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Determination of Poisson's Ratio in the Plane of the Paper

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Abstract
Within the framework of research work carried out at the Institute of Papermaking and Printing, the Technical University of Łódź, Poisson's ratios were determined in a paper plane. This feature presents a simple and easy method how to determine this material constant. The method proposed for the determination of Poisson's ratio utilises the results of typical procedures used when defining physical properties, such as Young's modulus, paper density and the TSI (tensile stiffness index). For practical verification of the method proposed, the author tested papers for the production of corrugated board and compared the values of ratios obtained for the propagation velocity of sonic waves in the machine and cross directions.

Key words: *paper, Poisson's ratio, orthotrophy.*

Introduction

Paper, like other fibrous structures, is a porous and non-homogeneous body. Fibre orientation in paper depends on the method of its manufacture. Hand made papers are characterised by random mechanical anisotropy in the plane. Some paper grades, such as filtering paper, can be treated as isotropic materials in their planes.

Papers made in industrial conditions (on paper machines), so-called machine papers, show an oriented distribution of mechanical properties.

The symmetry axes of this distribution, for which extreme values of breaking stresses, deformations at the moment of break and Young's moduli are achieved, usually overlap the main directions in the paper. Sometimes the main orthotrophy axes in the paper plane vary from the machine or cross direction by a small angle. Diversification of paper properties in the plane depends on the way fibres are arranged. During the papermaking process, both in the wire and press end, fibres are oriented in the web. However, analysing the fibre structure shown in ***Figure 1***, it is difficult to notice the orientation of fibers, but during the process of paper formation, their "roll axes" tend to be placed in the travel direction of the web.

When the term „fibre axis" is used, it refers to a symbolic axis coinciding with the largest measurement of the fibre, whose shapes are very diverse. In typical paper machines when papermaking stock reaches the wire, fibrous material tends to be aligned in the papermaking direction, which is the machine direction. The final fibre orientation is also influenced by both drying and pressing processes. The average orientation of the fibre axis in the ready paper web is much closer to the machine than to the cross direction, and it is this factor that decides the distribution of a paper's mechanical properties.

Such a structure allows the paper to be treated as an orthotropic elastic material. In order to use the theory of elasticity in practice to describe the behaviour of papers under different stress states, it is necessary to determine values of material constants. Poisson's ratios are numbered among the constants which are the most difficult to determine for paper.

Poisson's ratios for various bodies may have both the positive and negative values [3 - 5]. In the case of paper, the values of Poisson's ratio are positive, and I can agree with Marcinkowski [6], who concludes that they are in the range of 0 - 1.

Since paper shows orthotropic material properties, we need to know six Poisson's ratios in order to fully describe paper behaviour. In practice, when we treat the paper as a thin film, it is enough to determine two Poisson's ratios – between the machine and cross directions.

Figure 1. *Microscope image of a paper surface of approx. 1 mm^2 [14].*

Current methods of measuring Poisson's ratio

The determination of Poisson's ratio for paper has been explored by many researches. Brecht and Wanka determined its value with a microscope mounted above a strained paper piece [6]. They found that the value of Poisson's ratio remains constant as a function of the load until the tension in the test piece exceeded 1%. At a strain above 1%, they noticed an increase in the ratio value.

Ranger and Hopkins [7] used a measurement method in which small glass beads were glued to the paper surface and used to calculate the desired strains. The values of Poisson's ratio reported ranged from 0.4 to 0.95.

This technique was also used by Jones [8] who used a camera to register the strains. Initial measurements of Poisson's ratio gave values ranging from 0.5 to 1.2. Such high values were caused by cockles, which were finally eliminated with the use of a frame. The results obtained with the frame ranged from 0.10 to 0.57. Jones also noticed that the value of Poisson's ratio rises along with increased tensile strains.

The cockling of large test pieces during unidirectional stretching was also analysed in other research works [5, 9], where the authors used frames with improved construction. The values of Poisson's ratio obtained ranged from 0.1 to 1.

The method of strain determination with the use of a movable microscope used by Brecht and Wanka requires long-lasting maintenance of the test piece under stress, which causes a flowing effect in the paper. Taking photographs seems to be a better method for strain registration. It allows to register the shape of the test piece in a very short time, and owing to this it reduces the share of permanent deformations appearing in the stretched material.

The latest testing methods register strains much more easily and precisely with the use of a video extensometer of high resolution.

Interesting methods for the determination of Poisson's ratio were presented by Uesaka [10] and Urruty [11], who tested papers under a bidirectional state of stress in the plane. However, they needed complicated, non-standard instruments to make measurements.

As we know, the behaviour of paper resembles the behaviour of an elastic body at low levels of stress, which is why the values of Poisson's ratio for measurements based on the theory of elasticity should be determined in tests where low stress is used.

The methods described above for the determination of Poisson's ratio are time- and labour-consuming and require specialist testing equipment. In addition, during tests. test pieces of different measurements, various types of fixing, different levels of stress, and the time of their interaction with the tested paper are used. The modification of any mentioned factor may have a crucial effect on the result of measurement and finally on the value of the material constant determined.

For the above reasons, when determining Poisson's ratio, the methods based on the measurement of ultrasonic wave propagation in paper seem to be very useful. Such research works as [12, 15, 16 - 18] were carried out for years and their results have found practical applications.

Castagnede [18] made interesting observations while comparing the values of material constants determined acoustically and mechanically. Young's moduli evaluated with acoustic tests were higher and Poisson's ratios were lower compared to those which were determined based on the results of mechanical tests.

Determination of Poisson's ratio based on the results of acoustic and mechanical tests

In order to simplify the measurement of Poisson's ratio, the author utilised the results of measurements of the propagation velocity of ultrasonic waves in a paper plane obtained when determining the TSI (tensile stiffness index) [12]. The method presented allows to calculate the material constant on the basis of values of Young's moduli for the main direction in the paper plane, the density and propagation velocity of ultrasonic waves in one of the main directions.

The formula for the propagation velocity of ultrasonic waves in the direction compatible with the direction of the main orthotrophy axis can be determined using the formula below describing the propagation velocity of waves in the paper plane treated as a thin orthotropic film (***Equation 1***) [1]:

$$\rho = V_l^2 = A + B \qquad (1)$$

where:

A and B are given by Equations (2) and (3),

V_l – propagation velocity of longitudinal waves,

α – inclination angle of the wave propagation direction in relation to the main orthotrophy axis,

C_{ij} – stiffness coefficients,

ρ – paper density.

For the direction of wave propagation compatible with MD treated as the main orthothropy direction in the paper plane $\alpha = 0$, which, after considering (2) and (3), gives:

$$A = (C_{11} + C_{66})/2 \qquad (4)$$

$$B = (C_{11} - C_{66})/2 \qquad (5)$$

After substituting (4) and (5) with (1) and performing the transformation we have

$$V_{MD} = \sqrt{\frac{C_{11}}{\rho}} \qquad (6)$$

The relationship obtained is analogous to the formula describing the propagation velocity of ultrasonic waves in the main directions of other orthotropic materials [13].

Stiffness coefficient C_{11} in the paper plane can be determined using the formula

$$C_{11} = \frac{E_{MD} \cdot E_{CD}}{E_{CD} - E_{MD} \cdot \nu^2_{MDCD}} \qquad (7)$$

where:

E - Young's modulus (the index indicates the direction for which the modulus was determined),

ν - Poisson's ratio, (the first index shows the direction of transverse strain, and the other index indicates the stress),

MD - machine direction,

CD - cross direction.

Substituting (7) to (6) and transforming it we obtain the relationship allowing to determine the value of Poisson's ratio based on the paper density of Young's moduli in MD and CD and the propagation velocity of waves in MD

$$\nu_{MDCD} = \sqrt{\frac{E_{CD}}{E_{MD}}\left(1 - \frac{E_{MD}}{\rho \cdot V^2_{MD}}\right)} \qquad (8)$$

where:

V_{MD} - propagation velocity of ultrasonic waves in *MD*.

Knowing coefficient ν_{MDCD}, coefficient ν_{CDMD} can be calculated from the relationship

$$\nu_{CDMD} = \frac{E_{MD} \cdot \nu_{MDCD}}{E_{CD}} \qquad (9)$$

or, from the relationship obtained by substituting (11) with (12)

$$\nu_{CDMD} = \sqrt{\frac{E_{MD}}{E_{CD}}\left(1 - \frac{E_{MD}}{\rho \cdot V^2_{MD}}\right)} \qquad (10)$$

Performing the analogical operation for CD, we obtain

$$\nu_{CDMD} = \sqrt{\frac{E_{MD}}{E_{CD}}\left(1 - \frac{E_{CD}}{\rho \cdot V^2_{CD}}\right)} \qquad (11)$$

$$A = \frac{1}{2}\left(\cos^2\alpha \cdot C_{11} + \sin^2\alpha \cdot C_{22} + C_{66}\right) \qquad (2)$$

$$B = \frac{1}{2}\sqrt{\left[\cos^2\alpha\left(C_{11} - C_{66}\right) + \sin^2\alpha\left(C_{66} - C_{22}\right)\right]^2 + 4 \cdot \cos^2\alpha \cdot \sin^2\alpha\left(C_{12} - C_{66}\right)} \qquad (3)$$

Equations 2, and 3.

$$\nu_{MDCD} = \sqrt{\frac{E_{CD}}{E_{MD}}\left(1 - \frac{E_{CD}}{\rho \cdot V_{CD}^2}\right)} \quad (12)$$

where:

V_{CD} - propagation velocity of ultrasonic waves in CD.

Test methods

The test pieces were initially conditioned for 24 hours in air of 23 °C and relative humidity of 50%.The tests were carried out in a conditioned room with the same conditions.

To determine the propagation velocity of waves in the machine and cross directions, the author used the results of a TSI test performed on apparatus from Lorentzen & Wettre which measures the velocity directly.

After carrying out measurements of the propagation velocity of ultrasonic waves using the same paper test pieces, basis weight measurements were performed. Measurements of Young's moduli were carried out in accordance with the PN-EN ISO 1924-2 standard. Paper density was determined in accordance with Standard PN-EN 20534. The tests were made for paper and board grades listed in ***Table 1***.

On the basis of the measurement results, the author calculated Poisson's ratios with the use of the propagation velocity of ultrasonic waves in the machine direction. Then calculations of the same ratios were made with the use of propagation velocity of ultrasonic waves in the cross direction. The error made when determining constant C_{11} with the simplified relationship (5) for the papers tested ranged from 1.5% to 2.4% of the measured value.

Test results

Figure 2 shows values of Poisson's ratio determined on the basis of the propagation velocity of ultrasonic waves in the machine and cross directions, along with measurement errors.

Any discrepancy between results obtained on the basis of the velocity measurements in the machine and cross direction is in the area of 10% for all the papers. In all the test cases, those differences are within the margin of measurement error.

Table 1. *Papers used in tests.*

Paper grade	Symbol	Thickness, mm	Basis weight, g/m²
Liner	L220	0.347	220
Fluting	F123	0.241	123
Liner	L140	0.230	140
Fluting	F130	0.191	130
Fluting	F198	0.302	198
Fluting	F124	0.203	124
Liner	L158	0.183	158
Liner	L130	0.150	130

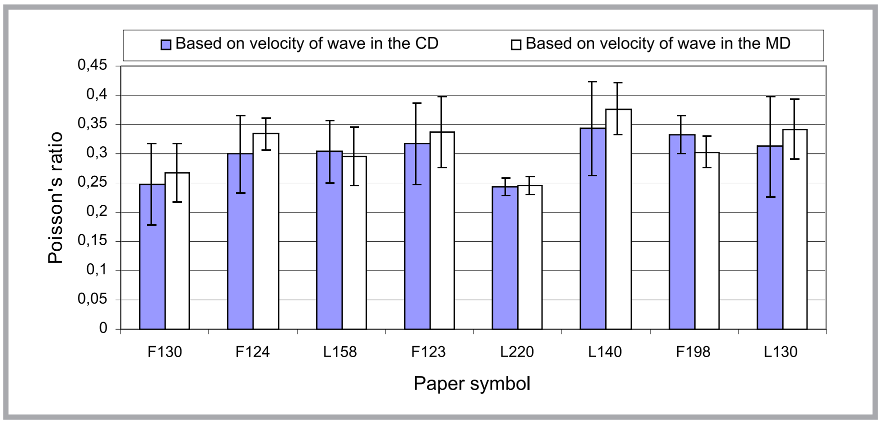

Figure 2. *Comparison of values of Poisson's ratio* ν_{MDCD}.

The main reason for the above discrepancy and high values of errors made when determining Poisson's ratio is connected with the inhomogeneity of paper structure and measurement errors made when determining the values used for calculations. When determining Young's moduli on the basis of the tension curve, its first part is omitted, most often due to measurement errors. This causes the underrating of Young's moduli determined by mechanical methods when compared with moduli determined on the basis of acoustic measurements. Consequently, the value of expression $(1 - E/(\rho V^2))$, appearing in the formulas for calculation of Poisson's ratio, increases, and for this reason, in some cases, calculated values of Poisson's ratio may be higher than the real ones.

If the TSO index has a high value, of several degrees, the machine and cross directions cannot be treated as the main orthotrophy directions in the paper plane. In such a situation, the method presented can be used for the determination of Poisson's ratio for the main orthotrophy axes.

Conclusions

- Determination of Poisson's ratios with the use of propagation velocity measurements of ultrasonic waves is based upon the results of standard tests used in papermaking science, and for this reason they are easy and quick to perform.
- The method presented, using the measurement of the propagation velocity of waves reduces the values of measurement errors connected with the cockling of test pieces, variable levels of strains used during the tests or the different times of their interaction with the test piece.
- The test method using the measurement results of TSI and TSO (tensile stiffness orientation) indices allows to carry out the simultaneous determination of the inclination angle of the main orthotrophy axes from the main directions in the paper plane (*MD*, *CD*), which has a significant impact on the practical applications of material constants determined.

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