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Numerical Modelling of Geometrical Parameters of Textile Composites

Abstract
This paper describes a simple sinusoidal geometrical model of textile composites. On the basis of this model, how to obtain certain geometrical parameters that fully characterize the geometry of the composite structure will be presented. Using geometrical considerations, it is possible to obtain from this model basic mechanical parameters that are very useful for further strength analysis. Such mechanical considerations and a method for calculating mechanical parameters will be presented in future articles. On the basis of the above-mentioned theoretical considerations, a special computer program was developed. The method of calculation presented in this paper can be applied to more complicated models of textile composites.

Key words: *textile composites, textile mechanics, numerical methods, woven fabric, unit cell model.*

Introduction

Textile composites represent a class of advanced materials that are reinforced with textile preforms for structural or load-bearing applications. In general, composites can be defined as a select combination of dissimilar materials with a specific internal structure and external shape. The unique combination of two material components leads to particular mechanical properties and superior performance characteristics not possible with any of the components alone [1-4].

Many authors in their works have discussed the classical textile studies performed by Peirce, Kawabata and Leaf. Scida and others [5] presented a micromechanical model called MESOTEX for the prediction of the elastic behaviour of composites reinforced with non-hybrid weave (plain weave, satin weave and twill weave) and hybrid weave (hybrid plain weave and hybrid twill weave) fabrics. By using the classical thin laminate theory applied to each woven structure, this analytical model takes into account the strand undulations in the two directions and also integrates the geometrical and mechanical parameters of each constituent (resin, fill and warp strands). Woo and Whitcomb in [6] analysed macroelements for detailed stresses of woven textile composites. A symmetric plain weave unit cell configuration was studied to evaluate the performance of the procedure. The stress results were compared with those from other analyses.

Lomov, Gusakov and others [7] used a mathematical model of the internal geometry of 2D and 3D woven fabrics as a unit cell geometry preprocessor for meso-mechanical models of composite materials. The model computes a spatial placement of all the yarns in a fabric repeat for a given weave structure (a special coding algorithm is employed) and the given warp and weft yarns' geometrical and mechanical parameters.

In [8], Jiang, Tabiei and Simitses presented a stress and strain averaging procedure for local and global analysis of plain weave fabric composites. Osada, Nakai and Hamada [9] considered the final fractures of composites to be caused by cumulation of the microfractures, so that the initiation of a microfracture, namely, the initial fracture, is an important factor of which to know the mechanical properties. Microfracture behaviours in textile composites were decided by the geometry of the textile fabric quantitatively. First, in order to investigate the geometry of the textile fabric, the crimp ratio and aspect ratio were measured. Tensile testing was performed and the knee point on the stress-strain curve was identified. The arrangement, properties and structure of the fibres within the yarn and the yarns within the fabric generate a complex mechanism of deformation. Therefore, Turfaoui and Akesbi [10] developed a theoretical model of the mechanical behaviour of the plain weave. The modelling of textile structures by the finite element method was a new approach based on the combination of geometric and mechanical models. The finite element method permits a construction and representation of fabrics by taking into consideration the yarn undulation, the existence or not of symmetries in the basic cell and the type of contact between the warp and weft yarns.

The modelling of textile structures was also considered by Milašius in [12] and [13], where considerations are presented concerning the principles of woven fabric 3D structure formation and cross-section shapes. Kobza [14] presented the bending theory of multi-layered composites filled with soft materials.

The range of applications for composite materials appears to be limitless. Textile composites can be defined as the combination of a resin system with a textile fibre, yarn or fabric system. They may be either flexible or quite rigid. This paper will present how to build a simple geometrical model of textile composites and how to obtain from this model certain geometrical and mechanical parameters that are very useful for further strength analysis. As a novelty, in this study, the validity and limitations of the presented geometric model will be studied carefully. Besides, the usability of new iterative procedures in the Mathematica environment will be tested to extend the range of applications.

The geometrical model of textile composites

Let us consider a balanced plain weave textile composite in which the warp and fill yarns contain the same number of fibres n, with all the filaments having the same diameter d_f , and with the warp and fill yarns having the same yarn

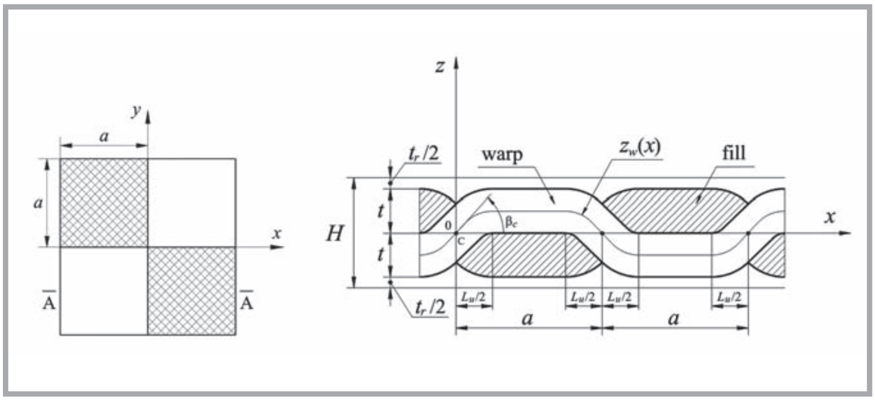

Figure 1. Cross section of the representative unit cell (RUC) along the warp yarn.

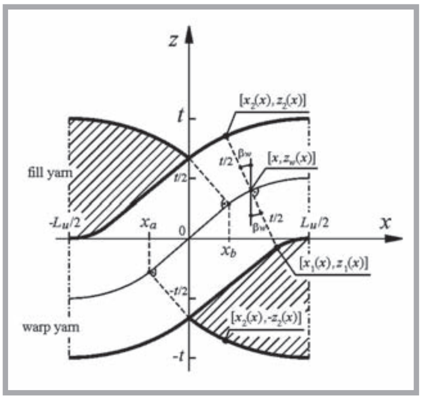

Figure 2. Geometry of an undulation region along the warp yarn.

packing density p_d. The yarn packing density is defined as $p_d = n \cdot A_f/A$, where $A_f = \pi\, d_f^2/4$ is the cross-sectional area of the filament. The cross-sectional area of the yarns is given by $A_f = (\pi\, d_f^2 n)/(4 p_d)$. The representative unit cell (RUC) is a rectangle consisting of two warp yarns interlaced with two fill yarns with resin matrix filling the remaining portion of the volume. Its dimensions are $2a \times 2a$ and its thickness is denoted by H. The other parameters are denoted and shown in ***Figure 1***, which shows the idealization of the real model. The idealizations are relatively simple because most warp and weft yarns are nominally straight. The thickness of the yarn along the centreline of the yarn path is denoted by t. If the fibre volume fraction specified for the unit cell is too small, it is necessary to add an additional resin layer of thickness t_r to the unit cell. The geometry of the path of the warp or fill yarn is modelled using two assumptions: 1 – the centreline of the yarn path consists of undulation portions and straight portions, with the centreline of the undulating portions described by the sine function, 2 – the cross-sectional area and the thickness of the yarn normal to its centreline are uniform along the arc length of the centreline. The centreline of the warp yarn path in an undulating region is specified by

$$z_w(x) = \frac{t}{2}\sin(\pi x / L_u)\,, \quad -L_u/2 \le x \le L_u/2\,. \tag{1}$$

The trigonometric functions of the angle β_w in terms of function $z_w(x)$ are

$$\cos\beta_w = [1 + (z'_w)^2]^{-1/2}\,,\quad \sin\beta_w = z'_w[1 + (z'_w)^2]^{-1/2}\,,\quad \tan\beta_w = dz_w/dx = z'_w\,. \tag{2}$$

$$\tan\beta_w = [\pi t/(2L_u)]\cos(\pi x/L_u)\,,$$
$$\text{but } \tan\beta_w = \tan\beta_c\cos(\pi x/L_u)\,,$$

where β_c is the so-called crimp angle (see ***Figure 1***).

$$\tan\beta_c = \tan\beta_w(x=0) = \pi t/(2L_u)\,. \tag{3}$$

The volume fraction of the fibres in the RUC is given by $V_f = (4 p_d A l)/(4a^2 H)$. For the warp yarns, the arc length is given by $l = 2\int_{-L_u/2}^{L_u/2}\sqrt{1+(z'_w)^2}\,dx + 2(a - L_u)$.

Using this and integrating, we have a relationship for the volume fraction:

$$V_f = [A\,p_d/(a\,t\,h_r)][1 - (L_u/a)\,g_1(\beta_c)]\,, \tag{4}$$

$$g_1(\beta_c) = 1 + 2\sqrt{1+\tan^2\beta_c}\;E(m)/\pi\,,\quad m = \tan^2\beta_c/1+\tan^2\beta_c\,,\quad (0 \le m \le 1 \quad \text{for} \quad 0 \le \beta_c \le \pi/2)\,, \tag{5}$$

where $h_r = 1 + t_r/(2t)$ (t_r is the thickness of the resin layer) and $E(m)$ is a complete elliptic integral of the second kind.

The geometry of the undulation region

On the basis of a constant thickness normal to the centreline path, we can determine the equations of the lower and upper curves of the warp yarn in the cross section of the RUC along the undulating portion of the path. These lower and upper curves are depicted in the detail of the undulation region shown in ***Figure 2***. The parametric equations of the lower and upper curves are:

$$x_1(x) = x + (t/2)\sin\beta_w(x)\,,\quad z_1(x) = z_w(x) - (t/2)\cos\beta_w(x)\,, \tag{7}$$

$$x_2(x) = x - (t/2)\sin\beta_w(x)\,,\quad z_2(x) = z_w(x) + (t/2)\cos\beta_w(x)\,. \tag{8}$$

Using eqs. (1) to (3) and setting in the first of eq. (7) $x_1(x_a) = 0$, it can be shown that the value of is the root of the equation

$$\phi_a + \tan^2\beta_c\cos(\phi_a)\cdot \cdot\,[1+\tan^2\beta_c\cos^2(\phi_a)]^{-1/2} = 0\,, \tag{9}$$
where $\phi_a = \pi x_a/L_u$ and $x_b = -x_a$.

Using complicated mathematical considerations, we can obtain the relationship for the cross-sectional area of a tip of the fill yarns A_{tip} in the undulating region, determined by integration using the functions given in eqs. (7) to (9)

$$A_{tip} = \int_0^{L_u/2} z_1\,dx_1 + \int_0^{L_u/2} z_2\,dx_2 = \int_{x_a}^{L_u/2} z_1(x)(dx_1/dx)\,dx + \int_{x_a}^{L_u/2} z_1(x)(dx_1/dx)\,dx\,.$$

Now, the cross-sectional area of the fill yarn, which is the same as the warp yarn, is $A = 2A_{tip} + t(a - L_u)$. Finally,

$$A = a\,t\,[1 - (L_u/a)\,g_2(\beta_c)]\,,\quad \text{where} \quad g_2(\beta_c) = 1 - G_2(\phi_a, \beta_c)/\pi\,. \tag{10}$$

$$G_2(\phi_a,\beta_c) = I_1 - (I_2 + I_3)(1+\tan^2\beta_c)^{-1/2} + I_4(1+\tan^2\beta_c)^{-1} \quad \text{where}$$
$$I_1 = 2\cos\phi_a \qquad I_2 = -2F(\phi_a, m)$$
$$I_3 = [2E(\phi_a,m) + 2(m-1)F(\phi_a,m) - \sqrt{2}\,m\sin 2\phi_a(2 - m + m\cos 2\phi_a)^{-1/2}]\big/(m-1) \tag{12}$$
$$I_4 = m\cos\phi_a\big/[(1-m)(1-m+m\cos^2\phi_a)] + \sqrt{m}\,\mathrm{atan}[\sqrt{m/(1-m)}\cos\phi_a]\big/[(1-m)\sqrt{1-m}]$$

Set of Equations 12.

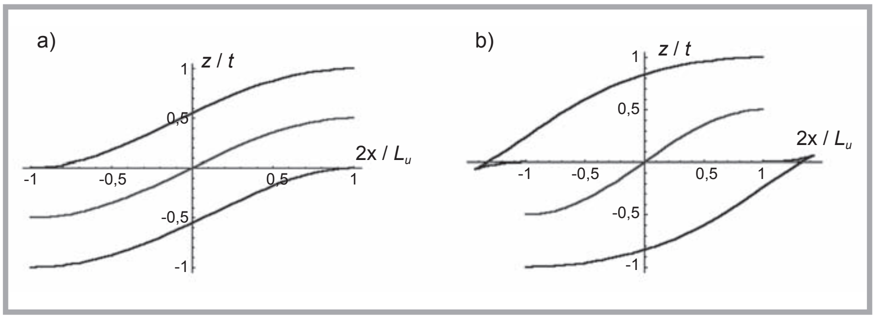

Figure 3. *Geometry of an undulation region along the warp yarn calculated numerically: a) $\beta_c \le \pi/4$ – model is valid, b) $\beta_c > \pi/4$ – model is not valid (a cusp forms).*

Table 1. *The output data.*

n	*a*, mm	*t*, mm	L_u, mm	β_c, deg	No. of iterations
2000	1.4110	0.0857	0.5869	12.90	4
10000	1.4110	0.4727	0.7812	43.54	6
14000	1.6260	0.5804	0.9117	45.00	23

Substituting eq. (10) for A in eq. (4), we can rearrange the result as a quadratic equation in the ratio of L_u / a:

$$g_1 g_2 (L_u/a)^2 - + (g_1 + g_2)(L_u/a) + 1 - V_f h_r / p_d = 0. \qquad (11)$$

Equation 11 is used in the iterative procedure to determine parameters .
The function of the crimp angle is defined by Equation 12 (see page 50).

$F(\phi_a, m)$ and $E(\phi_a, m)$ are the incomplete elliptic integrals of the first and second kind.

Assuming that $0 \le \beta_c^2 << 1$, we obtain approximations to the volume fraction equation (4) and the yarn shape equation (10). We neglect terms of order β_c^2 and higher in the series expansions to obtain the final results

$$t = A\, p_d / (V_f\, h_r\, a),$$
$$L_u = a(1 - V_f\, h_r / p_d)/(1 - 2/\pi). \qquad (13)$$

Small crimp angle approximations (13) are used in the iterative procedure at the beginning of the calculations.

Algorithm to determine the architecture parameters

Input parameters: n, d_f, p_d, a, V, tol, $imax$.
Output parameters: t, L_u, β_c, t_r, a_{new} – new yarn spacing only if $\beta_c = 45°$, V – vol. of ¼ of the RUC, V_{yarn} – vol. of the yarns in ¼ of the RUC, V_r – vol. of the resin in ¼ of the RUC.

1. ***Compute the cross-sectional area* A** of the yarn from $A = (\pi d_f^2 n)/(4 p_d)$. Set $h_r = 1$.

From the small crimp angle equations (13), compute the initial values of the thickness t and length L_u. The initial crimp angle $(\beta_c)_0$ is computed from eq. (3).

2. ***Begin the iteration loop: for to* imax *in steps of one.***

a. Set $t_r = 0$ or $h_r = 1$ and for the crimp angle $(\beta_c)_{i-1}$ solve the quadratic equation (11) for the two roots of the ratio L_u / a.
b. If no positive real root L_u / a exists, then STOP the program and write a warning message.
c. Take the smallest positive real root, and multiply the root by the yarn spacing a to obtain a new value of the undulation length L_u.
d. If $0 < L_u \le a$, then go to step 2e, else, if $L_u > a$, then set $L_u = a$ and solve eq. (11) for h_r.
e. Determine a new thickness t from the fibre volume fraction equation (4).
f. Calculate a new crimp angle $(\beta_c)_i$ from eq. (3) using the new L_u and t determined in steps 2c to 2e.
g. Calculate the difference in the iterates using the measure $\varepsilon = \left|(\beta_c)_i - (\beta_c)_{i-1}\right|$.
h. If $\varepsilon > tol$, then increase the index $i \rightarrow i + 1$.
i. If $i \le imax$, then go to step 2a, else, if $i > imax$, then STOP and print a non-convergence message.
j. If $\varepsilon > tol$, then the fixed point iteration is judged to have converged to the crimp angle $\beta_c = (\beta_c)_i$.

3. ***Check the constraint on the crimp angle.***

a. If $0 < \beta_c \le \pi / 4$, then STOP. Convergence of the geometric parameters t, L_u and t_r has been achieved and the geometry of the RUC is established.
b. If $\beta_c > \pi / 4$, then set $\beta_c = \pi / 4$ and assume that the specified value of the yarn spacing a is incorrect. Solve for t, L_u, t_r and a in steps from 3c to 3f.
c. Set $t_r = 0$ or $h_r = 1$ and solve the quadratic equation (11) for the two roots L_u/a.
Take the positive real root for the ratio of L_u/a.
d. If $0 < L_u/a \le 1$, then go to step 3e, else, if $L_u/a > 1$, then set $L_u/a = 1$ and solve eq. (11) for h_r.
e. Because $\beta_c = 1$, we determine from eq. (3) that $t = (2/\pi)\,(L_u/a)a$. Substitute this expression for t into the volume fraction equation (4) and solve for the new yarn spacing a_{new} to obtain

$$a_{new} = \sqrt{\pi\, d_p\, A[1 - g_1(\beta_c)(L_u/a)] / [2(L_u/a) V_f\, h_r]},$$

where $\beta_c = \pi/4$.
f. For a_{new} from step 3e, compute $L_u = (L_u/a)\cdot a_{new}$, $t = (2/\pi) L_u$ and $t_r = 2t(h_r - 1)$.

Input data $d_f = 0.007$, $V_f = 0.64$, $a = 1.411$ mm, $p_d = 0.75$, $tol = \pi/4 \cdot 10^{-6}$, $imax = 30$. The output data is presented in ***Table 1***.

For $n = 14000$, a maximum crimp angle condition, the yarn spacing a is recomputed.

In ***Figure 3***, a numerically calculated shape of the undulation region along the warp yarn is shown for two cases: a) model is valid ($\beta_c \le \pi/4$), b) model is not valid because a cusp forms in the filament in a region close to the end of the undulating region ($\beta_c > \pi/4$).

Conclusions

The presented method of analysis was very useful for the modelling of textile composites. It can of course be applied for more complicated models (not only with a sinusoidal centreline of the yarn). On the basis of this model, certain geometrical parameters were obtained. These parameters fully characterize the geometry of the considered composite structure. The method has shown that the presented model is valid only for a crimp angle of less than 45° ($\beta_c \le \pi/4$). If the crimp angle $\beta_c > \pi/4$, the model is not valid because a cusp forms in the filament in a region close to the end of the undulating region. Using geometrical considerations, it is possible to obtain from this model basic mechanical parameters that are very useful for further strength analysis. Such mechanical considerations and a method for calculating mechanical parameters will be presented in a future article. Using the Mathematica environment [11], a special computer program was developed to calculate the geometrical parameters.

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