**Ryszard Korycki**

Department of Technical Mechanics
and Informatics,
Technical University of Łódź
ul. Żeromskiego 116, 90-924 Łódź, Poland;
e-mail: Ryszard.Korycki@p.lodz.pl

# Method of Thickness Optimization of Textile Structures During Coupled Heat and Mass Transport

**Abstract**

*The transient, coupled heat and mass transport within textile structures is defined by means of state equations,and a set of boundary and initial conditions. The state equations according to Henry/David and Nordon describe the mass balance within a textile structure, the conservation of energy, as well as the relationship between water vapour concentration within fibrefibres and relative humidities of the air and fibre. An arbitrary behavioral functional is introduced and its first-order sensitivity is analyzed. Simple numerical examples of both the optimisation and identification of the material thickness are presented.*

**Key words:** *optimization, sensitivity analysis, heat and mass transport.*

## Introduction

Physically speaking, the problem of heat and mass transfer within textile structures is complicated and is solved by means of different methods. Henry [10] proposed a mechanism for coupled water vapour and heat transfer into an assembly of textile fibres. This description was supplemented by David and Nordon [3]. An improved mathematical model by Henry/David and Nordon taking into account the water vapour sorption mechanism of fibres was developed by Li, Luo [17] and Li [18] to describe and predict the coupled transport in fabric. The same authors, as well as Haghi [8], discuss the boundary and initial conditions typical for textile structures. A similar model of dynamic heat and water transfer through layered fabrics is discussed by Fohr, Couton and Treguier [6]. The one-dimensional transfer within porous media is characterised by energy and mass balance, which leads to a system of differential equations. Different physical phenomena characterising coupled heat and mass transfer are described by Szekeres and Engelbrecht [22], cf., for example cross-coupled heat and moisture diffusion, cross-coupled heat and moisture convection, and the coupled diffusion and convection of heat and moisture. Chitrphiromsri and Kuznietsov [1] discuss the energy, enthalpy and equations of solid and gas phase continuities for the heat and moisture transport in firefighter protective clothing during fire exposure. The model is complicated and does not completely describe the normal conditions of personal protective clothing.

Boundary conditions are important for the solution of differential equations. The microclimate formed by textiles was discussed by Więźlak et al. [24]. Theoretical and experimental investigations were carried out on single-layer clothing material packs. One of the most important material parameters is thermal conductivity. Jirsak et al. [11] compared dynamic and static methods for measuring the thermal properties of different textile materials. Numerical calculations of the effective thermal conductivity of fibrous composite materials with an interfacial thermal resistance was discussed by Rocha and Cruz [20]. The heat transfer coefficient is determined on the basis of the immutable parameters of the textile layer, which was studied by Ziegler and Kucharska-Kot [25]. To have a comprehensive understanding of the protection and thermal comfort of protective clothing, Lee and Obendorf [15] examined the barrier and air/moisture vapour permeabilities of materials commonly used for protective clothing. Some effects of the outwear moisture transfer rate during bicycle exercise was developed by Sato et al. [21]. Coupled heat and moisture transfer within textile structures is important for understanding the dynamic thermal comfort of clothing, see Kaasjager [12]. The mathematical model introduced is used to determine the coupled transfer within fabrics as well as the thermal comfort of the user. Effective composite properties as a result of different homogenisation methods are discussed by Golański, Terada, Kikuchi [7]. The effective thermal conductivity of gas-solid composite materials was determined by Liang and Qu [16] by means of a specially developed homogenisation method. Basic physical principles of the body's mechanism for heat transfer were discussed by Turgul Ogulata [23],and the thermal insulation properties caused by different interlayers were determined by Nadzeikiene et al. [19].

The main idea of the presented paper is to optimise the thickness of textile materials subjected to coupled heat and mass transfer. The model by Henry/David and Nordon is introduced to describe the physical phenomena. Mathematically speaking, we introduce the first-order sensitivity of an arbitrary behavioral functional as well as determine the sensitivities of state fields and the sensitivity expression within the textile structure. The state variables are the temperature, water vapour concentration within the fibrefibres and water vapour concentration of the air in the interfibrefibre spaces. The material derivative concept and direct approach to sensitivity analysis are considered. This paper constitutes an extension of the previous work in the area of sensitivity analysis by Dems, Mróz [4], Dems, Korycki [5] and Korycki [13]. The optimisation of material thickness by means of sensitivity analysis of the coupled heat and moisture transfer of textiles is not considered in the literature analysed for the compled heat and moisture transfer of textiles.

## Problem of primary transient heat and mass transfer

Let us introduce a complex textile structure packed with fibres and gas within the free spaces between the fibres subjected to coupled heat and mass transfer. The heat is transported by the conduction within the fibres as well as by convection on the external surfaces of the fibres. The radiation, according the Stefan-Boltzmann Law, is proportional to the fourth power of the temperature. The values of temperature are small and the heat transfer by radiation can be now neglected. We then introduce a change the water vapour phase, i.e. the sorption of water vapour by the textile material and desorption into the interfibrefibre spaces. The water vapour diffuses through the interfibre spaces, absorbed and desorbed by the fibre, and then transferred to the surrounding by convection. To simplify both the physical and mathematical models, Li [18] introduces the following assumptions: (i) the volume changes of fibres caused by different moisture content can be neglected; (ii) the moisture is transferred through the fibres by sorption from the surrounding to the material and desorption from the fibre to the surroundings; (iii) the orientation of fibres within the textile structure plays a minimum role in the mass transport because the diameters of the fibres are small and water can travel much more rapidly in the air than in the fibres; (iv) the instantaneous thermal equilibrium between the fibres and gas in the interfibre spaces is achieved during the process of mass transfer, as most textile fibres are of very small diameter and have a very large surface/volume ratio.

The first state equation constitutes the mass balance, cf. Li [18]. The left-hand side describes the accumulation of water vapour in the interfibre spaces filled by air, as well as within the material of fibres. The right-hand side determines the transport of water vapour through the air within the interfibre spaces, cf. Henry [10]. The second state equation determines the conservation of heat energy. The left-hand side describes the energy caused by the heat conduction within the element packed with fibres and the air between the fibre, as well as by the change in the water vapour phase, i.e. the sorption of water vapour by the textile material and desorption into the interfibre spaces. The right-hand side determines the energy caused by the temperature of the fibres and air in the interfibre spaces. Both equations have the form

$$\begin{cases} \varepsilon\dfrac{dw_a}{dt} + (1-\varepsilon)\dfrac{dw_f}{dt} = \dfrac{h_a\varepsilon}{\varsigma}\dfrac{d^2w_a}{dx^2}; \\ c\dfrac{dT}{dt} - \lambda_w\dfrac{dw_f}{dt} = \lambda\dfrac{d^2T}{dx^2}; \end{cases} \quad (1)$$

where $w_a$ is the water vapour concentration in the air filling the interfibre void space, $w_f$ denotes the water vapour concentration in the fibres, $\varepsilon$ is the porosity of the fabric, t denotes real time, $h_a$ is the diffusion coefficient of water vapour in the air, $\zeta$ is the effective tortuousness of the fabric, T is the temperature, $\lambda$ is the thermal conductivity of the fabric, $\lambda_w$ denotes the heat sorption of water vapour by fibres, c is the volumetric heat capacity of the fabric.

The problem is described by means of state variables $w_a$, $w_f$, & T, and we next introduce the third state equation to solve this problem. Let us assume, according David and Nordon [3], that the content of water in the fabric is below the saturation point within the fibre for normal conditions. We also consider the sorption of water vapour by the fibres from the surrounding and desorption to the air within the free spaces between fibres, which is a typical situation for dry protective clothing during normal work. The equation proposed is an experimental, exponential relationship cf. Henry [10]; Li, Luo [17]

$$\frac{1}{\varepsilon}\frac{dw_f}{dt} = (H_a - H_f)k_1\left[1 - \exp\left(k_2|H_a - H_f|\right)\right] \quad (2)$$

where $H_a$ is the relative humidity of air, $H_f$ is the relative humidity of the fibres, $k_1$ and $k_2$ are adjustable parameters that are predicted or evaluated by comparing the model and measured mass of the fabric, cf. Haghi [8], Li and Luo [17], Li [18]. For simplicity we assume that the water vapour concentration has the same value as that measured in practice and $k_1 = k_2 = 1$. The main difficulty is to determine correlations between the water vapour concentrations $w_a$, $w_f$ and relative humidities $H_a$, $H_f$ for both the fibres and air.

The content of water within the fabric is below the saturation point within the fibre, and the water vapour is absorbed from the air, which describes the absorption coefficient $\eta$. The water vapour concentration within the porous structure is characterised acc. [2] by means of the following physical parameters: (i) water vapour pressure e; (ii) relative humidity (the water vapour concentration) w; (iii) absolute humidity H; (iv) temperature T. Following [2], we have

$$H_a > H_f;\ H_f/H_a = \eta < 1; \qquad H = e/E\cdot 100\%;\ w = 38582.80e/T \quad (3)$$

According to Henry/David and Nordon [3], we can assume the following: (i) the temperature of the surrounding air is the same as that on the surface of the fibres $T_a$=$T_f$; (ii) the saturated vapour pressure of the water has the same value in case of air and fibre surface $E_a$=$E_f$. Based on the above assumptions, we formulate the proportions of the physical parameters as follows

$$\frac{H_f}{H_a} = \frac{\frac{e_f}{E_f}\cdot 100\%}{\frac{e_a}{E_a}\cdot 100\%} = \frac{e_f}{e_a} = \eta \qquad \frac{w_f}{w_a} = \frac{38582.80\frac{e_f}{T_f}}{38582.80\frac{e_a}{T_a}} = \frac{e_f}{e_a} = \eta \quad (4)$$

Let us next introduce the absorption coefficient $\eta$ as time-independent. Differentiating Equations (4) with respect to the real time t we obtain $dH_f/dt = \eta dH_a/dt$; $dw_f/dt = \eta dw_a/dt$.

$$\frac{dH_f}{dt} = \eta\frac{dH_a}{dt}; \quad \frac{dw_f}{dt} = \eta\frac{dw_a}{dt}.$$

Introducing Equation (2) into the set of Equations (1), we formulate the correlations

$$\begin{cases} \left(1-\varepsilon+\dfrac{\varepsilon}{\eta}\right)\dfrac{dw_f}{dt} = \dfrac{h_a\varepsilon}{\varsigma\eta}\dfrac{d^2w_f}{dx^2}; \\ c\dfrac{dT}{dt} - \lambda_w\dfrac{dw_f}{dt} = \lambda\dfrac{d^2T}{dx^2}. \end{cases} \quad (5)$$

Let us next consider that the textile structure analysed undergoes normal conditions. The isolation layer of air comes into contact with the skin and undergoes slight changes in the assumed atmosphere. The values of state variables on the skin's surface are known i.e. first-kind conditions exist on the boundaries $\Gamma_T$ and $\Gamma_1$ for x = 0. Both the temperature T and moisture concentration $w_f$ are characterised, for example, by Więźlak et. al. [24]. Let us next consider the convective nature of the fabric as well as the heat and mass exchange between the fabric and surrounding air. The boundaries $\Gamma_C$ and $\Gamma_3$ for x = L are characterised by third-kind conditions. We formulate the mixed conditions at both ends of the model; compare ***Figure 1.***

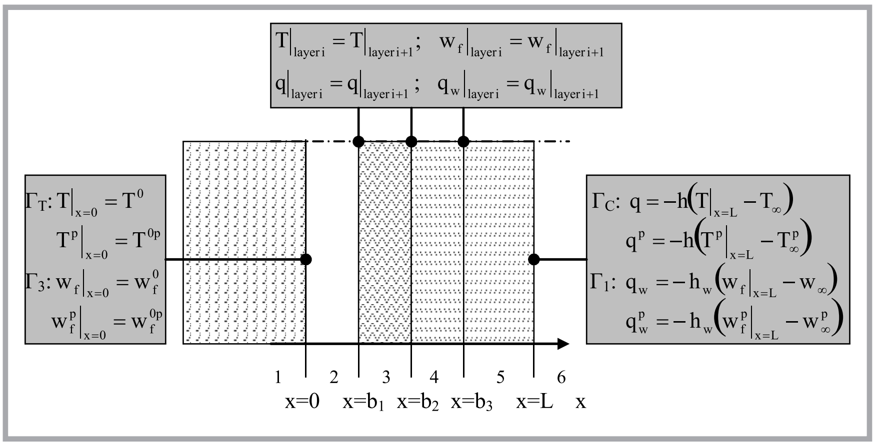


***Figure 1.*** *Schematic representation of a cross-section of a textile structure and boundary conditions for primary problem1 – human skin; 2 – isolation layer of air between skin and textile structure; 3, 4, 5 – layers of textile structure; 6 – surrounding.*

$$T|_{x=0} = T^0 \quad x \in \Gamma_T; \qquad w_f|_{x=0} = w_f^0 \quad x \in \Gamma_1;$$
$$q = \lambda\varepsilon \left.\frac{dT}{dx}\right|_{x=L} = -h\left(T|_{x=L} - T_\infty\right) \quad x \in \Gamma_C; \tag{6}$$
$$q_w = \lambda_w \varepsilon \left.\frac{dw_f}{dx}\right|_{x=L} = -h_w\left(w_f|_{x=L} - w_\infty\right) \quad x \in \Gamma_3$$

where h is the heat convection coefficient, $h_w$ the mass convection coefficient, $T_\infty$ the surrounding temperature, and $w_\infty$ the water vapour concentration of the surrounding. Fourth-kind boundary conditions are defined on the common surface of two structures or two material phases in contact. Thus the conditions are of practical importance for multilayer composite structures. The state variables as well as the heat and mass flux densities are continuous. The initial conditions are the prescribed temperature and water vapour concentration at the beginning of the process. All conditions have the form

$$\begin{aligned}
&T(x,t)|_{layer\,i} = T(x,t)|_{layer\,i+1};\\
&w_f(x,t)|_{layer\,i} = w_f(x,t)|_{layer\,i+1};\\
&q(x,t)|_{layer\,i} = q(x,t)|_{layer\,i+1};\\
&q_w(x,t)|_{layer\,i} = q_w(x,t)|_{layer\,i+1};\\
&T(x,0) = T_0; \quad w_a(x,0) = w_{a0};\\
&w_f(x,0) = w_f(w_\infty, T_0)
\end{aligned} \tag{7}$$

The optimisation can be simplified by introducing the same cross-section of the textile structure and conditions of heat and mass transfer. The problem is analysed in an optional cross-section of the structure as a 1D problem, cf. ***Figure 1***. Let us assume that the textile is an integrated composite structure i.e. the isolation layers of air between the internal material layers can be neglected. Only one isolation layer of air comes into contact with the human skin and first layer of clothing, cf. ***Figure 1***. The thickness of the layer was assumed as equal to $\delta = 10^{-3}$m.

## First-order sensitivity of an arbitrary functional

Let us next locate the material phases by means of the vector of design parameters **b,** cf. ***Figure 1***. We define an arbitrary behavioral functional associated with the problem as the Equation 8, where $\Psi_1$, $\Psi_2$, $\gamma_1$, $\gamma_2$ are continuous and differentiable functions of the listed arguments. The integrands $\gamma_1$, $\gamma_2$ are considered as the sum at both ends of the 1D model analysed.

The first variation of the objective functional with respect to design parameters has the form $\delta F = DF/Db_p \cdot \delta b_p$; p = 1…P; where $F_p = DF/Db_p$ is the material derivative of the functional F, assumed as the first-order sensitivity of this functional.

The shape modification is described for 1D model of the structure by means of the transformation velocity field $v^p(x,t,\mathbf{b})$, associated with the design parameter $b_p$, treated as a time-like parameter, cf. Korycki [13]. We next formulate the following correlations for any continuous function g and its gradient, cf. Haug, Choi, Komkov [9]

$$\begin{aligned} g_p &= g^p + g_{,x}\, v^p;\\ (g_{,x})_p &= (g_p)_{,x} - g_{,x}\, v^p_{,x} \end{aligned} \tag{9}$$

where $g_p = Dg/Db_p$ is the global (or material) derivative of the function g with respect to the design parameter $b_p$ and $g^p = \partial g/\partial b_p$ denoting the local (or partial) derivative of the function g with respect to the design parameter $b_p$. The first-order sensitivity of an arbitrary functional F can be expressed with respect to Equation (8), Equations (9) and the material derivative of the length element in the basic form, cf. Dems, Mróz [4], Dems, Korycki [5] (Equation 10).

The sensitivity is analysed by means of the direct approach, which is the most useful for calculating the sensitivities of the entire response field with respect to a few design variables. Dems, Mróz [4], Dems, Korycki [5] and Korycki [13] analysed the direct approach. The first-order sensitivities are formulated by means of additional heat and mass transfer problems associated with the variation of each design parameter $b_p$; p = 1,…P. The additional structure has the same shape as well as properties of heat and mass transfer as the primary one. The correlations are determined by the differentiation of primary equations with respect to design

$$F = F_1 + F_2 = \int_0^{t_f}\left[\int_L \Psi_1(T, T_{,x}, q, f, \dot{T})\,dL + \sum_{i=1}^{2}\gamma_1(T, q, T_\infty)\right]dt + \int_0^{t_f}\left[\int_L \Psi_2(w_f, w_{f,x}, q_w, \dot{w}_f)\,dL + \sum_{i=1}^{2}\gamma_2(w_f, q_w, w_{f\infty})\right]dt \tag{8}$$

$$\begin{aligned}
F_p = \int_0^{t_f}\Bigg\{&\int_L\left[\Psi_{1,T}\,T_p + \nabla_{Tx}\Psi_1 (T_{,x})_p + \Psi_{1,q}\,q_p + \Psi_{1,f}\,f_p + \Psi_{1,\dot{T}}\,\dot{T}_p + \Psi_1 v^p_{,x}\right]dL + \\
&+ \int_L\left[\Psi_{2,w_f}\,W_{fp} + \nabla_{wx}\Psi_2 (w_{f,x})_p + \Psi_{2,q_w}\,q_{wp} + \Psi_{2,\dot{w}_f}\,\dot{w}_{fp} + \Psi_2 v^p_{,x}\right] + \\
&+ \sum_{i=1}^{2}\left[\gamma_{1,T}\,T_p + \gamma_{1,q_n}\,q_p + \gamma_{1,T_\infty}(T_\infty)_p\right] + \sum_{i=1}^{2}\left[\gamma_{2,w_f}\,w_{fp} + \gamma_{2,q_{nw}}(q_w)_p + \gamma_{2,w_{f\infty}}(w_{f\infty})_p\right]\Bigg\}dt
\end{aligned} \tag{10}$$

(8)

where: $\Psi_{1,T} = \dfrac{\partial \Psi_1}{\partial T}$; $\nabla_{Tx}\Psi_1 = \dfrac{\partial \Psi_1}{\partial T_{,x}}$; $\Psi_{1,q} = \dfrac{\partial \Psi_1}{\partial q}$; $\Psi_{1,f} = \dfrac{\partial \Psi_1}{\partial f}$; $\Psi_{2,\dot{w}_f} = \dfrac{\partial \Psi_2}{\partial \dot{w}_f}$;

$\gamma_{1,T} = \dfrac{\partial \gamma_1}{\partial T}$; $\gamma_{1,q} = \dfrac{\partial \gamma_1}{\partial q}$; $\gamma_{1,T_\infty} = \dfrac{\partial \gamma_1}{\partial T_\infty}$; $\gamma_{2,w_f} = \dfrac{\partial \gamma_2}{\partial w_f}$; $\gamma_{2,q_w} = \dfrac{\partial \gamma_2}{\partial q_w}$; $\gamma_{2,w_{f\infty}} = \dfrac{\partial \gamma_2}{\partial w_{f_\infty}}$

***Equations 8, 10.***

parameters. The state variables are the temperature $T^p = \dfrac{\partial T}{\partial b_p}$ and water vapour concentration $w_f^p = \dfrac{\partial w_f}{\partial b_p}$.

Let us first determine the state equations by the differentiation of Equations (5). Assuming that the material parameters are independent design parameters, we obtain the correlations

$$\begin{cases} \left(1-\varepsilon+\dfrac{\varepsilon}{\eta}\right)\dfrac{dw_f^p}{dt} = \dfrac{h_a\varepsilon}{\varsigma\eta}\dfrac{d^2w_f^p}{dx^2}; \\ c\dfrac{dT^p}{dt} - \lambda_w\dfrac{dw_f^p}{dt} = \lambda\dfrac{d^2T^p}{dx^2}; \end{cases} \tag{11}$$

Our next goal is to determine the boundary and initial conditions for the additional structure by means of the mixed conditions, cf. Equations (6) and Equations (7), cf. ***Figure 1.***

$$\begin{aligned} &T^p\big|_{x=0} = T^{0p} = T_p^0 - T^0{,_x}\, v^p \quad x \in \Gamma_T; \\ &w_f^p\big|_{x=0} = w_f^{0p} = w_{fp}^0 - w_f^0{,_x}\, v^p \quad x \in \Gamma_1; \\ &q^p = \lambda\varepsilon \dfrac{dT^p}{dx}\Big|_{x=L} = -h\left(T^p\big|_{x=L} - T_\infty^p\right) \quad x \in \Gamma_C; \\ &q_w^p = \lambda\varepsilon \dfrac{dw_f^p}{dx}\Big|_{x=L} = -h_w\left(w_f^p\big|_{x=L} - w_\infty^p\right) \quad x \in \Gamma_3. \\ &T_0^p(x,0) = T_{0p} - T_0{,_x} \cdot v^p; \\ &w_{a0}^p(x,0) = w_{a0p} - w_{a0}{,_x} \cdot v^p; \\ &w_{f0}^p(x,0) = w_{f0p} - w_{f0}{,_x} \cdot v^p. \end{aligned} \tag{12}$$

We next formulate the sensitivity expression of the functional described by Equation (8). Applying Equations (9) to Equation (10) and integrating by parts and in time adequate terms within the correlation obtained, we formulate the first-order sensitivity presented by Equation (13).

To determine the state variables, we solve one primary and the set of additional problems. The number of problems is the same as the number of parameters, i.e. for design parameters P we should solve (P + 1) problems of the heat and mass transfer. Each additional problem is defined by Equations (11) and Equations (12).

## Problems of optimisation and identification

Shape optimisation is the minimisation of the objective functional with a constraint imposed on the material cost. Assuming the homogeneous 1D textile of the unit cost u, the cost is proportional to the width of structure L, which is determined

$$\text{minimise } F \text{ or minimise } (-F) \text{ for} \quad C - C_0 = uL - C_0 = 0 \tag{14}$$

We introduce the Lagrange functional $F' = F + \chi(C - C_0)$, where $\chi$ is the Lagrange multiplier. Following the stationarity of the above functional, the stationarity conditions are defined as

$$\frac{DF}{Db_p} = -\chi uL \quad \text{and} \quad uL - C_0 = 0 \tag{15}$$

Consequently, we introduce the first-order sensitivity expression, cf. Equation (13) and define the most used objective functionals. We can introduce the following measures

$$F_1 = \int_0^{t_f} q\big|_{\Gamma_T} dt; \quad F_2 = \int_0^{t_f} q_w\big|_{\Gamma_1} dt. \tag{16}$$

where q is the heat flux density and $q_w$ the mass flux density. Minimisation of the above functional corresponds to the design of both an optimal heat and mass isolator, whereas for a model of a heat radiator and mass transporter, the functionals should be maximised. The objective functional can be the global measure of maximum local state variables

$$\begin{aligned} F_1 &= \int_0^{t_f} \left\{ \left[ \left( \frac{T}{T_0} \right)_{\Gamma_T}^n \right]^{\frac{1}{n}} \right\} dt \;;\; n \to \infty; \\ F_2 &= \int_0^{t_f} \left\{ \left[ \left( \frac{w_f}{w_{f0}} \right)_{\Gamma_1}^n \right]^{\frac{1}{n}} \right\} dt \;;\; n \to \infty \end{aligned} \tag{17}$$

Mathematically speaking, the problem of identification is defined as the minimisation of the objective functional without constraints. Stationary conditions of the problem $DF/Db_p = 0$ are defined by means of the first-order sensitivity, cf. Equation (13). The temperature of the real structure $T_m$ and water vapour concentration within the fibres $w_{fm}$ are measured during the identification at point $\Gamma_m$ of the 1D structure. The objective functional most used are the "distances" between the state variables of the model and real structure identified

$$\begin{aligned} F_1 &= \frac{1}{2}\int_0^{t_f} \left[ (T - T_m)^2 \right]_{\Gamma_m} dt \;; \\ F_2 &= \frac{1}{2}\int_0^{t_f} \left[ (w_f - w_{fm})^2 \right]_{\Gamma_m} dt \end{aligned} \tag{18}$$

The functionals can be the measures of state equations at the point $\Gamma_m$, defined as follows

$$\begin{aligned} F_1 &= \int_0^{t_f} \left\{ \left[ \left( \frac{T}{T_m} \right)_{\Gamma_m}^n \right]^{\frac{1}{n}} \right\} dt \;;\; n \to \infty; \\ F_2 &= \int_0^{t_f} \left\{ \left[ \left( \frac{w_f}{w_{fm}} \right)_{\Gamma_m}^n \right]^{\frac{1}{n}} \right\} dt \;;\; n \to \infty \end{aligned} \tag{19}$$

These functionals are homogeneous and can be used during the modification by expansion or contraction of the boundary. Minimisation of the functionals reduces the "distance" between temperatures T and $T_m$ and concentrations $w_f$ and $w_{fm}$ as well as minimises maximum local tem-

$$\begin{aligned} F_p = &\left[\int_L \Psi_{1,\dot{T}}\, T^p dL\right]_0^{t_f} + \left[\int_L \Psi_{2,\dot{w}_f}\, w_f^p dL\right]_0^{t_f} + \int_0^{t_f}\left(\sum_{i=2}^{2}\Psi_1 v^p\right)dt + \int_0^{t_f}\left(\sum_{i=2}^{2}\Psi_2 v^p\right)dt + \\ &+ \int_0^{t_f}\int_L \Big\{ \nabla_{Tx}\Psi_1 (T,_x)^p + \Psi_{1,q}\, q^p + \Psi_{1,f}\, f^p + \left[\Psi_{1,T} - \frac{d}{dt}\left(\Psi_{1,\dot{T}}\right)\right] T^p \Big\} dL + \\ &+ \int_L \Big\{ \nabla_{wx}\Psi_2 (w_f,_x)^p + \Psi_{2,q_w}\, q_w^p + \left[\Psi_{2,w_f} - \frac{d}{dt}\left(\Psi_{2,\dot{w}_f}\right)\right] w_f^p \Big\} dL + \\ &+ \sum_{i=1}^{2}\Big[\gamma_{1,T}\, T_p^0 + \gamma_{1,q}\left(q^p + q,_x v^p\right) + \gamma_{1,T_\infty}\left(T_\infty^p + T_\infty,_x v^p\right)\Big]_{\Gamma_T} + \\ &+ \sum_{i=1}^{2}\Big[\gamma_{1,T}\left(T^p + T,_x v^p\right) + \gamma_{1,q}\left(q^p + q,_x v^p\right) + \gamma_{1,T_\infty}\left(T_\infty\right)_p^0\Big]_{\Gamma_C} + \\ &+ \sum_{i=1}^{2}\Big[\gamma_{2,w_f}\left(w_f^0\right)_p + \gamma_{2,q_w}\left(q_w^p + q_w,_x v^p\right) + \gamma_{2,w_{f\infty}}\left(w_{f\infty}^p + w_{f\infty},_x v^p\right)\Big]_{\Gamma_1} + \\ &+ \sum_{i=1}^{2}\Big[\gamma_{2,w_f}\left(w_f^p + w_f,_x v^p\right) + \gamma_{2,q_w}\left(q_w^p + q_w,_x v^p\right) + \gamma_{2,w_{f\infty}}\left(w_{f\infty}^0\right)_p\Big]_{\Gamma_3} \Big\} dt. \end{aligned} \tag{13}$$

***Equation 13.***

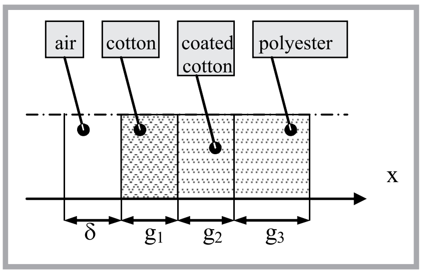


*Figure 2. Design variables for optimisation problem.*

perature and water vapour concentration values within fibres.

The third form of identification functional can be the adaptation of Damage Location Assurance Criterion (DLAC), discussed for mechanical problems, for example, by Krawczuk, Żak, Ostachowicz [14]. The criterions now have the following form

$$F_1 = \int_0^{t_f} \left[ \frac{(T_m T)^2_{\Gamma_m}}{(T_m T_m)_{\Gamma_m} (T\ T)_{\Gamma_m}} \right] dt \ ;$$

$$F_2 = \int_0^{t_f} \left[ \frac{(w_{fm} w_f)^2_{\Gamma_m}}{(w_{fm} w_{fm})_{\Gamma_m} (w_f\ w_f)_{\Gamma_m}} \right] dt \qquad (20)$$

The correlations between temperatures T and $T_m$ as well as concentrations $w_f$ and $w_{fm}$ range from 0 (no correlation between state variables) to 1 (full correlation).

## Numerical examples

### Shape optimisation

Let us optimise the thickness of the composite textile by means of the 1D model made of three different materials, cf. ***Figure 2***. The internal layer #1 is made of cotton and is characterised according to Li [18] by the thermal conductivity of fibres $\lambda = 54.1 \cdot 10^{-2}$ W/(mK); the heat sorption of water vapour by fibres $\lambda_w = 3552.9 \cdot 10^3$ J/kg; the volumetric heat capacity of dry fibres $c = 1863 \times 10^3$ J/(m$^3$K), the effective porosity $\varepsilon = 0.95$; the effective tortuousness of fabric $\zeta = 1.50$; the density of fibres $\rho = 1350$ kg/m$^3$; the mass convection coefficient $h_w = 0.17$ m/s; and the heat convection coefficient $h = 105$ W/(m$^2$K).

***Table 1.*** *Initial and optimal thickness of material layers within a textile structure.*

| Initial/ optimal | Material thickness ·10$^{-3}$, m | | | |
|---|---|---|---|---|
| | d | $g_1$ | $g_2$ | $g_3$ |
| Initial | 1.0 | 3.0 | 2.0 | 2.0 |
| Optimal | 1.0 | 2.12 | 1.89 | 2.99 |

Isolation layer #2 is made of coated cotton which is the result of the finishing procedure. Some parameters are different, i.e. the thermal conductivity of fibres $\lambda = 58 \times 10^{-2}$ W/(mK); the heat sorption of water vapour by fibres $\lambda_w = 3100 \times 10^3$ J/kg; the volumetric heat capacity of dry fibres $c = 1910 \times 10^3$ J/(m$^3$K). The other values are the same as for layer #1.

External layer #3 is made of polyester and the material parameters are the following, cf. Li [18]: the thermal conductivity of fibres $\lambda = 50 \times 10^{-2}$ W/(mK); the heat sorption of water vapour by fibres $\lambda_w = 2522 \times 10^3$ J/kg; the volumetric heat capacity of dry fibres $c = 1531 \times 10^3$ J/(m$^3$K), the effective porosity $\varepsilon = 0.98$; the effective tortuousness of fabric $\zeta = 1.95$; the density of fibres $\rho = 1255$ kg/m$^3$; the mass convection coefficient $h_w = 0.01$ m/s; and the heat convection coefficient $h = 120$ W/(m$^2$K).

Let us assume that the temperature of skin changes with time according to the function $T = 309 + \exp(0.0005\ t)$; for the time parameters $t_0 = 0$; $t_k = 360$ s; $\Delta t = 90$ s. At the same end of the structure, the water vapour concentration within fibres changes with time by means of the function $w_f = 0.05 + 0.25t - 0.001t^2$; the time parameters have the same values.

The optimisation problem can be determined as the search for the material thickness, which secures the minimum heat and mass transfer throughout the textile structure. Mathematically speaking, we maximise the objective functional F, i.e. minimise (-F). The optimisation functional Equations (16) are defined on the external boundary. The first-order sensitivity of the optimisation functional is determined by Equation (13).

The constraints are the constant values of the maximal and minimal thickness of each material layer. The first stage of optimisation is the analysis procedure performed by means of the Finite Element Net of 50 nodes. The calculations at the synthesis stage were determined by applying the external penalty function. The locations of the material phases are determined in ***Table 1***. The optimal configuration was found in 9 iterations.

The polyester is not laminated with cotton, and due to the not smooth textile surface of the material, a small air layer between the coated cotton and polyester arises. This layer has a very small thickness and can be neglected in relation to the layer between the skin and textile. From ***Table 1*** we conclude that the layer made of polyester is the best to secure the minimisation of heat and mass transfer through the multilayer textile structure. The thickness of both cotton layers is reduced in relation to initial values. Consequently, the optimal value of the objective functional is equal to 87% of the initial one.

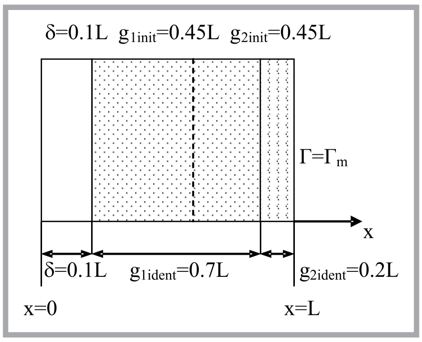


***Figure 3.*** *Initial and identified locations of the material phases.*

### Shape identification

We assume that the structure is made of wool and the textile material has two layers, which is the result of the finishing process, cf. ***Figure 3***. Let us assume that the first layer is made of pure wool, whereas the second layer is of wool coated by a special substance during the finishing process.

The pure wool which creates layer #1 is characterised according to Li and Luo [17] by the thermal conductivity of fibres $\lambda = 38.49 \cdot 10^{-2}$ W/(mK); the heat sorption of water vapour by fibres $\lambda_w = 4124.5 \cdot 10^3$ J/kg; the volumetric heat capacity of dry fibres $c = 1609.7 \times 10^3$ J/(m$^3$K), the effective porosity $\varepsilon = 0.925$; the effective tortuousness of fabric $\zeta = 1.20$; the density of fibres $\rho = 1320$ kg/m$^3$; the mass convection coefficient $h_w = 0.137$ m/s; and the heat convection coefficient $h = 99.4$ W/(m$^2$K).

The coating substance on the surface of layer #2 changes the material characteristics. The thermal conductivity of the fibres is equal to $\lambda = 42 \times 10^{-2}$ W/(mK); the heat sorption of water vapour by fibres $\lambda_w = 4500 \times 10^3$ J/kg; and the volumetric heat capacity of dry fibres $c = 1700 \times 10^3$ J/(m$^3$K). Other parameters have the same values as those of pure wool in layer #1.

The diffusion coefficient of water vapour in air is equal to $h_a = 2.49 \cdot 10^{-5}$ m/s, cf. Li [18].

The outer part of the textile structure has prescribed values of state variables. Temperature changes with time according to the exponential function $T_m = 320 + \exp(0.002t)$; for time parameters $t_0 = 0$; $t_k = 240\,0$ s; $\Delta t = 60$ s. Similarly, water vapour concentration within fibres changes with time according to the function $w_{fm} = 0.05 + 0.25t - 0.001t^2$; for the same time parameters.

The problem of identification is solved by means of the "distances" between the state variables, cf. Equations (18). Both state variables are measured at boundary point $\Gamma_m$, described by the coordinate $x = L$. The first-order sensitivity is defined by Equation (13).

The analysis stage was performed using a Finite Element Net of 50 nodes. The calculations at the synthesis stage were determined by the external penalty function. The locations of the initial and identified phases are shown in ***Figure 3***. The structure was identified in 8 iterations.

## Conclusions

Physically speaking, a multilayer textile structure packed with fibres and gas within free spaces gives different conditions of transport phenomena. In the paper presented we have analysed the mass balance, the conservation of energy and the experimental relationship for the content of water vapour within fabric situated below the saturation point. Consequently, we obtain the model introducing the sorption and desorption of water during transport. The complicated description of the transient coupled heat and mass transfer can be simplified. The equations are difficult to solve for transient problems by means of analytical methods, and the state variables should be determined numerically.

The steady problem of heat and mass transfer is easy to determine because we integrate the differential equations separately with respect to the length parameter and introduce a set of boundary conditions. The results are functions describing the distribution of temperature and the distribution of water vapour concentration within fibres. Both are linear with respect to the length parameter x.

The first-order sensitivity of an arbitrary behavioral functional is analysed here by means of the direct approach. The first-order sensitivity expression obtained can be applied to solve stationary conditions of identification. Additionally, we should introduce the objective functional which defines the necessary form of the first-order sensitivity correlation.

Thus, the methods discussed can be an effective tool for generating state variables in 1D problems of coupled heat and mass transfer as well as for identifying the shape.

The problem should be verified by means of laboratory tests. Detailed analysis of such implementations is beyond the scope of this paper and will be studied in detail in a consecutive publication.


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