

# Modelling of the Change in Structure of Woven Fabric under Mechanical Loading

## Abstract

The aim of this work was to study a new way of modelling the behaviour of structures of woven fabrics after mechanical loading. As the basis of our considerations, on the beginning of this working, a classical Painter nomogram was introduced. The modelling was done through the realisation of the following scientific - research aims: analysis of the behaviour of the structures of woven fabrics subjected to different static loads, study of methods of expectation of mechanical property at designing of fabrics, study of techniques enabling the analysis of the structure of fabrics subjected to deforming strengths, and a analysis of the property of fabrics with respect to the basic parameters of fibres and yarns. The devising of new, modified Painter nomograms made it possible to model the change in structure of fabric subjected to a static load, thanks to which it is possible to reflect the structure of internal real fabrics recreated, eliminating the generalisation of the Painter nomogram.

**Key words:** modeling, weave fabric, mechanical properties, internal structure of fabric, cross-section.

## Introduction

Woven fabrics are the most common example of flat textile materials used in the manufacturing of clothing, decorative, technical, and special purpose products. Increasing expectations regarding the variety of fabric uses have prompted researchers to seek the optimal applied properties of fabrics as well as their internal structure. Woven fabric as a complex textile product, with advantages and disadvantages of the fibres and yarns on one side as well as of the way of manufacturing and finishing the other, is an interesting, however, not well-known study case.

In this work we attempt to explain what links exist between the internal structure of fabrics, the properties of their fibres and yarns, and the properties of model fabrics under a mechanical load.

The results will make it easier for designers to choose appropriate parameters for fibres and yarns as well as to make adjustments to the loom in order to obtain a product of specific mechanical properties. To achieve this, the behaviour of fabrics subjected to a cutting and bending load in one and two directions are analysed with respect to geometric changes in the product.

The pragmatic justification for choosing this subject matter is the necessity of modelling the behaviour of woven structures under a mechanical load. The modelling was conducted through the

implementation of the following scientific - research aims of the paper:

- The analysis of the behaviour of woven structures subjected to different mechanical loads,
- The study of methods of predicting mechanical features when designing fabrics,
- The study of techniques enabling the analysis of the structures of fabrics subjected to deforming strengths,
- The analysis of the property of fabrics taking the basic parameters of fibres and yarns into consideration.

The starting point for the implementation of the goals mentioned above was the analysis of the geometric model of fabric of plain weave published in 1937 by Peirce [1,7], as well as of the Painter nomogram constructed on the basis of the model, published in 1952 [2, 7]. Analysis of the structural changes in selected, previously woven fabrics subjected to stretching showed that the Peirce model, projected on the Painter nomogram to show the structural changes in the fabric being stretched, can be up to 60% in error.

The analysis resulted in the formulation of 3 propositions for modification of the Pierce model, conventionally defined as type 'A', type 'B' and type 'C'.

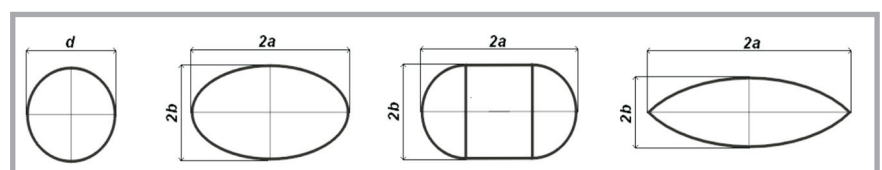
## Theoretical analysis of the problem

### Modelling of the internal structure of fabric

All real fabrics can be specified by the deformation of the circular cross-section of warp and weft threads. The scale of the deformation depends on the kind of thread, material used, the way of weaving, the weave, the weaving strengths and the susceptibility of the thread to cross-section deformation. The shape of the cross-section of threads in fabric can only be constructed using different geometric figures of the plane (*Figure 1*), such as a circle (Peirce) [1, 6 - 8], an ellipse, a hippodrome shape (Kemp) [5 - 7], a convex lens (Milasius) [11] etc. The shape of the cross-section in real fabrics is diverse, which is the result of pressing and bending strengths effecting the areas of contact between crossing warp and weft threads [9, 10, 12].

The most crucial measures of the cross-section are  $2b$  as a shorter axis of the thread section, which equals the thickness of the thread measured towards the pressing strength, and  $2a$  as the longer axis of the thread section, which equals the perpendicular direction of the pressing strength [3, 4] (*Figure 1*).

Peirce's geometric model [1] and the Painter nomogram [2] served as a start-



**Figure 1.** Studied cross-sections of threads in fabric.

ing point for modelling the internal structure of a fabric. Details of tests verifying the basic assumptions and simplifications used in modelling fabrics over the past 60 years are introduced beneath.

### Modifications of nomogram 'A'

The simplest but fully justified modification of the Painter nomogram is replacing the sum of diameters  $D=d_o+d_w$  from the Painter nomogram [2] with the sum of the smallest cross-section measures of the yarn, that is  $2b_o+2b_w$ . This results from the assumption that threads are most intensely deformed in places of mutual contact, therefore in modification 'A' the theoretical measure of the cross-section of threads is replaced with the real measure of the thread deformed (Figure 2.a).

Such an approach is a kind of simplification of a problem which requires mathematical analysis. In fact, pressing strengths deform the yarn, creating something like two ellipses, one lying on the other, which is in the picture of the cross-section of one yarn, which is the effect of the pressing strengths operating on both sides during the thickening of the fabric. (Figure 2.b).

Taking the above calculations into account, the nomogram created for the

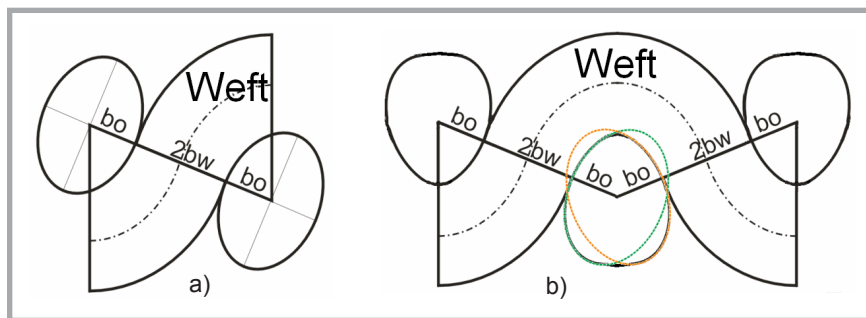


Figure 2. a) Cross-section of the fabric with minimal distance between the axes of the yarns. b) Real cross-section of the fabric repeat with minimal distance between the axes of the yarns.

$$\text{For } \frac{l}{2b_o+2b_w} = \text{const} \quad w_o = \frac{E_w'}{0.01395} \cdot \frac{l_o}{2b_o+2b_w} - 100 \quad (1)$$

$$\text{For } \frac{h}{2b_o+2b_w} = \text{const} \quad w_o = 27.79 \cdot \left( \frac{h_o}{2b_o+2b_w} \right)^2 (E_w')^2 \quad (2)$$

$$\text{For } \Theta = \text{const} \quad w_o = \frac{100(1-\cos\theta_o)}{\cos\theta_o} - \frac{71.68 \cdot (\sin\theta_o - \theta_o \cdot \cos\theta_o)}{\cos\theta_o} \cdot E_w' \quad (3)$$

### Equations 1, 2 and 3.

family of lines can be illustrated by the equations 1 - 3.

Taking real cross-measures of threads of a certain fabric, marks O and W were put onto the modified nomogram. (Figure 3).

Comparing this to the Painter nomogram, two parameters were changed, i.e. the position of marks O and W, and the course of tracks on which the marks will move during the one-way stretching of the fabric.

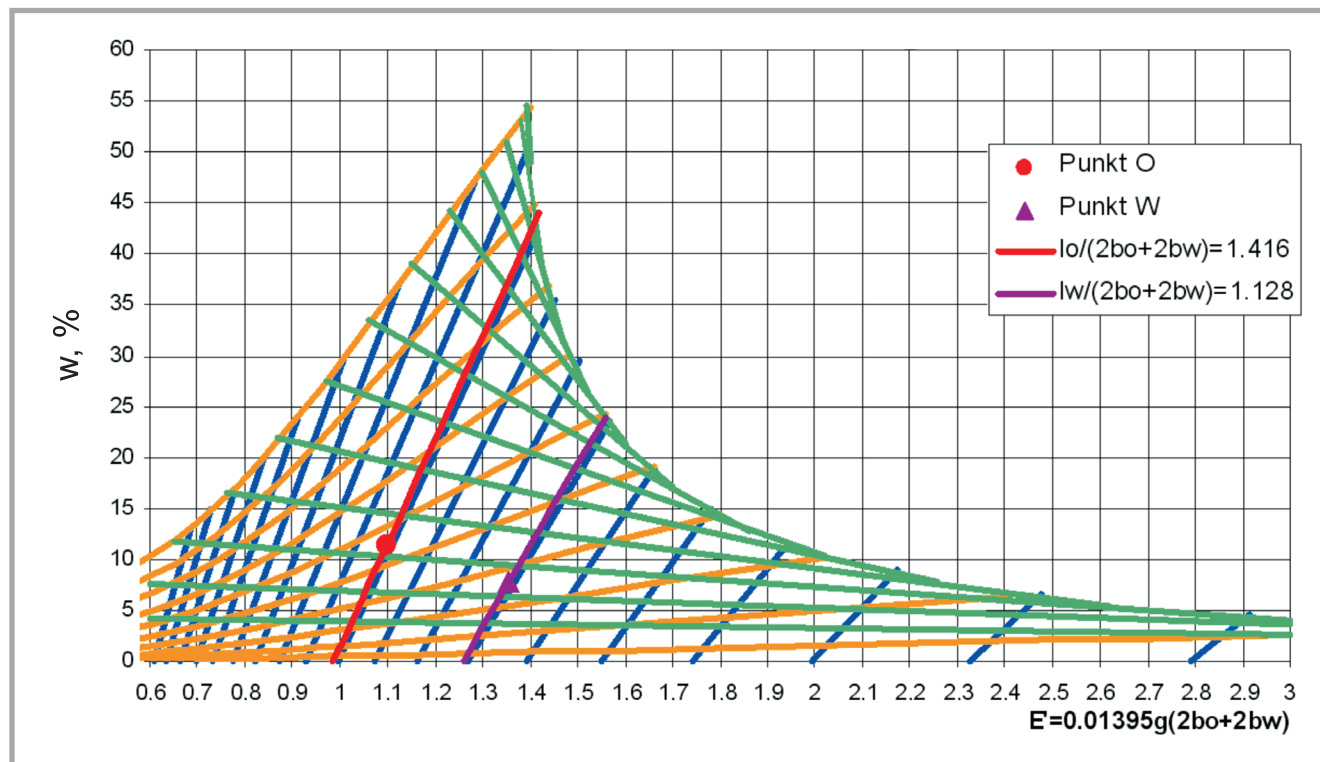


Figure 4. Modified nomogram where the sum of diameters was replaced with the sum of the smallest axes of the elliptic cross-section of the warp and weft threads.

The modification, taking into consideration the assumption that the smallest measure of the axis of the cross-section of yarn is taken instead of the diameter, is true only in extreme cases, i.e. at the moment of jamming the threads of one system by another. However, the case when the fabric is 100% filled with threads, is just theoretical.

Therefore, the formulae for families of lines were introduced. Cross-sections of threads are commonly used in the shape of the ellipse. Its positive effect lies in lessening the mistake that occurs in the case of the shape of the cross-section of thread being circular.

### Modification of nomogram 'B'

In modification 'B' the circular cross-section of threads is replaced with an elliptic one, which refers to known Kemp's tests using the Peirce model, where the shape of the cross-section of threads is more similar to the shape occurring in real fabrics; in the case studied, to the elliptic one (Figure 4).

The assumption was made that deformation in the shape of a cross-section of thread, from circular to elliptic, does not cause a change in the value of the section area.

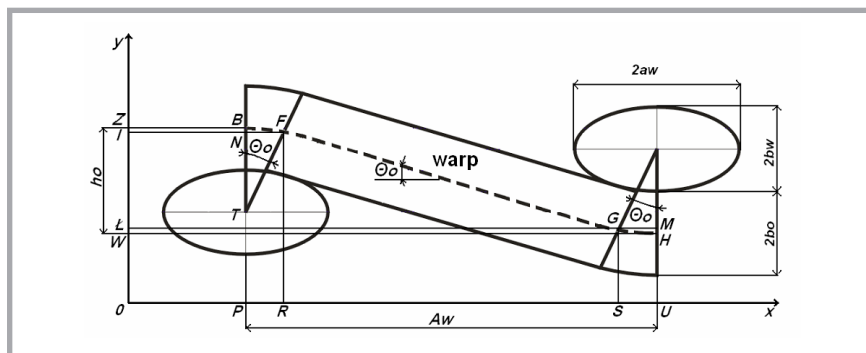


Figure 4. Section of fabric with elliptic sections of the yarn.

$$w_o = \frac{E_w'}{0.01395} \cdot \frac{l_o}{(2a_o\sqrt{e_o} + 2a_w\sqrt{e_w})} - 100 = \frac{E_w'}{0.01395} \cdot \frac{l_o}{U_{pe}} - 100 \quad (5)$$

$$w_o = 27.782 \cdot \left( \frac{h_o}{2a_o\sqrt{e_o} + 2a_w\sqrt{e_w}} \right)^2 (E_w')^2 = 27.782 \cdot \left( \frac{h_o}{U_{pe}} \right)^2 (E_w')^2 \quad (6)$$

$$w_o = \frac{100(1 - \cos\theta_o)}{\cos\theta_o} - \frac{71.6846 \cdot (\sin\theta_o - \theta_o \cdot \cos\theta_o) \cdot E_w'}{\cos\theta_o} \cdot \frac{(e_o 2a_o + e_w 2a_w)}{\cos\theta_o \cdot (2a_w\sqrt{e_w} + 2a_o\sqrt{e_o})} \quad (7)$$

Equations 5, 6, and 7.

$$\text{So: } \frac{\pi d^2}{4} = \pi ab \Rightarrow d = \sqrt{4ab} \quad (4)$$

$b$  – half of shorter axis of the cross-section of the thread.

where:

$d$  – diameter of thread,

$a$  – half of longer axis of the cross-section of the thread,

According to Painter, the equations of three families of lines are introduced. They are described by the equations 5, 6, 7.

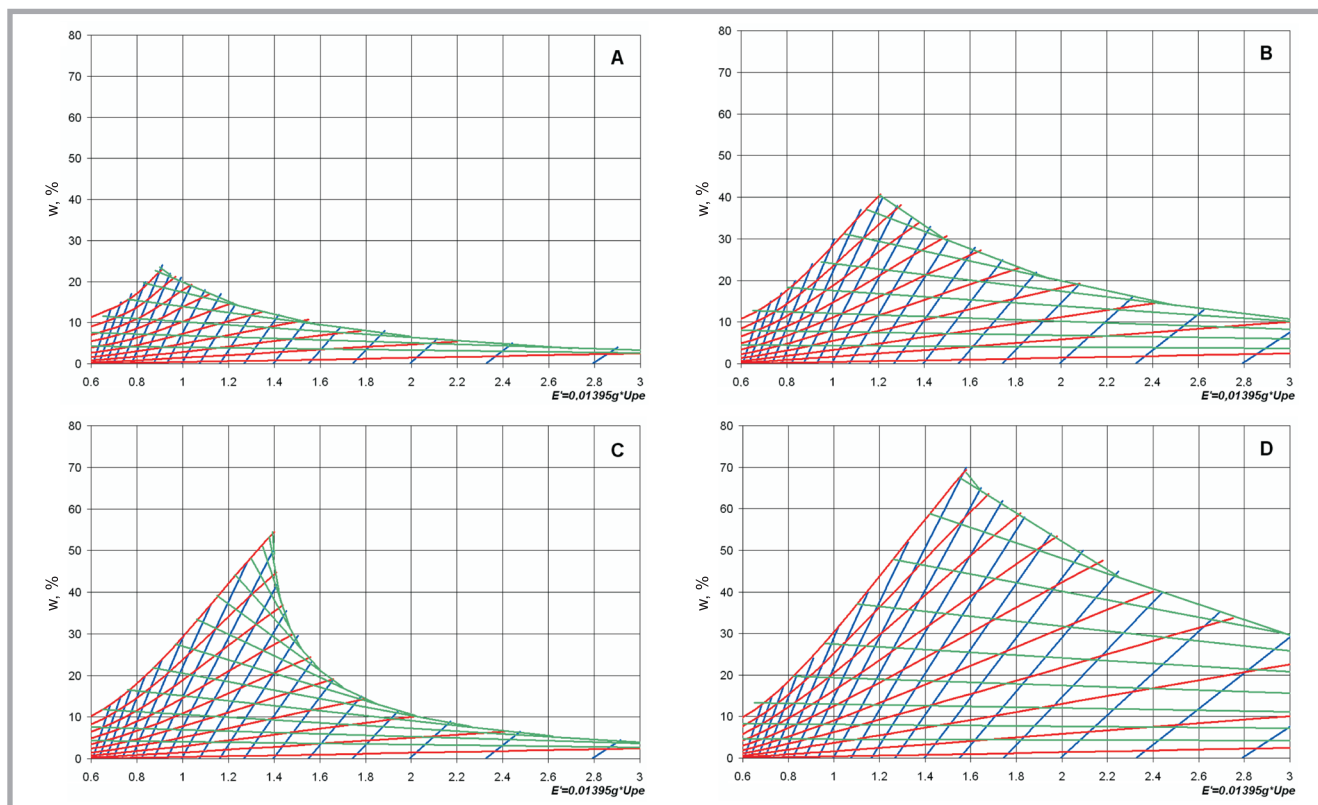


Figure 5. Example of nomogram modified for different values  $A$  of  $(2b_o + 2b_w)/U_{pe} = 1$ ; B),  $(2b_o + 2b_w)/U_{pe} = 0.5$ ; C),  $(2b_o + 2b_w)/U_{pe} = \cos\theta$ ; D) and  $(2b_o + 2b_w)/U_{pe} = 0.25$ .

$$w_w = \frac{E_o'}{0.00805 \cdot \sqrt{4 - \left(\frac{\Phi-1}{4}\right)^2}} \cdot \frac{l_w}{2b_{Osr}} - 100 \quad (8)$$

$$w_o = \frac{E_w'}{0.00805 \cdot \sqrt{4 - \left(\frac{9-\Phi}{4}\right)^2}} \cdot \frac{l_o}{2b_{Wsr}} - 100 \quad (11)$$

$$w_w = 83.34 \cdot \left( \frac{h_w}{2b_{Osr}} \right)^2 \left( \frac{E_o'}{\sqrt{4 - \left(\frac{\Phi-1}{4}\right)^2}} \right)^2 \quad (9)$$

$$w_o = 83.34 \cdot \left( \frac{h_o}{2b_{Wsr}} \right)^2 \left( \frac{E_w'}{\sqrt{4 - \left(\frac{9-\Phi}{4}\right)^2}} \right)^2 \quad (12)$$

$$w_w = \frac{100(1 - \cos\theta_w)}{\cos\theta_w} - \frac{124.15 \cdot (\sin\theta_w - \theta_w \cdot \cos\theta_w) \cdot E_o'}{\sqrt{4 - \left(\frac{\Phi-1}{4}\right)^2} \cos\theta_w} \quad (10)$$

$$w_o = \frac{100(1 - \cos\theta_o)}{\cos\theta_o} - \frac{124.15 \cdot (\sin\theta_o - \theta_o \cdot \cos\theta_o) \cdot E_w'}{\sqrt{4 - \left(\frac{9-\Phi}{4}\right)^2} \cos\theta_o} \quad (13)$$

**Equations 8 - 13.**

On the basis of the proportions:

$$l_0/U_{pe} = const, h_0/U_{pe} = const \text{ and } \Theta = const$$

for elliptic cross-sections of threads, when applying the formulae for the family of lines, it is possible to create the following nomograms (Figure 5) with different values of the proportion:

$$\frac{(2b_o + 2b_w)}{U_{pe}}$$

where  $U_{pe} = (2a_w \sqrt{e_w} + 2a_o \sqrt{e_o})$

Depending on the material used and the mechanical properties of the yarn, the

value of the above proportion fluctuates within the range of:

$$0 < \frac{(2b_o + 2b_w)}{U_{pe}} \geq 1$$

For  $\frac{(2b_o + 2b_w)}{U_{pe}} = \cos\theta$  the classic Painter nomogram is obtained (Figure 5).

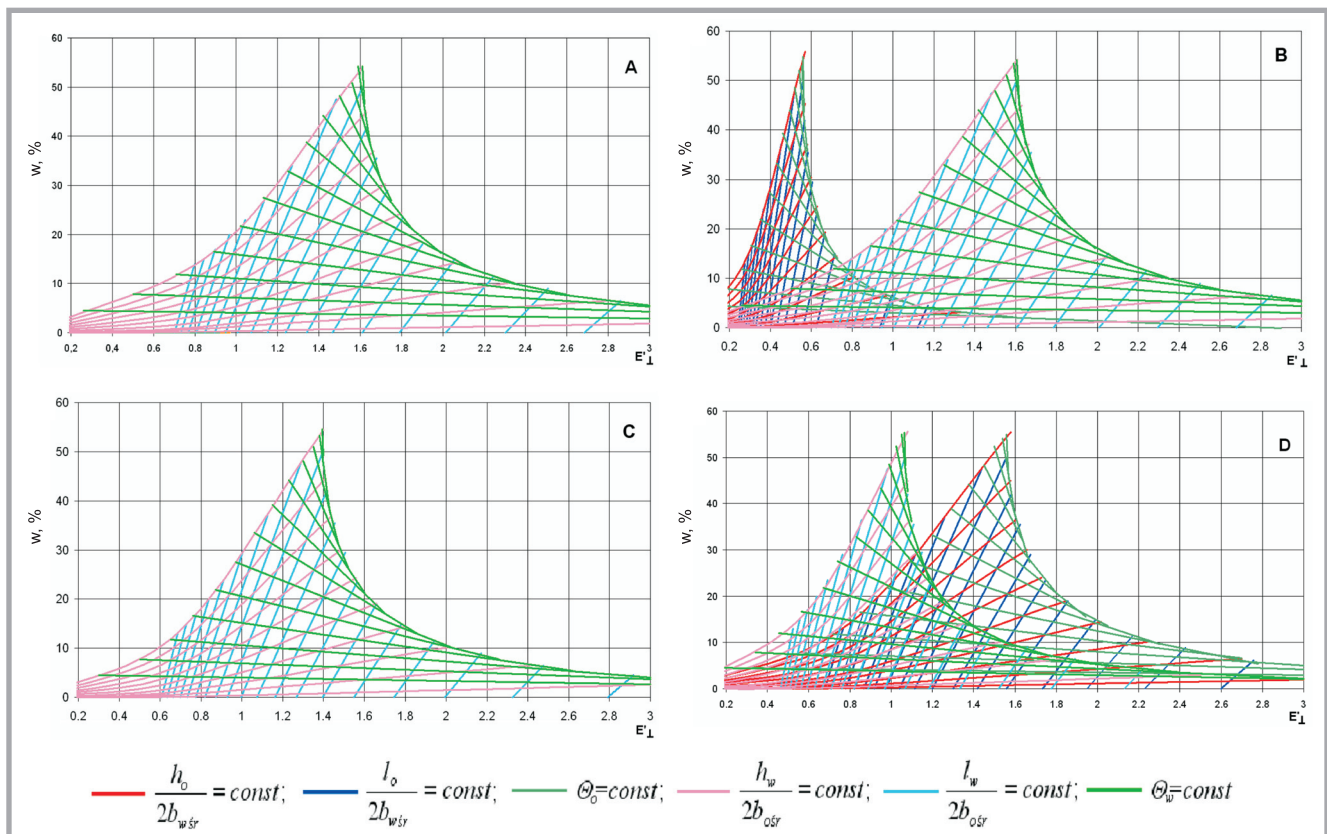
**Modification of nomogram 'C'**

In modification 'C', modification 'B' with respect to the shape of the thread sections is employed simultaneously with the modified proportion (used for calculating the scales of threads) describing the

maximum density of threads in the fabric, taking the phase of its structure into account. The equations of three families of lines for warp and weft systems were introduced, respectively. They are described by the Equations 8 - 13.

Having establishing one co-ordinate ( $w, E'_{\perp}$ ), all families of lines for function  $w_o = f(E'_w)$  and  $w_w = f(E'_o)$ , depending on the value of fabric phase, nomograms were drawn. (Figure 6).

For phase 1, where the warp is straight (the fabric towards the warp having



**Figure 6.** Modified nomogram for A)  $\Phi=1$ , B)  $\Phi=1,5$ , C)  $\Phi=5$ , D)  $\Phi=7$ .

reached the structure limit), the equations of lines  $w_o = f(E'_w)$  are reset, which excludes the drawing of this part of the nomogram. Only a part of the nomogram for function  $w_w = f(E'_o)$  appears. The symbol  $W$  can only be put in such a nomogram. Subsequent nomograms for  $\Phi = 1.5$  do  $\Phi = 8.5$  contain all families of lines  $w_o = f(E'_w)$  and  $w_w = f(E'_o)$ . For phase  $\Phi = 5$ , the families of lines for warp and weft overlap, creating Painter nomogram.

For phase 9, where the weft is straight (the fabric towards the weft having reached the border structure), the equations of lines  $w_w = f(E'_o)$  are reset, which excludes the drawing of this part of the nomogram. Only a part of the nomogram for function  $w_o = f(E'_w)$  appears. Mark  $O$  can only be put on such a nomogram. Nomograms for subsequent, selected phases of the fabric structure are shown below (Figure 6).

### Conclusions of the theoretical analysis of the problem

Comparing the modelling of the structure of woven fabrics on one side with three new nomograms on the other using the Painter nomogram, a conclusion can be made that different kinds of the shape of the internal structure of fabrics, depending on the designer's attitude and expectations, can be created. Moreover, it is possible to produce various models of the changes in the internal geometry of fabric under a mechanical load. Therefore, using these nomograms, the designer has an opportunity to realise their assumptions concerning the internal structure of fabrics.

Applying the Painter nomogram to circular sections of threads is possible when modelling the most general form of fabric. However, it has to be stated that the model is idealised and, thus, does not project the real shape of the internal geometry of fabric correctly. With the elliptic section of yarn, modelling both the internal structure of fabric and the different phases of its changes under deformation strengths is more precise.

Making an assumption for the elliptic cross-section of yarn for fabric, nomogram 'B' can be applied to analyse the internal structure of fabrics. When the fabric has a maximum possible density of threads, it means that the threads were deformed by pressing strengths, and

therefore nomogram 'A' should be applied. However, both nomograms work properly only for the two-system area of resistance because of the formula adapted for filling the fabric with threads. Consequently, when the fabric is of a one-system of resistance, a new 'C' nomogram including the value of phase is applied to analyse its structure. The model created is elastic, which allows the researcher to adjust it to the changing phase of the fabric structure and, thus, facilitates the prediction of the behaviour of fabric, performed at the design stage. What is important is that the new instrument of modelling and analysing fabric structure takes into account the fact that both warp and weft systems of fabric can behave differently.

### Experimental part

The research presented in this chapter covers the practical verification of the nomograms' utility, as well as includes an analysis of the changes in the structure parameters during stretching of the 16 woven cotton fabrics designed to show that differens as between them are caused by different features. All the fabrics used the same cotton warp of 40 tex. The weft differed in its linear density (20 tex and 30 tex yarns were used), the coefficient of twist of the weft yarn was equal to  $\alpha_m = 100$  and  $\alpha_m = 120$ . Ring spinning and open-end spinning were applied simultaneously. The next changes in the weaving conditions were accepted by taking into account the position of the back rest roller the distance from the base of the 100 cm, 86 cm and 94 cm), the moment of closing the shed (300° and 0°) and the initial tension of the warp (30.5 cN per end; and 20.5 cN per end). It gives six inputs ( $x_i$ ) on the two levels. The poliselective fractional two - leveled plan of the experiment generated 16 kinds of fabrics.

The stretching tests were carried out using a classical tensile tester working in one direction.

The images of fabrics' cross-sections were taken at eight points of the stretching process, which means after an elongation of 0%, 15%, 30%, 45%, 60%, 75%, and 90% (Figure 7). This is an original technique named by us 'step by the step technique'. It enabled an analysis of the geometrical shapes of the fabrics' cross-sections (Figure 8).

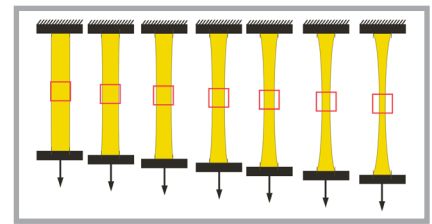


Figure 7. The experimental measurements of the cross-section of fabrics subjected to stretching by the step by step technique.

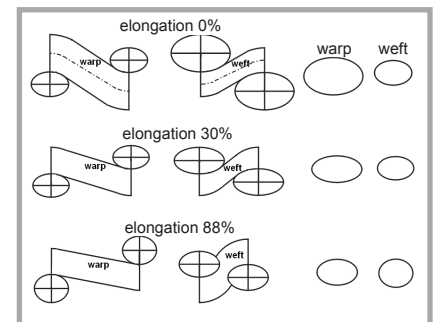


Figure 8. Pictures of cross-sections of fabric 3 under stretching along the warp.

The position of the tested fabrics was fixed by glue at each stage. Next, an analysis of their structure was conducted by an optical microscope. Such kind of research was carried out for the first time. It creates opportunity to build a database which describes the knowledge on the changes in structure of fabrics under stretching tension.

It could be stated that during stretching, the degree of deformation of the shape of the threads cross-sections increases and the area of their mutual contact enlarges. It happens as a result of growing tension pressing till the moment when the second system seize up. After that moment, the section gradually returns to its circular shape. The crimp of stretched threads decreases. However, due to stretching, the threads become thinner till they finally break. At the same time, the cross-sections of the second system change their shape from elliptic to more circular for all studied fabrics. During increasing in crimp of the weft, single filaments burst in bent yarns causing at the same time an enlargement of the cross-section area of yarn.

The above-mentioned research aiming at verification of the utility of the modified nomograms indicated that the modification 'C' is more precise than the classical Painter nomogram in describing the changes in fabric structure during stretching, and moreover is the most precise of all three modifications presented.

## 27<sup>th</sup> General Assembly of the Polish Textile Association and Seminar 'Science for Industry'

13 February 2009,  
Łódź, Poland

### Seminary moderators:

#### ■ **Małgorzata Cieślak**

Ph.D. – Scientific  
Secretary of the Textile  
Research Institute,

#### ■ **Katarzyna Grabowska**

Ph.D., Dsc. – V-ce Dean  
of the Faculty of Material  
Technologies and Textile  
Design

*Lectures concerning new scientific achievements, research problems and new product development were held by representatives of the Technical University of Łódź; Textile Research Institute, Łódź; Institute of Natural Fibres, Poznań; The Institute of Security Technology MORATEX and the POLMATEX-CENARO Research Centre.*

*Association Representatives were chosen during the second part of the assembly. Prof. Witol Łuczyński Ph.D., Dsc. will remain as President of the Polish Textiles Association.*

For more information please contact:

Polish Textile Association  
General Management  
Tel. 48 42 632 71 15;  
e-mail: swp.zg@wp.pl

## ■ Conclusions

- In this study a instrument for modelling the changes in the structure of woven fabric under a mechanical load was created.
- The research conducted proved that applying the classical Painter nomogram, with its simplifications, as a diagram describing changes in the structure of fabric under one-directional stretching, can lead to 60% error. It is because this nomogram does not include changes in the cross-sections of the yarns, the one-system area of resistance, the stiffness of threads and the variable area of contact of two mutually crossing threads.
- A new way of modelling changes in the structure of fabric, including its construction phase, was drawn up. It enabled the analysis of the behaviour of woven structures under a mechanical load.
- The new way of modelling the structure of fabric allows to predict its mechanical properties, to evaluate the possibilities of manufacturing certain yarn, as well as evaluate the accuracy of the choice of yarn for use at the design stage. At the same time, it reduces, by at least four times, the margin of error compared to previous methods.
- The experiment also showed that during stretching for one system of threads, while reducing the cross-section sizes of the yarn, the threads gradually take the shape of a flattened ellipse till the moment when the second system seizes up, after which the section gradually returns to its circular shape.

## Acknowledgments

This work was financed by promoter grant No.: 3 T08E08826 and a 10 monthly scholarship - Marie Curie (HPMT-CT-2000-00030) at the Katholieke Universiteit Leuven (Belgium), the Department of Metallurgy and Materials Engineering in a Composite group.

## References

1. Peirce F. T. *Journal Textiles Institute*, 21,1409,1937.
2. Painter E. V. „Mechanics of Elastic Performance of Textile Materials – Part VIII: Graphical Analysis of Fabric Geometry” *Textile Research Journal*; Vol XXII, No.3, Page153-169, March 1952.
3. Masajtis J. „Budowa i projektowanie tkanin – cz. I”, skrypt Politechniki Łódzkiej, Łódź 1991.
4. Masajtis J. „Analiza strukturalna tkanin”, Łódź 1999.
5. Janusz T. „Tkactwo, część III”, WPLiS, Warszawa 1963.
6. Szosland J. „Laboratorium podstaw budowy i technologii tkanin”, praca zbiorowa, skrypt Politechniki Łódzkiej, Łódź 1984.
7. Szosland J. „Podstawy budowy i technologii tkanin”, WNT, Warszawa, 1991.
8. Frontczak-Wasiak I. „Laboratorium podstaw mechanicznej technologii włókna – tkactwo”, skrypt Politechniki Łódzkiej, Łódź, 1987.
9. Damianow G. B.; Baciew C. Z.; Surnina N.F. „Strojenije tkani i sowlremennyje metody jeje projektowania” Moskwa, Liegkaja i piściewaja promyszljennost, 1984.
10. Aleksejew K. G. „Osnowy rascieta paramajetrow strojenija i iformirowanija tkaniej”, Moskwa, Ljiegkaja industrija 1973.
11. Milašius V. “ Woven Fabric’s Cross-Section: Problems, Theory, and Experimental Data” *Fibres and Textiles in Eastern Europe* No 4(23)/98, pp. 48-50.
12. Skljannikow W.P. “Strojenije i kaciestwo tkaniej”, Moskwa 1984.
13. Ariadurai, S. A., P. Polturi, et al, “Modeling the in-plane permeability of woven composites”, *Textile Research Journal* 69(5): 345-351, 1999.
14. Chen X., W-Y.Yo, A. E. Tayyar, R. J. Day, “Mouldability of Angle-Interlock woven fabrics for Technical Application” *Textile Research Journal* 72(3) 195-200; 2002.
15. Dastoor P. H., S. P. Hersh, et al.; “Computer-assisted structural design of industrial woven fabrics, Part II System operation, heuristic design.” *Journal of the Textile Institute* 85(2): pp. 110-134. 1994.
16. Handolescu-C; Nica-I; Handolescu-I, „Analytical relations between the structure and the mechanical properties of the woven fabrics”, *Industria-Textila.*; 45(3): pp. 144-150; 1994.
17. Hearle J. W. S., P. Grosberg, S. Backer; “Structural Mechanics of Fibers, Yarn and Fabrics” Wiley – Interscience, New York, pp. 371-387; 1969.
18. Lomov S. V., A. V. Gusakov C, G. Huy-smans A, A. Prodromou A., I. Verpoest, “Textile geometry preprocessor for meso-mechanical models of woven composites”, *Composites Science and Technology* 60; pp. 2083-2095; 2000.
19. Oloffson B.; „A general model of a fabric as a geometric mechanical structure” *J. Textiles Isntn*. Nr11,55, pp. 541-557; 1964.

Received 20.05.2008 Reviewed 14.01.2009