of corrugated board for the production of packaging is bending stiffness, BS. There are normalised methods of bending stiffness measurements in the *MD* and *CD* main directions described in PN-ISO 5628: 1995. There are also known methods for calculating the bending stiffness in the main directions in the plane of corrugated board based on knowledge of the geometrical parameters of its cross-section and mechanical properties of the materials used for particular layers [2]. In most cases, these methods are sufficient to compare corrugated board when assessing their suitability as a construction material, or to calculate board packaging resistance to static pressure, *BCT*. This is due to the fact that usually the directions determined by the height, width and length of boxes coincide with the main directions in the board. For this reason, when calculating the packaging strength, the properties of board in the *MD* and *CD* directions are used. An example of this is the commonly used Mc'Kee [3] formulas or the other relationships used to calculate *BCT* [4, 5].

If the boards are subjected to bending moments in directions other than the main ones, information on the bending stiffness distribution in the cardboard plane is needed to calculate the strength of the packaging designed. In industrial practice, this information is not available, as *BS* measurements are not made in other than *MD* and *CD* directions.

This study proposes a method for calculating bending stiffness *BSa* in any direction in the board plane based on calculated or measured *BS* values in the machine direction, BS_{MD} and cross direction BS_{CD}.

Methodology

In papermaking, paper and cardboard bending stiffness is determined in relation to the width unit of the bent testing piece e, which in the case of bending in the direction inclined to the machine direction at angle α, in a simplified manner referred to as the direction α, can be written with the *Equation (1)*:

$$BS\alpha = \frac{E\alpha \cdot I\alpha}{b} \qquad (1)$$

where:

Eα – Young's modulus against *α*,
Iα – moment of inertia of a cross-section perpendicular to direction α, against the neutral axis when bending,
b – width of testing piece

In the case of corrugated board with *n* number of layers, its stiffness can be calculated from the relationship *Equation (2)*

$$BS_{\alpha} = \frac{1}{b} \sum_{i=1}^{n} E\alpha_i \cdot I\alpha_i \qquad (2)$$

where:

i – index denoting layer number,
$E\alpha_i$ – Young's i-th layer in the direction of *α*,
$I\alpha_i$ – moment of inertia of the i-th layer cross-section against the neutral axis of the board during bending.

In the method presented, it is difficult to determine the moment of inertia of the corrugated layer with respect to the axis of

the neutral cross-section of the bent corrugated board. This is due to the inability to accurately describe the flute type, which can be simplified, for example, into the form of a triangle [2] *Figure 1*.

This simplification allows the BS_{CD} value to be calculated with an accuracy of 10% [4], sufficient in industrial practice. In the machine direction, the influence of corrugated layers on the bending stiffness of the board is negligibly small, and by calculating the BS_{MD} using relationship (2), it can be omitted. If the corrugated layer is omitted when determining bending stiffness in the cross direction, it may cause the BS_{CD} value of the three-layer board to be understated by approx. 15% of the real value, and in the case of board with a larger number of layers, the error is even higher. Using the method discussed to calculate BS in directions other than main ones in the board plane, the problem is to determine the value of Young's modulus. In the case where the board is bent in a direction other than MD or CD, the values of Young's moduli of the corrugated layer in directions that are not parallel to the surface of the material for the corrugated layer are required to calculate the bending stiffness, and such tests are not performed. This causes that the use of relationship *Equation (2)* to determine bending stiffness in a direction other than MD and CD is difficult.

For this reason, the board model has been simplified by subjecting its cross-section to homogenisation, as a result of which a board with any number of layers can be treated as a homogeneous orthotropic plate with the main axes' orthotrophy overlapping the machine and the cross direction.

According to [6], taking into account the results of works [7, 8], approximate *Equations (3)* and *(4)* can be introduced to transform the BS stiffness values for rotated co-ordinates Ox y having data in the $O_{MD\ CD}$ system (*Figure 2*).

Equations (3) and *(4)*, where: α – rotation angle of the coordinate system, – correction factor determined experimentally.

Seven board types were used for the tests, the parameters of which are described in *Table 1*.

In order to verify the calculation methods, BS_a measurements were carried out in seven directions in the plane of each of the boards tested (*Figure 3*), ie in the
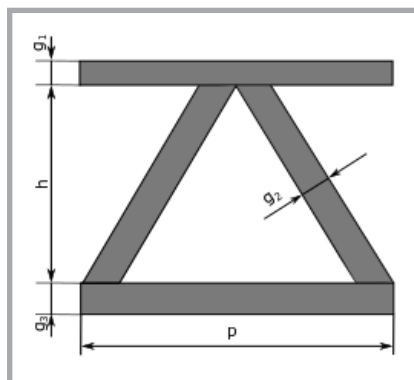
Figure 1. *Simplified cross-section of corrugated board: g_i – thickness of i-th layer, p – flutescale, h – flute height.*
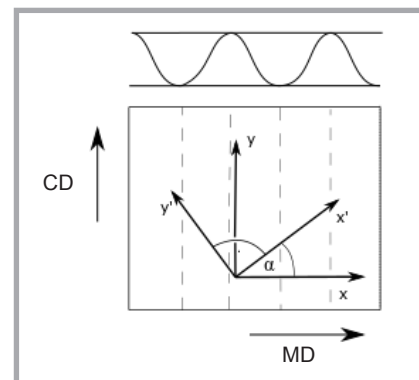
Figure 2. *Diagram of rotation of the coordinate system in the plane of the board.*
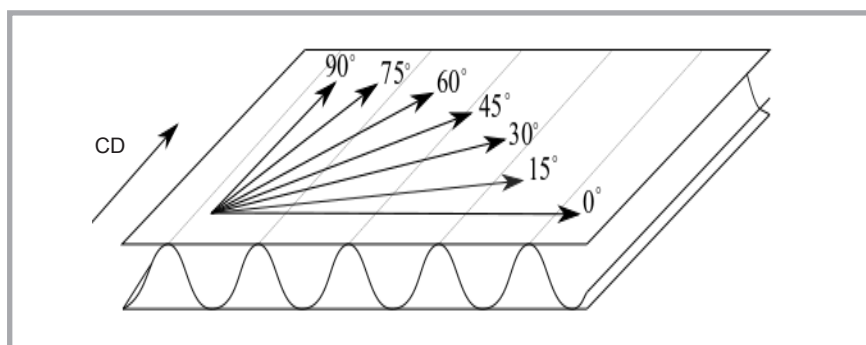
Figure 3. *Directions of cutting testing pieces for measurements of BSα values.*

MD, CD and five indirect directions deviating from the machine direction by 15°, 30°, 45°, 60° and 75°.

The following measurements were made in laboratory conditions:
– bending stiffness of corrugated boards, according to PN-ISO 5628:1995,
– Young's modulus of materials for layers of all boards, according to PN-EN ISO 1924-2:2010,
– thicknesses of board layers, according to Standard PN-EN ISO 534:2012,
– thicknesses of cardboard in accordance with PN-EN ISO 534:2012,
– scale and flute heights according to PN-90/P-50527.

Before testing, all test pieces were air-conditioned in accordance with the PN-EN 20187: 2000 standard in air at 23 °C and 50% relative humidity.

Table 1. *Board parameters.*

Board	Number of layers	Flute type	Grammage, g/m²
T1	3	B	443
T2	3	C	434
T3	3	C	560
T4	5	B/E	730
T5	5	B/E	854
T6	5	B/E	710
T7	5	C/C	760

$$\frac{1}{BS\alpha} = \frac{1}{BS_{MD}}\,cos^4\alpha + \beta\,\frac{1}{\sqrt{BS_{MD}BS_{CD}}}\,sin^2\alpha \cdot cos^2\alpha + \frac{1}{BS_{CD}}\,sin^4\alpha \qquad (3)$$

$$\frac{1}{BS\alpha} = \frac{1}{BS_{MD}}\,cos^4\alpha + \beta\,\frac{1}{\sqrt{BS_{MD}BS_{CD}}}\,sin^2\alpha \cdot cos^2\alpha + \frac{1}{BS_{CD}}\,sin^4\alpha \qquad (4)$$
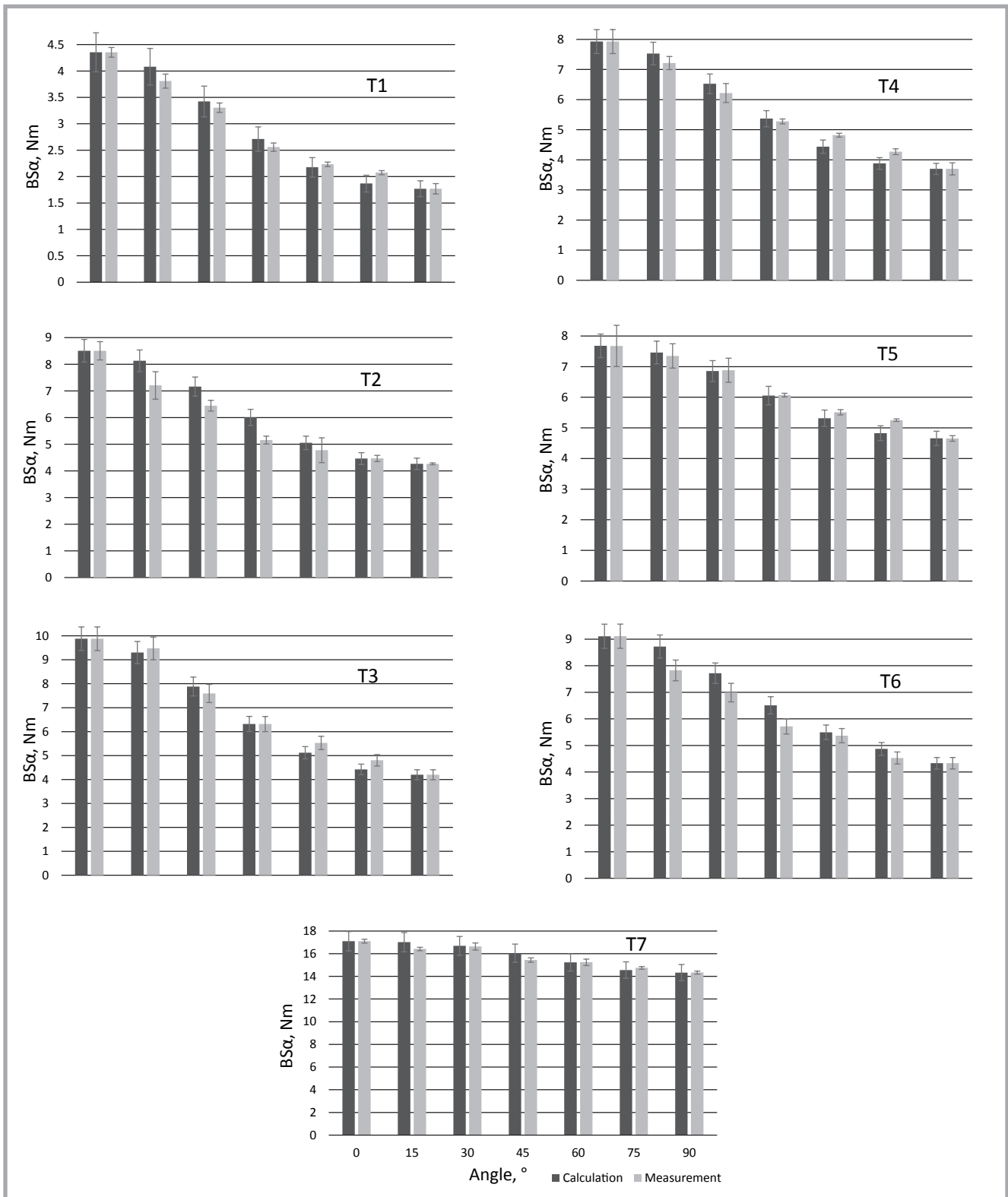
Equations (3) and (4).

Figure 4. *Results of measurements and calculations of bending stiffness made using measured values of BS_{MD} and BS_{CD}.*

By solving (3) against *ß* and for BSα substituting values obtained in the measurements, the values of coefficients for a particular board were determined, which ranged from 1.5 to 2.7. There was no dependence of the value of the coefficient on the number of cardboard layers,

and for this reason the value of coefficient ß = 1.9 was used, which was the same for all boards, giving the best compliance of the results of measurements and calculations. The value was then used for theoretical calculations of bending stiffness in selected directions.

By solving ***Equation (3)*** against *BSα*, a relationship was obtained, by means of which theoretical values of bending stiffness of all boards tested in all intermediate directions were determined.
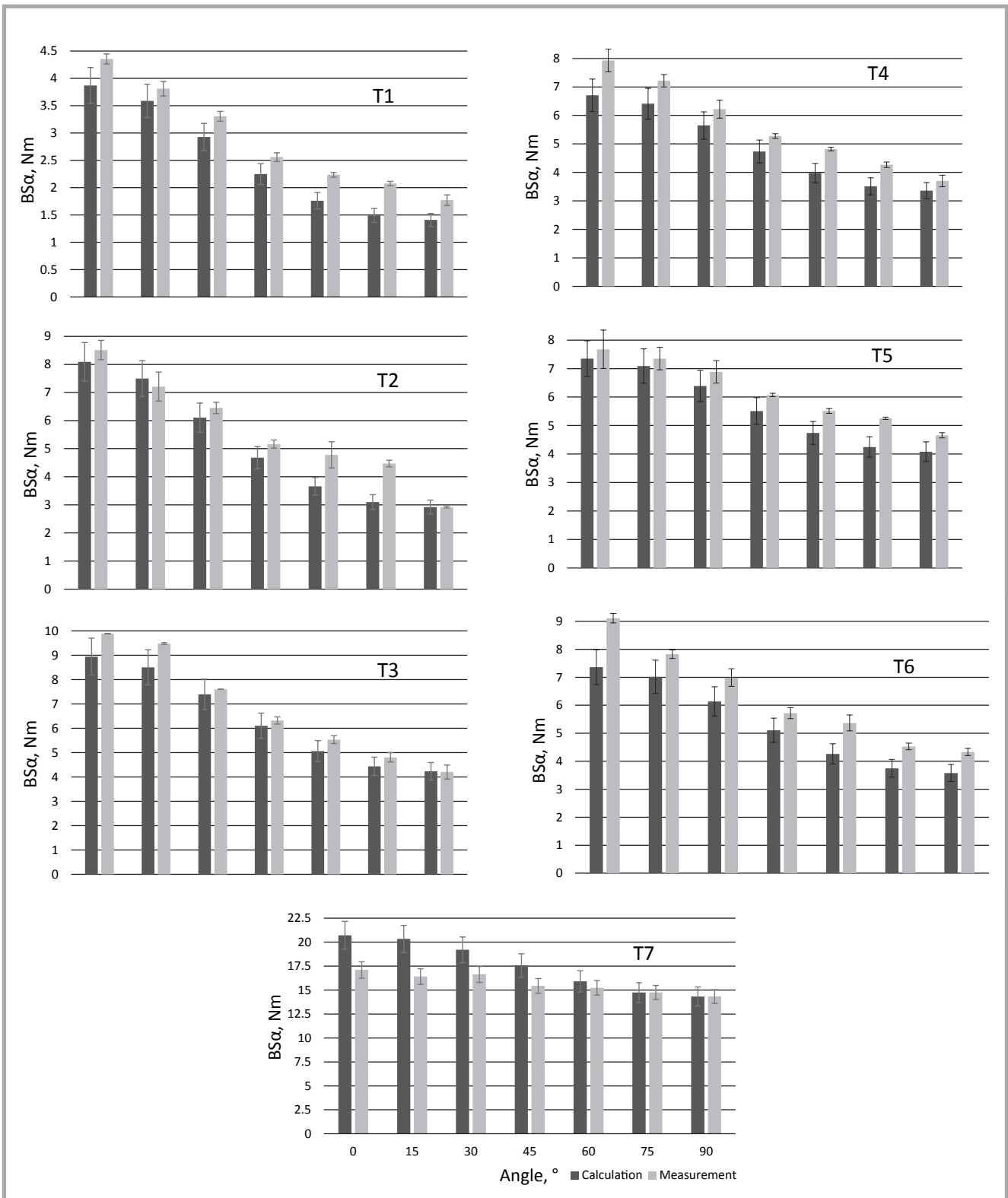
Figure 5. *Results of measurements and calculations of bending stiffness made using calculated values of BS_{MD} and BS_{CD}.*

The calculations were made twice to facilitate their differentiation in further parts of the study and will be referred to as the first and second series of calculations.

In the first series of calculations, BS_{MD} and BS_{CD} values determined during measurements were used. In the second series of calculations, BS_{MD} and BS_{CD} were calculated from relationship (2) using data on mechanical properties of the materials of individual layers and geometrical parameters of board cross-sections.

■ **Test results**

A comparison of measurement results and calculations made in the first series is illustrated in ***Figure 4***.

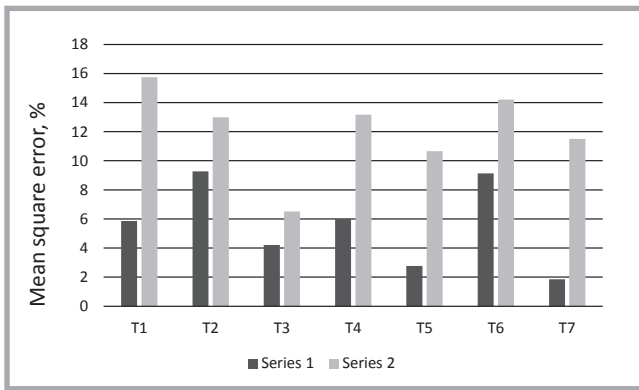A comparison of measurement results and calculations made in the second series is illustrated in ***Figure 5***.

In all cases in which the calculation of bending stiffnesses in intermediate directions was made using measured values of BS_{MD} and BS_{CD}, differences between the measurement results and calculations are within the limits of measurement and calculation errors.

The maximum difference between the measured and calculated values of bending stiffness in the intermediate directions in the case of calculations made using the measured values of BS_{MD} and BS_{CD} is about – 16% of measured values. The maximum difference between the measured and calculated values of bending stiffness in the intermediate directions in the case of calculations made using the calculated values of BS_{MD} and BS_{CD} is about 31% of measured values.

For each board and series of calculations, the mean square error of the BSα calculation in five intermediate directions was determined, the results of which obtained are shown in ***Figure 6***.

High discrepancies in the measured and theoretically determined values in the second series of calculations result from errors in the BS_{MD} and BS_{CD} bending stiffness calculations, which can be seen by comparing the measured and calculated values in the main directions in ***Figure 5***.

◼ Summary

The test results confirmed the practical applicability of the proposed method of calculating the bending stiffness of board in the intermediate directions between *CD* and *MD*, using known *BS* values in the main directions in the board plane.

In the case where the bending stiffness in the intermediate directions was calculated on the basis of measured values of BS_{MD} and BS_{CD}, the average square error of *BSα* calculation values in all intermediate directions in a given cardboard did not exceed 9.5%.

The use of the method for calculating the board stiffness in the design phase, connected with the need to calculate the BS_{MD} and BS_{CD} theoretically, gives worse results. In the cases examined, the average square error of BSα calculation values in all intermediate directions in a given board did not exceed 16%. ☐

References

1. Szewczyk W, Marynowski K, Tarnawski W. An Analysis of Young's Modulus Distribution in the Paper Plane. *FIBRES & TEXTILES in Eastern Europe* 2006; 14, 4: 91-94.
2. Szewczyk W. Nowe metody oceny nośności wielowarstwowych laminatów z papierów i tektur, Zeszyty naukowe nr 1027, Politechnika Łódzka, 2008.
3. Aloumi B, Alnashwan W, Malasri S, Othmani A, Kist M, Sampson N, Fotso R. Experimental Verification of McKee Formula. *International Journal of Advanced Packaging Technology* 2015; *3*(1): 129.
4. Schaffrath H J, Reichenbach F, Schabel S, Schaffrath H J, Fachgebiet PMVTU. Prediction of Box-Failure by Using Paper Data with Enhanced Mckee-Formula. *Progress in Paper Physics Seminar* 2016; p. 54.
5. Schaffrath H J, Reichenbach F, Schabel S. Prediction of Box Failure from Paper Data for Asymmetric Corrugated Board. *TAPPI JOURNAL* 2018; 17, 8: 429-434.
6. Jones R M. Mechanics of Composite Materials. 2/E, Taylor & Francis, USA, 1998.
7. Łęcka M, Mania R,. Marynowski K. Nowa metoda wyznaczania wytrzymałości na ściskanie pudeł z tektury falistej. *Przegląd Papierniczy* 2006; 62, 3: 150-153.
8. Kolakowski Z, Łęcka M, Mania R, Marynowski K. Forecasting of the Corrugated Board Buckling Based on the ECT and BCT Indexes, Polsko-Ukraińskie Seminarium *Theoretical Foundations of Civil Engeering*, 2002.