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Theoretical Study of Yarn Torque Caused by Fibre Tension in the Spinning Triangle

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Abstract
The spinning triangle is a critical region in the spinning process of staple yarn, the geometry of which plays an important role in determining the physical performance and qualities of spun yarns, especially yarn torque. It has been shown that the fibre tension in the spinning triangle is the most influential factor governing the magnitude of yarn torque. Therefore, in this paper, a theoretical study of yarn torque caused by fibre tension in the spinning triangle is presented. Cases of yarn with one, two, three, four and five feeding strands were investigated, in which the arrangement of fibres was assumed as hexagonal close packing with a single core fibre, two core fibres, three core fibres, four core fibres, and five core fibres at the centre, respectively. Theoretical models of the fibre tension in spinning triangles and corresponding yarn residual torque due to the fibre tension are presented. As an application of the method proposed, 26.5 tex cotton yarns were taken as an example for numerical simulations. Fibre tension in the spinning triangles and corresponding yarn torque were simulated numerically using Matlab software.

Key words: *ring spinning, spinning triangle, yarn torque, fibre tension.*

Introduction

Yarn torque is one of the properties of most concern in evaluating yarn performance and further textile processing, which is determined by the mechanical state of the constituent fibres and their configuration in yarns [1 - 4]. Existing results indicate that fibre tension, torsion and bending are the three main components contributing to the yarn torque, and the fibre tension within a taut yarn is the most influential factor governing the magnitude of yarn torque [4] while fibre tension is produced in the spinning triangle, determined by its geometry. Therefore yarn torque caused by fibre tension in the spinning triangle is discussed in this paper.

The spinning triangle is a critical region in the spinning process of staple yarn, in which yarns are finally formed by twisting an assembly of short fibres [13]. Therefore the geometric and mechanical performances of the spinning triangle play an important role in determining the distribution of fibre tension in the spinning triangle and the physical performance of spun yarns correspondingly [5].The subject of spinning triangle has been one of the most important research topics and attracted more and more attention recently. A number of theoretical investigations have been carried out [6 - 10], in which the energy method plays an important role. In order to predict the distribution of tension forces of the fibres in a symmetric spinning triangle, a theoretical model was developed primarily using the energy method [8]. Then this model was further extended to the asymmetric spinning triangle by considering the migration of the axis fibre at the front roller nip [1] and the inclination angle of the spinning tension [6], respectively. Furthermore quantitative relationships between the mechanical performance of a ring spinning triangle and the spinning parameters were investigated using the Finite Element Method (FEM) [5].

Since fibre tension in the spinning triangle is the most influential factor for yarn torque, there have been lots of theoretical researches linking fibre tension in the spinning triangle to yarn residual torque [1, 7, 11]. In general, the research is carried out in three steps: Firstly fibre tension distributions in the spinning triangle should be given. Secondly the average fibre tension in each layer is presented according to the fibre arrangement within a yarn. Finally the yarn torque caused by fibre tension can be obtained. Therefore the fibre arrangement within a yarn would also influence the yarn torque. In the general case, for ease of analysis, all fibres are assumed to be ideally packed in the yarn cross-section in concentric circular rings [1]. There are two ideal packing models used in the general case: open packing and the hexagonal close packing model [12]. Therefore, in this paper, a theoretical study of yarn torque caused by fibre tension in the spinning triangle is presented, and the arrangement of fibres in the yarn is assumed as open packing and hexagonal close packing with a single core fibre, two core fibres, three core fibres, four core fibres, and five core fibres at the centre, respectively.

Theoretical analysis

In this section, yarn torque caused by fibre tension in the spinning triangle will be investigated theoretically. The following assumptions are made [1, 11].

Assumption 1. The ends of all fibres gripped in the front roller nip distribute evenly, i.e. $w_i = w_{i'} = iw/(2n)$, where w_i is the distance between the end of the i-th fibre on the right side and point O, $w_{i'}$ - the corresponding i'-th fibre on the left side, and $2n + 1$ is the total number of fibres in the spinning triangle.

Assumption 2. The central fibre in the spinning triangle runs into the core of the yarn, and fibres near the central fibre prefer to be arranged close to the core of the yarn successively.

Assumption 3. During the yarn forming process, the fibre is elastic and its tension created in the spinning triangle will be kept completely when the fibres are transferred to the yarn from the spinning triangle.

Assumption 4. All the fibres are ideally packed in the yarn cross-section in concentric circular rings. The arrangement of fibres in the yarn is open packing or hexagonal close packing with a single core fibre, two core fibres, three core fibres, four core fibres or five core fibres at the center, respectively, and around the center six fibres are arranged [12]. If the arrangement is open packing, the number of fibres arranged in each layer is given by:

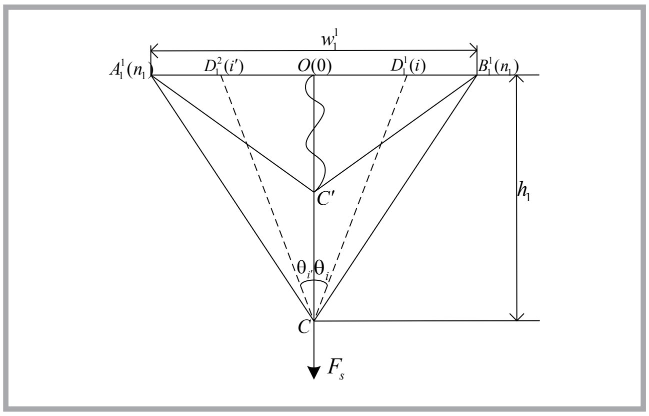

Figure 1. *Model of symmetrical spinning triangle.*

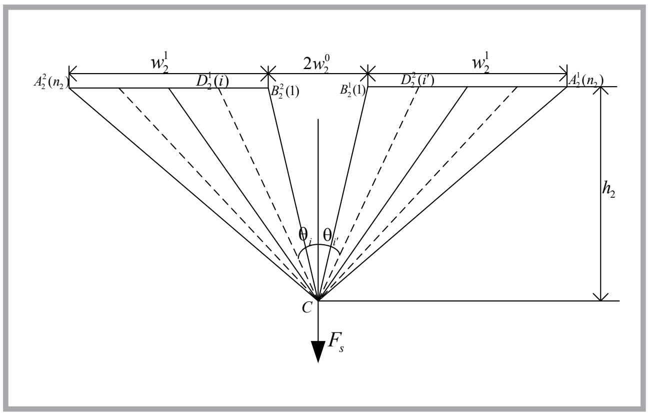

Figure 2. *Spinning triangle model with two strands.*

$$\begin{cases} N_j = 1 & j = 1 \\ N_j = \mathrm{Int}\left(180/\arcsin\dfrac{1}{2(j-1)}\right) & j \geq 2 \end{cases} \quad (1)$$

where Int(x) is the integral function.

If the arrangement is hexagonal close packing, the number of fibres arranged in each layer is given by

$$\begin{cases} N_j = 1 & j = 1 \\ N_j = 6(j-1) & j \geq 2 \end{cases} \quad (2)$$

or

$$\begin{cases} N_j = n_0 & j = 1 \\ N_j = 6(j-1) + n_0 & j \geq 2 \end{cases} \quad (3)$$

where $n_0 = 2, 3, 4, 5$ is the number of core fibres at the centre.

Theoretical analysis of yarn torque with one feeding strand

A model of a symmetrical spinning triangle with one feeding strand is shown in ***Figure 1***. Here, O is the middle point of the nip line, C' the initial convergence point without any force, and C is the twisting point with constant force F_s. w_1^1 and h_1 are the height and width of the spinning triangle, respectively. θ_i is the angle between the right i-th fibre and central fibre, and $\theta_{i'}$ the corresponding angle between the left i'-th fibre and central fibre. Let us suppose that the number of fibres at each side of the central fibre is n_1, i.e. there are $2n_1 + 1$ fibres in the spinning triangle.

Then using the principle of minimum potential energy and basing on the analysis in [6, 7], the distribution of fibre tension in the spinning triangle, as shown in ***Figure 1***, can be given as ***Equation 4*** where, A is the cross-section of fibre, and E is the fibre tensile Young's modulus, $M_i = \dfrac{1}{\cos\theta_i}$, $M_{i'} = \dfrac{1}{\cos\theta_{i'}}$.

According to ***Figure 1***, we have

$$\tan\theta_i = \frac{w_i}{h_1}, \tan\theta_{i'} = \frac{w_{i'}}{h_1} \quad (5)$$

Where $w_i = iw_1^1/(2n_1)$, $w_{i'} = i'w_1^1/(2n_1)$. The yarn torque caused by fibre tension in the spinning triangle can be attained if the arrangement of fibres in the yarn is hexagonal close packing with a single core fibre [1, 11]. Similarly the yarn torque can be obtained if the arrangement of fibres in the yarn is open packing.

Theoretical analysis of yarn torque with two feeding strands

A model of a spinning triangle with two symmetrical feeding strands is shown in ***Figure 2***. Here, w_2^1 is the width of the spinning triangle for each strand, and $2w_2^0$ is the distance between two strands. θ_i is the angle between the θ_i-th fibre in the left (first) strand and the axisymmetric line of the spinning triangle, and $\theta_{i'}$ is the corresponding angle between the i'-th fibre in the right (second) strand and the axisymmetric line. Let us suppose that the number of fibres in each strand is n_2, i.e. there are $2n_2$ fibres in the spinning triangle.

Then, using the principle of minimum potential energy, the distribution of fibre tension in the spinning triangle, as shown in ***Figure 2***, can be given as ***Equation 6*** where:

$$M_i = \frac{1}{\cos\theta_i},\ M_{i'} = \frac{1}{\cos\theta_{i'}},$$

$$\tan\theta_i = \frac{w_i + w_2^0}{h_2},\ \tan\theta_{i'} = \frac{w_{i'} + w_2^0}{h_2},$$

$$w_i = \frac{iw_2^1}{n_2},\ w_{i'} = \frac{i'w_2^1}{n_2}.$$

$$\begin{aligned} F_{i=0,1,\cdots,n_1} &= \frac{F_s - 2AE\sum_{i=1}^{n_1} M_i\left(M_i - 1\right)}{2\sum_{i=1}^{n} M_i^2 + 1} M_i + AE\left(M_i - 1\right) \\ F_{i'=0,1,\cdots,n_1} &= \frac{F_s - 2AE\sum_{i=1}^{n_1} M_i\left(M_i - 1\right)}{2\sum_{i=1}^{n_1} M_i^2 + 1} M_{i'} + AE\left(M_{i'} - 1\right) \end{aligned} \quad (4)$$

$$\begin{aligned} F_{i=1,\cdots,n_2} &= \frac{F_s - 2AE\sum_{i=1}^{n_2} M_i\left(M_i - 1\right)}{2\sum_{i=1}^{n} M_i^2} M_i + AE\left(M_i - 1\right) \\ F_{i'=1,\cdots,n_2} &= \frac{F_s - 2AE\sum_{i=1}^{n_2} M_i\left(M_i - 1\right)}{2\sum_{i=1}^{n_2} M_i^2} M_{i'} + AE\left(M_{i'} - 1\right) \end{aligned} \quad (6)$$

Equations 4, 6.

Then, based on the assumption above, yarn residual torque caused by fibre tension in the spinning triangle can be given as follows, for two cases investigated.

Case 1. $2n_2 = \sum_{j=1}^{N}[6(j - 1) + 2]$

In this case, all $2n_2$ fibres are arranged in hexagonal close packing in a yarn with two core fibres at the centre, and $2n_2$ - 2 fibres are arranged from the second layer to the Nth layer exactly, with $3(j - 1) + 1$ fibres in each strand distributed in the jth layer. The average fibre tension of each layer within a yarn T_j can be given as follows:

$$\begin{cases} T_j = \dfrac{F_{i=1} + F_{i'=1}}{2} & j = 1 \\ T_j = \dfrac{\sum_{i=1+\alpha_j}^{\beta_j} F_i + \sum_{i'=1+\alpha_j}^{\beta_j} F_{i'}}{6(j-1)+2} & 2 \le j \le N \end{cases} \quad (7)$$

where

$$\alpha_j = \sum_{k=1}^{j}[3(k - 1) + 1] - 3(j-1) - 1,$$

$$\beta_j = \sum_{k=1}^{j}[3(k - 1) + 1].$$

Then, the contribution from the tension of all fibres in each layer to the yarn torque is given by ***Equation 8*** where r is the fibre radius and θ the yarn surface helix angle.

Finally the total yarn torque due to fibre tension in the spinning triangle is given as follows:

$$L = \sum_{j=1}^{N} L_j \quad (9)$$

Case 2.

$$\sum_{j=1}^{N-1}[6(j - 1) + 2] < 2n_2 < \sum_{j=1}^{N}[6(j - 1) + 2]$$

In this case, the first $\sum_{j=1}^{N-1}[6(j - 1) + 2]$ fibres are arranged in hexagonal close packing within a yarn from the first layer to the $(N - 1)$layer exactly. The number of fibres in the Nth layer does not fill in the layer completely. The average fibre tension of each layer within a yarn T_j can be given as ***Equation 10***:

$$L_j = 12(j-1)^2 T_j r \sin\left(\arctan\left(\frac{2(j-1)}{2N-1}\tan\theta\right)\right) \quad (8)$$

$$L_N = 2(N-1)\sigma_2 T_N r \sin\left(\arctan\left(\frac{2(N-1)}{2N-1}\tan\theta\right)\right) \quad (11)$$

$$F_{i=1,\cdots,n_3} = \frac{F_s - 3AE\sum_{i=1}^{n_3} M_i(M_i - 1)}{3\sum_{i=1}^{n_3} M_i^2} M_i + AE(M_i - 1)$$

$$F_{i'=1,\cdots,n_3} = \frac{F_s - 3AE\sum_{i=1}^{n_3} M_i(M_i - 1)}{3\sum_{i=1}^{n_3} M_i^2} M_{i'} + AE(M_{i'} - 1) \quad (13)$$

$$F_{i''=1,\cdots,n_3} = \frac{F_s - 3AE\sum_{i=1}^{n_3} M_i(M_i - 1)}{3\sum_{i=1}^{n_3} M_i^2} M_{i''} + AE(M_{i''} - 1)$$

Equations 8, 11, 13.

$$\begin{cases} T_j = \dfrac{F_{i=1} + F_{i'=1}}{2} & j = 1 \\ T_j = \dfrac{\sum_{i=1+\alpha_j}^{\beta_j} F_i + \sum_{i'=1+\alpha_j}^{\beta_j} F_{i'}}{6(j-1)+2} & 2 \le j \le N-1 \\ T_j = \dfrac{\sum_{i=1+\tau_2}^{n_2} F_i + \sum_{i'=1+\tau_2}^{n_2} F_{i'}}{\sigma_2} & j = N \end{cases} \quad (10)$$

where $\tau_2 = \sum_{j=1}^{N-1}[3(j - 1) + 1]$, $\sigma_2 = 2(n_2 - \tau_2)$.

Then, the contribution from the tension of all fibres in the Nth layer to the yarn torque is given by ***Equation 11***.

Finally the total yarn torque caused by fibre tension in the spinning triangle is given as follows:

$$L = \sum_{j=1}^{N-1} L_j + L_N \quad (12)$$

where L_j is the yarn torque caused by the tension of all fibres in the first layer to the $(N - 1)$th layer, as shown in ***Equation 8***.

Theoretical analysis of yarn torque with three feeding strands

A model of a spinning triangle with three symmetrical feeding strands is shown in ***Figure 3***, were, w_3^1 is the width of the spinning triangle for each strand, and w_3^0 is the distance between each strand to the axisymmetric line of the spinning triangle. θ_i, $\theta_{i'}$ *and* $\theta_{i''}$ are the angles between the *i*-th fibre in the first strand, the *i'*-th fibre in the second strand, the *i''*-th fibre in the third strand and the axisymmetric line of the spinning triangle, respectively. Let us suppose that the number of fibres in each strand is n_3, i.e. there are $3n_3$ fibres in the spinning triangle.

Then, using the principle of minimum potential energy, the distribution of fibre tension in the spinning triangle, as shown in ***Figure 3*** (see page 44), can be given as ***Equation 13*** where:

$$M_i = \frac{1}{\cos\theta_i},\ M_{i'} = \frac{1}{\cos\theta_{i'}},$$

$$M_{i''} = \frac{1}{\cos\theta_{i''}},\ \tan\theta_i = \frac{w_i + w_3^0}{h_3},$$

$$\tan\theta_{i'} = \frac{w_{i'} + w_3^0}{h_3},\ \tan\theta_{i''} = \frac{w_{i''} + w_3^0}{h_3},$$

$$w_i = \frac{iw_3^1}{n_3},\ w_{i'} = \frac{i'w_3^1}{n_3},\ w_{i''} = \frac{i''w_3^1}{n_3}.$$

Then, yarn residual torque caused by fibre tension in the spinning triangle can be given by the following two cases.

Case 1. $3n_3 = \sum_{j=1}^{N}[6(j - 1) + 3]$

In this case, all $3n_3$ fibres are arranged in hexagonal close packing in a yarn with three core fibres at the centre, and $3n_3$ - 3 fibres are arranged from the second layer to the Nth layer exactly, with $2(j - 1) + 1$ fibres in each strand distributed in the jth layer. The average fibre tension of each

$$
\begin{cases}
T_j = \dfrac{F_{i=1} + F_{i'=1} + F_{i''=1}}{3} & j = 1 \\
T_j = \dfrac{\sum\limits_{i=1+\alpha_j}^{\beta_j} F_i + \sum\limits_{i'=1+\alpha_j}^{\beta_j} F_{i'} + \sum\limits_{i''=1+\alpha_j}^{\beta_j} F_{i''}}{6(j-1)+3} & 2 \le j \le N
\end{cases}
\qquad (14)
$$

$$
\begin{cases}
T_j = \dfrac{F_{i=1} + F_{i'=1} + F_{i''=1}}{3} & j = 1 \\
T_j = \dfrac{\sum\limits_{i=1+\alpha_j}^{\beta_j} F_i + \sum\limits_{i'=1+\alpha_j}^{\beta_j} F_{i'} + \sum\limits_{i''=1+\alpha_j}^{\beta_j} F_{i''}}{6(j-1)+3} & 2 \le j \le N-1 \\
T_j = \dfrac{\sum\limits_{i=1+\tau_3}^{n_3} F_i + \sum\limits_{i'=1+\tau_3}^{n_3} F_{i'} + \sum\limits_{i''=1+\tau_3}^{n_3} F_{i'}}{\sigma_3} & j = N
\end{cases}
\qquad (15)
$$

$$
F_{i=1,\cdots,n_4} = \frac{F_s - 4AE\sum\limits_{i=1}^{n_4} M_i\left(M_i - 1\right)}{4\sum\limits_{i=1}^{n_4} M_i^{\,2}} M_i + AE\left(M_i - 1\right)
$$

$$
F_{i'=1,\cdots,n_4} = \frac{F_s - 4AE\sum\limits_{i=1}^{n_4} M_i\left(M_i - 1\right)}{4\sum\limits_{i=1}^{n_4} M_i^{\,2}} M_{i'} + AE\left(M_{i'} - 1\right)
$$

$$
F_{i''=1,\cdots,n_4} = \frac{F_s - 4AE\sum\limits_{i=1}^{n_4} M_i\left(M_i - 1\right)}{4\sum\limits_{i=1}^{n_4} M_i^{\,2}} M_{i''} + AE\left(M_{i''} - 1\right) \qquad (A)
$$

$$
F_{i'''=1,\cdots,n_4} = \frac{F_s - 4AE\sum\limits_{i=1}^{n_4} M_i\left(M_i - 1\right)}{4\sum\limits_{i=1}^{n_4} M_i^{\,2}} M_{i'''} + AE\left(M_{i'''} - 1\right)
$$

Equations 14, 15, A.

layer within a yarn T_j can be given as ***Equation 14*** where

$$\alpha_j = \sum_{k=1}^{j} [2(k - 1) + 1] - 2(j-1) - 1,$$

$$\beta_j = \sum_{k=1}^{j} [2(k - 1) + 1].$$

Then, we can get the yarn torque as ***Equations 8*** and ***9***.

Case 2.

$$\sum_{j=1}^{N-1} [6(j - 1) + 3] < 3n_3 < \sum_{j=1}^{N} [6(j - 1) + 3].$$

Similarly, in this case, the average fibre tension of each layer within a yarn T_j can be given as ***Equation 15*** where

$$\tau_3 = \sum_{j=1}^{N-1} [2(j - 1) + 1],\ \sigma_3 = 3\left(n_3 - \tau_3\right).$$

Then, we can get the yarn torque as ***Equations 8, 11*** and ***12***.

Theoretical analysis of yarn torque with four feeding strands

A model of a spinning triangle with four symmetrical feeding strands is shown in ***Figure 4***. Here, w_4^1 is the width of the spinning triangle for each strand, and w_4^0 is the distance between each strand to the axisymmetric line of the spinning triangle. θ_i, $\theta_{i'}$, $\theta_{i''}$ and $\theta_{i'''}$ are the angles between the *i*-th fibre in the first strand, the *i'*-th fibre in the second strand, the *i''*-th fibre in the third strand, the *i'''*-th fibre in the fourth strand and the axisymmetric line of the spinning triangle, re-

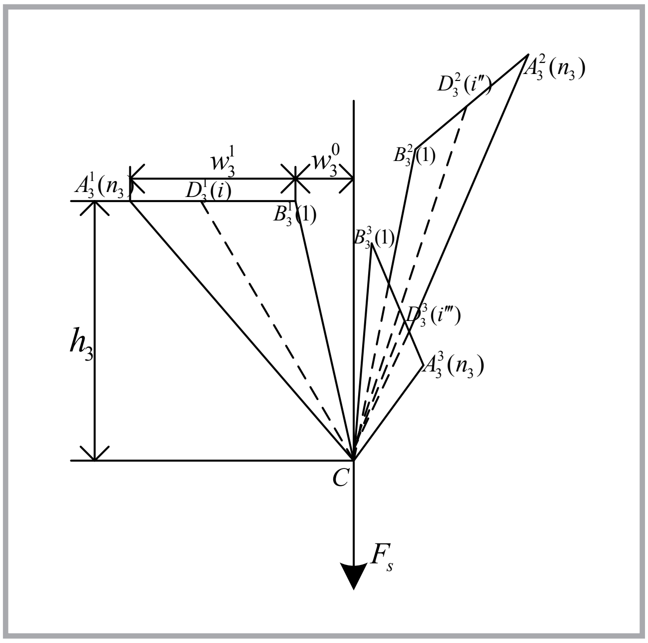

Figure 3. *Spinning triangle model with three strands.*

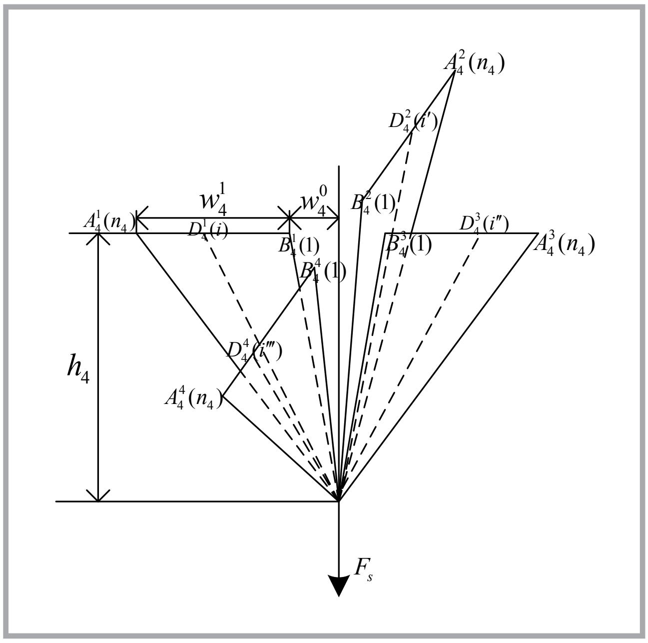

Figure 4. *Model of spinning triangle with four strands.*

spectively. Let us suppose that the number of fibres in each strand is n_4, i.e. there are $4n_4$ fibres in the spinning triangle.

Then, using the principle of minimum potential energy, the distribution of fibre tension in the spinning triangle, as shown in ***Figure 4***, can be given ***Equation A*** were:

$$M_i = \frac{1}{\cos\theta_i},\; M_{i'} = \frac{1}{\cos\theta_{i'}},$$

$$M_{i''} = \frac{1}{\cos\theta_{i''}},\; M_{i'''} = \frac{1}{\cos\theta_{i'''}},$$

$$\tan\theta_i = \frac{w_i + w_4^0}{h_4},\; \tan\theta_{i'} = \frac{w_{i'} + w_4^0}{h_4},$$

$$\tan\theta_{i''} = \frac{w_{i''} + w_4^0}{h_4},\; \tan\theta_{i'''} = \frac{w_{i'''} + w_4^0}{h_4},$$

$$w_i = \frac{i w_4^1}{n_4},\; w_{i'} = \frac{i' w_4^1}{n_4},\; w_{i''} = \frac{i'' w_4^1}{n_4},$$

$$w_{i'''} = \frac{i''' w_4^1}{n_4}.$$

Then, yarn residual torque caused by fibre tension in the spinning triangle can be given by the following two cases.

Case 1. $4n_4 = \sum_{j=1}^{N}[6(j - 1) + 4]$.

In this case, all $4n_4$ fibres are arranged in hexagonal close packing in a yarn with four core fibres at the centre, and $4n_4 - 4$ fibres are arranged from the second layer to the *N*th layer exactly. $2(j - 1)$ fibres in the first and second strands and $(j - 1) + 2$ fibres in the third and fourth strands will be distributed in the *j*th layer if *j* is even, while $(j - 1) + 2$ fibres in the first and second strands and $2(j - 1)$ fibres in the third and fourth strands will be distributed in the *j*th layer if *j* is odd. The average fibre tension of each layer within a yarn T_j can be given as ***Equation 16***.

Then, we can get the yarn torque as ***Equations 8, 11*** and ***12***.

Case 2.

$$\sum_{j=1}^{N-1}[6(j - 1) + 4] < 4n_4 < \sum_{j=1}^{N}[6(j - 1) + 4]$$

Similarly, in this case, the average fibre tension of each layer within a yarn T_j can be given as ***Equation 17.***

Then, we can get the yarn torque as ***Equations 8, 11*** and ***12***.

$$\begin{cases} T_j = \dfrac{F_{i=1} + F_{i'=1} + F_{i''=1} + F_{i'''=1}}{4} & j = 1 \\[2ex] T_j = \dfrac{\sum_{i=1+\alpha_j}^{\beta_j} F_i + \sum_{i'=1+\alpha_j}^{\beta_j} F_{i'} + \sum_{i''=1+\phi_j}^{\varphi_j} F_{i''} + \sum_{i'''=1+\phi_j}^{\varphi_j} F_{i'''}}{6(j-1)+4} & 2 \le j \le N \end{cases} \tag{16}$$

where, if *j* is even,

$$\alpha_j = 1 + \sum_{k=1}^{\frac{j-2}{2}}[2(2k - 1) + 1] + \sum_{k=1}^{\frac{j-2}{2}}\{[(2k + 1) - 1] + 2\} - \{[(j - 1) - 1] + 2\}$$

$$\beta_j = 1 + \sum_{k=1}^{\frac{j-2}{2}}[2(2k - 1) + 1] + \sum_{k=1}^{\frac{j-2}{2}}\{[(2k + 1) - 1] + 2\}$$

$$\phi_j = 1 + \sum_{k=1}^{\frac{j-2}{2}}\{[(2k + 1) - 1] + 2\} + \sum_{k=1}^{\frac{j-2}{2}}[2(2k - 1) + 1] - 2[(j - 1) - 1]$$

$$\varphi_j = 1 + \sum_{k=1}^{\frac{j-2}{2}}\{[(2k + 1) - 1] + 2\} + \sum_{k=1}^{\frac{j-2}{2}}[2(2k - 1) + 1]$$

if *j* is odd,

$$\alpha_j = 1 + \sum_{k=1}^{\frac{j-1}{2}}[2(2k - 1) + 1] + \sum_{k=2}^{\frac{j-1}{2}}\{[(2k + 1) - 1] + 2\} - 2[(j - 1) - 1]$$

$$\beta_j = 1 + \sum_{k=1}^{\frac{j-1}{2}}[2(2k - 1) + 1] + \sum_{k=2}^{\frac{j-1}{2}}\{[(2k + 1) - 1] + 2\}$$

$$\phi_j = 1 + \sum_{k=1}^{\frac{j-1}{2}}\{[(2k - 1) - 1] + 2\} + \sum_{k=2}^{\frac{j-1}{2}}[2(j - 1) + 1] - \{[(j - 1) - 1] + 2\}$$

$$\varphi_j = 1 + \sum_{k=1}^{\frac{j-1}{2}}\{[(2k - 1) - 1] + 2\} + \sum_{k=2}^{\frac{j-1}{2}}[2(2k - 1) + 1]$$

$$\begin{cases} T = \dfrac{F_{i=1} + F_{i'=1} + F_{i''=1} + F_{i'''=1}}{} & j = \\[2ex] T = \dfrac{\sum_{i=1+\alpha_j}^{\beta_j} F_i + \sum_{i'=1+\alpha_j}^{\beta_j} F_{i'} + \sum_{i''=1+\phi_j}^{\varphi_j} F_{i''} + \sum_{i'''=1+\phi_j}^{\varphi_j} F_{i'''}}{6(\;\;-1)+4} & 2 \le j \le N-1 \\[2ex] T = \dfrac{\sum_{i=1+\tau_4}^{n_4} F_i + \sum_{i'=1+\tau_4}^{n_4} F_{i'} + \sum_{i''=1+\nu_4}^{n_4} F_{i''} + \sum_{i'''=1+\nu_4}^{n_4} F_{i'''}}{} & j = N \end{cases} \tag{17}$$

where, if *N* is even,

$$\tau_4 = 1 + \sum_{k=1}^{\frac{N-2}{2}}[2(2k - 1) + 1] + \sum_{k=1}^{\frac{N-2}{2}}\{[(2k + 1) - 1] + 2\}$$

$$\nu_4 = 1 + \sum_{k=1}^{\frac{N-2}{2}}\{[(2k + 1) - 1] + 2\} + \sum_{k=1}^{\frac{N-2}{2}}[2(2k - 1) + 1]$$

if *j* is odd,

$$\tau_4 = 1 + \sum_{k=1}^{\frac{N-1}{2}}[2(2k - 1) + 1] + \sum_{k=2}^{\frac{N-1}{2}}\{[(2k - 1) - 1] + 2\}$$

$$\nu_4 = 1 + \sum_{k=1}^{\frac{N-1}{2}}\{[(2k - 1) - 1] + 2\} + \sum_{k=2}^{\frac{N-1}{2}}[2(2k - 1) + 1] \qquad \sigma_4 = 2\left(n_4 - \tau_4 + n_4 - \nu_4\right)$$

Equations 16 and 17.

$$F_{i=1,\cdots,n_4} = \frac{F_s - 5AE\sum_{i=1}^{n_5} M_i\left(M_i - 1\right)}{5\sum_{i=1}^{n_5} M_i^2} M_i + AE\left(M_i - 1\right)$$

$$F_{i'=1,\cdots,n_4} = \frac{F_s - 5AE\sum_{i=1}^{n_5} M_i\left(M_i - 1\right)}{5\sum_{i=1}^{n_4} M_i^2} M_{i'} + AE\left(M_{i'} - 1\right)$$

$$F_{i''=1,\cdots,n_4} = \frac{F_s - 5AE\sum_{i=1}^{n_5} M_i\left(M_i - 1\right)}{5\sum_{i=1}^{n_5} M_i^2} M_{i''} + AE\left(M_{i''} - 1\right) \qquad \text{(B)}$$

$$F_{i'''=1,\cdots,n_4} = \frac{F_s - 5AE\sum_{i=1}^{n_5} M_i\left(M_i - 1\right)}{5\sum_{i=1}^{n_5} M_i^2} M_{i'''} + AE\left(M_{i'''} - 1\right)$$

$$F_{i''''=1,\cdots,n_4} = \frac{F_s - 5AE\sum_{i=1}^{n_5} M_i\left(M_i - 1\right)}{5\sum_{i=1}^{n_5} M_i^2} M_{i''''} + AE\left(M_{i''''} - 1\right)$$

$$\begin{cases} T_j = \dfrac{F_{i=1} + F_{i'=1} + F_{i''=1} + F_{i'''=1} + F_{i''''=1}}{5} & j = 1 \\ T_j = \dfrac{\sum_{i=1+\alpha_{1j}}^{\beta_{1j}} F_i + \sum_{i'=1+\alpha_{2j}}^{\beta_{2j}} F_{i'} + \sum_{i''=1+\alpha_{3j}}^{\beta_{3j}} F_{i''} + \sum_{i'''=1+\alpha_{4j}}^{\beta_{4j}} F_{i'''} + \sum_{i''''=1+\alpha_{0j}}^{\beta_{0j}} F_{i''''}}{6(j-1)+5} & 2 \le j \le N \end{cases} \qquad (18)$$

$$\begin{cases} T_j = \dfrac{F_{i=1} + F_{i'=1} + F_{i''=1} + F_{i'''=1} + F_{i''''=1}}{5} & j = 1 \\ T_j = \dfrac{\sum_{i=1+\alpha_{1j}}^{\beta_{1j}} F_i + \sum_{i'=1+\alpha_{2j}}^{\beta_{2j}} F_{i'} + \sum_{i''=1+\alpha_{3j}}^{\beta_{3j}} F_{i''} + \sum_{i'''=1+\alpha_{4j}}^{\beta_{4j}} F_{i'''} + \sum_{i''''=1+\alpha_{0j}}^{\beta_{0j}} F_{i''''}}{6(j-1)+5} & 2 \le j \le N-1 \\ T_j = \dfrac{\sum_{i=1+\tau_{15}}^{n_5} F_i + \sum_{i'=1+\tau_{25}}^{n_5} F_{i'} + \sum_{i''=1+\tau_{35}}^{n_5} F_{i''} + \sum_{i'''=1+\tau_{45}}^{n_5} F_{i'''} + \sum_{i''''=1+\tau_{05}}^{n_5} F_{i''''}}{\sigma_5} & j = N \end{cases} \qquad (19)$$

Equations B, 18 and 19.

Theoretical analysis of yarn torque with five feeding strands

A model of a spinning triangle with five symmetrical feeding strands is shown in ***Figure 5*** (see page 46). Here, w_5^1 is the width of the spinning triangle for each strand, and w_5^0 is the distance between each strand to the axisymmetric line of the spinning triangle. θ_i, $\theta_{i'}$, $\theta_{i''}$, $\theta_{i'''}$ and $\theta_{i''''}$ are the angles between the *i*-th fibre in the first strand, the *i'*-th fibre in the second strand, the *i''*-th fibre in the third strand, the *i'''*-th fibre in the fourth strand, the *i''''*-th fibre in the fifth strand and the axisymmetric line of the spinning triangle, respectively. Let us suppose that the number of fibres in each strand is n_5, i.e. there are $5n_5$ fibres in the spinning triangle.

Then, using the principle of minimum potential energy, the distribution of fibre tension in the spinning triangle, as shown in ***Figure 5***, can be given ***Equation B*** were,

$$M_i = \frac{1}{\cos\theta_i},\ M_{i'} = \frac{1}{\cos\theta_{i'}},$$

$$M_{i''} = \frac{1}{\cos\theta_{i''}},\ M_{i'''} = \frac{1}{\cos\theta_{i'''}},$$

$$M_{i''''} = \frac{1}{\cos\theta_{i''''}},\ \tan\theta_i = \frac{w_i + w_5^0}{h_5},$$

$$\tan\theta_{i'} = \frac{w_{i'} + w_5^0}{h_5},\ \tan\theta_{i''} = \frac{w_{i''} + w_5^0}{h_5},$$

$$\tan\theta_{i'''} = \frac{w_{i'''} + w_5^0}{h_5},\ \tan\theta_{i''''} = \frac{w_{i''''} + w_5^0}{h_5},$$

$$w_i = \frac{iw_5^1}{n_5},\ w_{i'} = \frac{i'w_5^1}{n_5},\ w_{i''} = \frac{i''w_5^1}{n_5},$$

$$w_{i''''} = \frac{i''''w_5^1}{n_5}.$$

Then, yarn residual torque caused by fibre tension in the spinning triangle can be given by the following two cases.

Case1. $5n_5 = \sum_{j=1}^{N} [6(j-1) + 5]$

In this case, all $5n_5$ fibres are arranged in hexagonal close packing in a yarn with five core fibres at the centre, and $5n_5 - 5$ fibres are arranged from the second layer to the *N*th layer exactly. $2(j - 1) + 1$ fibres in the first strand and $(j - 1) + 1$ fibres in the second to fifth strands will be distributed in the *j*th layer if *j*mod5 = 1; $2(j - 1) + 1$ fibres in the second strand and $(j - 1) + 1$ fibres in the first and third to fifth strands will be distributed in the *j*th layer if *j*mod5 = 2; $2(j - 1) + 1$ fibres in the third strand and $(j - 1) + 1$ fibres in the first, second, fourth and fifth strands will be distributed in the *j*th layer if *j*mod5 = 3; $2(j - 1) + 1$ fibres in the fourth strand and $(j - 1) + 1$ fibres in the first to third, and fifth strands will be distributed in the *j*th layer if *j*mod5 = 4, and $2(j - 1) + 1$ fibres in the fifth strand and $(j - 1) + 1$ fibres in the first to fourth strands will be distributed in the *j*th layer if *j*mod5 = 0. The average fibre tension of each layer within a yarn T_j can be given as ***Equation 18*** where

$$_{sj} = \sum b_{sk} - b_{sj}\ ,\ \beta_{sj} = \sum_{k=1}^{j} b_{sk}\ ,$$

if *k*mod5 = *s*, $b_{sk} = 2(k - 1) + 1$, otherwise $b_{sk} = (k - 1) + 1$ for $s = 0, 1, 2, 3, 4$.

Then, we can get the yarn torque as ***Equations 8, 11*** and ***12***.

Case2.

$$\sum_{j=1}^{N-1}[6(j - 1) + 5] < 5n_5 < \sum_{j=1}^{N}[6(j - 1) + 5]$$

Similarly, in this case, the average fibre tension of each layer within a yarn T_j can be given as ***Equation 19*** where

$$\tau_{sj} = \sum_{k=1}^{N-1} b_{sk}\ ,\ \sigma_5 = \sum_{s=0}^{4}\left(n_5 - \tau_{s5}\right).$$

Simulations and discussions

To illustrate the effects of fibre tension within the yarn on yarn torque, 26.5 tex cotton yarns were taken as an example in the following simulation. The yarn and fibre parameters used in it are listed as follows: number of fibres in the yarn: 180, spinning tension: 30 cN, fibre linear density: 0.15 tex, fibre's Young's modulus: 50 cN/tex, twist direction: Z, and the fibre radius: 3.08 μm. The number of fibres packed in each layer within a yarn is given in ***Table 1***. We can see that the number of fibres in the last layer does not fill in the layer completely. The number of fibres in the left, right and raised parts of the spinning triangle packed in each layer is shown in ***Table 2***.

An image of the spinning triangle in traditional ring spinning can be captured by the high speed camera system OLYMPUS i-speed3 (Japan). After some measurements and calculations, we can get the parameters of the spinning triangle, shown in ***Figure 1***, as follows:

$$w_1^1 = 4.5 \text{ mm},\ h_1 = 3.9 \text{ mm} \quad (20)$$

Table 1. *Number of fibres in each layer within a yarn.*

Layer number	Number of fibres in each layer					
	Open packing	Hexagonal close packing				
		a single core fibre	two core fibres	three core fibres	four core fibres	five core fibres
1	1	1	2	3	4	5
2	6	6	8	9	10	11
3	12	12	14	15	16	17
4	18	18	20	21	22	23
5	25	24	26	27	28	29
6	31	30	32	33	34	35
7	37	36	38	39	40	41
8	43	42	40	33	26	19
9	7	11	0	0	0	0

Table 2. *Number of fibres in each strand in each layer.*

Layer number	Number of fibres in each spinning triangle part			
	$n_2 = 90$	$n_3 = 60$	$n_4 = 45$	$n_5 = 36$
1	(1, 1)	(1, 1, 1)	(1, 1, 1, 1)	(1, 1, 1, 1, 1)
2	(4, 4)	(3, 3, 3)	(2, 2, 3, 3)	(3, 2, 2, 2, 2)
3	(7, 7)	(5, 5, 5)	(4, 4, 4, 4)	(3, 5, 3, 3, 3)
4	(10, 10)	(7, 7, 7)	(6, 6, 5, 5)	(4, 4, 7, 4, 4)
5	(13, 13)	(9, 9, 9)	(6, 6, 8, 8)	(5, 5, 5, 9, 5)
6	(16, 16)	(11, 11, 11)	(10, 10, 7, 7)	(6, 6, 6, 6, 11)
7	(19, 19)	(13, 13, 13)	(8, 8, 12, 12)	(13, 7, 7, 7, 7)
8	(20, 20)	(11, 11, 11)	(8, 8, 5, 5)	(1, 6, 5, 4, 3)
9	(0, 0)	(0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0, 0)

In the following, numerical simulation of fibre tensions in the spinning triangle and corresponding yarn torque will be presented. For ease of analysis, all fibre in the spinning triangle will be relabeled. Taking the fibre in the spinning triangle with three strands, as shown in ***Figure 3***, as an example, the fibres from left to right in the first strand will be labelled 1, 2, ..., n_3 successively, those from left to right in the second strand - $n_3 + 1$, $n_3 + 2$, ..., $2n_3$ successively, and then the fibres from left to right in the third strand as $2n_3 + 1$, $2n_3 + 2$, ..., $3n_3$ successively, which can be seen in ***Figure 6***.

According to the analysis above, simulation of fibre distributions in the spinning triangle with different strands and corresponding yarn torque can be presented

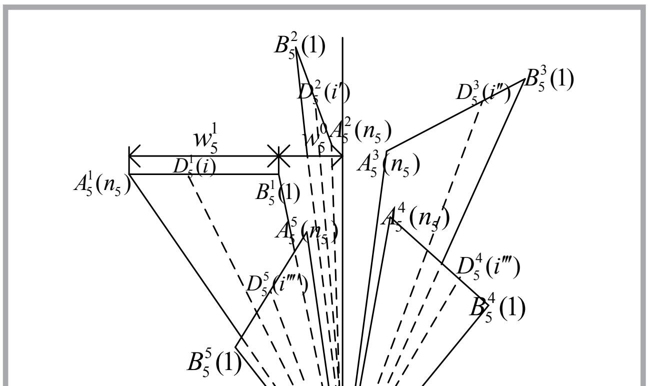

Figure 5. *Model of spinning triangle with five strands.*

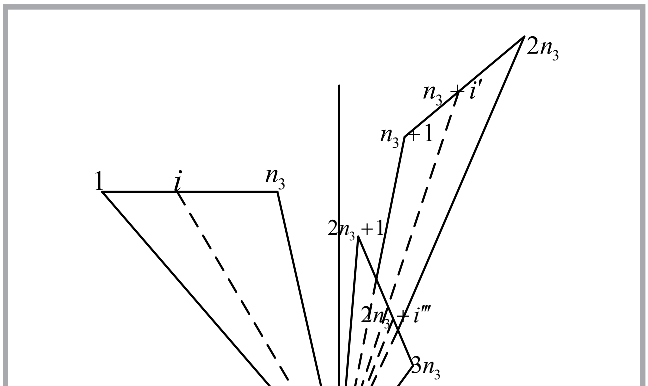

Figure 6. *Relabeling of fibres in the model of the spinning triangle with three strands.*

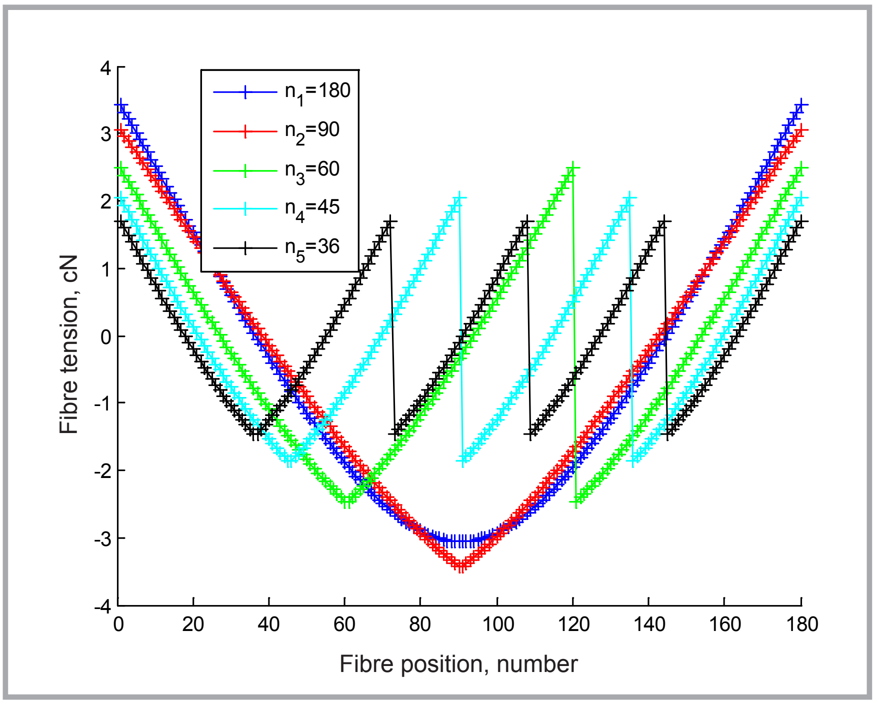

Figure 7. *Fibre tension in spinning triangles with different strands when the height of the spinning triangle is identical.* ***Attention****: The full lines does not present functions but only trends of the values.*

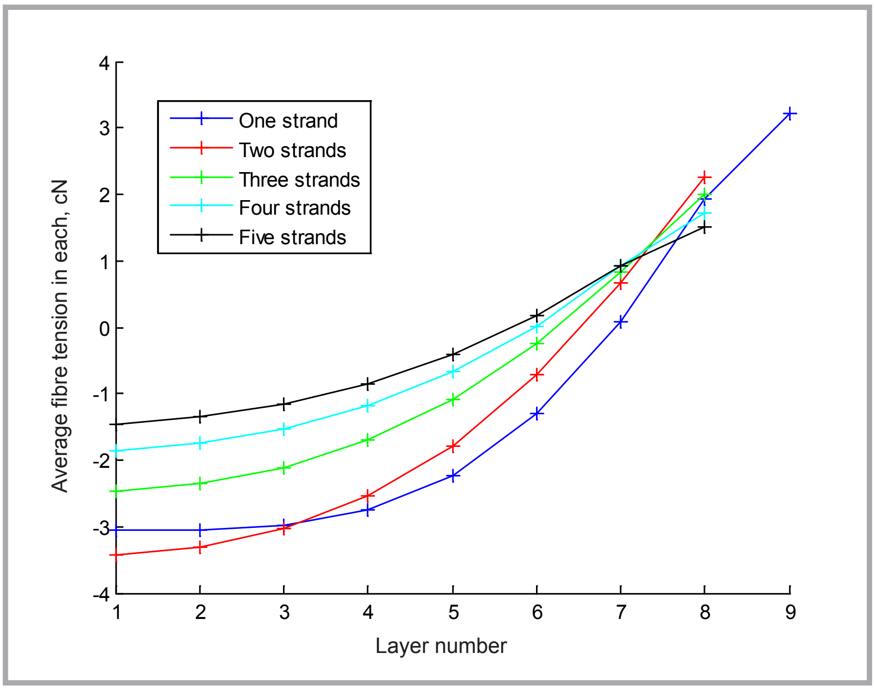

Figure 8. *Average fibre tension in each layer with different strands when the height of the spinning triangle is identical.* ***Attention****: The full lines does not present functions but only trends of the values.*

using Matlab software. Firstly the height of the spinning triangle is assumed to be identical, i.e. $h_1 = h_2 = h_3 = h_4 = h_5$ = 2.8 mm, with $w_2^0 = w_3^0 = w_4^0 = w_5^0 =$ = 2 mm, then numerical simulation results of fibre tension distributions in corresponding spinning triangles with different strands are shown in ***Figure 7***, and the corresponding average fibre tension in each layer is shown in ***Figure 8***. Here the width of each spinning triangle is assumed to be proportional to the fibre number in it, i.e. $w_1^j = w_1^1 n_j / 180$ for j = 2, 3, 4, 5. It is shown that the magnitude of fibre tension in the spinning triangle with two strands is almost identical to that of the fibre tension in the spinning triangle with one strand, and then decreases significantly with an increase in the strand number. It has been shown that the magnitudes of fibre tension in the spinning triangle will probably influence fibre tensile stresses within the yarn and thus affect the total torque of spun yarns, with larger magnitudes potentially leading to larger yarn torque [2, 6]. The calculated yarn torques caused by fibre tension in the spinning triangle is shown in the first line of ***Table 3***. It is shown that the yarn torque with two strands is a little larger than that with one strand, one possible explanation for which is that although the magnitudes of fibre tension in the two kinds of spinning triangles are almost identical, the asymmetry of fibre tension in the spinning triangle with two strands is a little worse than that with one strand, which may lead to larger yarn torque. Meanwhile the yarn torque decreases when the feeding strands are increased from two to five.

In the simulations above, the distances between each strand to the axisymmetric line of the spinning triangle w_j^0 are taken identically for j = 2, 3, 4, 5. Therefore the effects of w_j^0 on yarn torque are subsequently analysed , and numerical simulations of yarn torque with different w_j^0 are presented in ***Figure 9***, in which w_j^0 is changed from 0.5 mm to 4 mm. It is shown that with an increase in w_2^0, yarn torque with two feeding strands constantly decreases, whereas with an increase in w_3^0, w_4^0 or h_j, yarn torque with three, four or five feeding strands increases at first and then decreases, i.e. showing a parabola shape.

In the general case, the height of the spinning triangle is not identical with the different numbers of feeding strands. Therefore numerical simulations of fibre tensions and corresponding yarn torque with different numbers of feeding strands are also presented where the height of the spinning triangle h_j is changed proportionally to the width w_1^j, i.e. the ratio of the height of the spinning triangle h_j to the width w_1^j is identical, that is, $h_j = h_1 n_j/180$ for j = 1, 2, 3, 4, 5. Firstly, taking $w_2^0 = w_3^0 = w_4^0 = w_5^0 =$ 2 mm, numerical simulation results of fibre tension distributions in corresponding spinning triangles with different numbers of feeding strands are shown in ***Figure 10***. It is also shown that the magnitude of fibre tension in the spinning triangle decreases significantly when the feeding strand number is increased from two to five. Meanwhile the reduction is larger in this case than that in the assumption that h_j is identical, which may lead to smaller yarn torque, see the calculated yarn torques shown in the second line of ***Table 3***. Then numerical simulations of yarn torque with different w_j^0 are presented in ***Figure 11***. It is shown that with an increase in w_j^0, yarn torque with a corresponding number of feeding strands constantly decreases for j = 2, 3, 4, 5, that is, within a certain range, an increase in the distance between each strand to the axisymmetric line of the spinning triangle is beneficial for a reduction in yarn torque in theory.

Conclusions

This paper is devoted to theoretical studies of yarn torque caused by fibre tension in the spinning triangle. In order to investigate situations where the arrangement of fibres is hexagonal close packing with a single core fibre, two core fibres, three core fibres, four core fibres or five core fibres at the center, cases of yarn with one, two, three, four and five feeding strands were investigated, respectively. By using the energy approach, theoretical models of fibre tension in the spinning triangles and the corresponding yarn residual torque due to the fibre tension have been given.

As an application of the method proposed, 26.5 tex cotton yarns were taken as an example for numerical simulations. By using a high speed camera system - OLYMPUS i-speed3, an image of the spinning triangle in a traditional ring spinning with one feeding strand was captured, and the parameters of the spinning triangle w_1^1 and h_1 were obtained. Then numerical

Table 3. *Yarn torques with different strands.*

	Number of fibres in each spinning triangle with different strands, cN·μm				
	n_1 = 180	**n_2 = 90**	**n_3 = 60**	**n_4 = 45**	**n_5 = 36**
h identical	1323.8	1399.3	1353.3	1185.7	999.8
h changed	1323.8	1352.8	1134.7	853.5	621.7

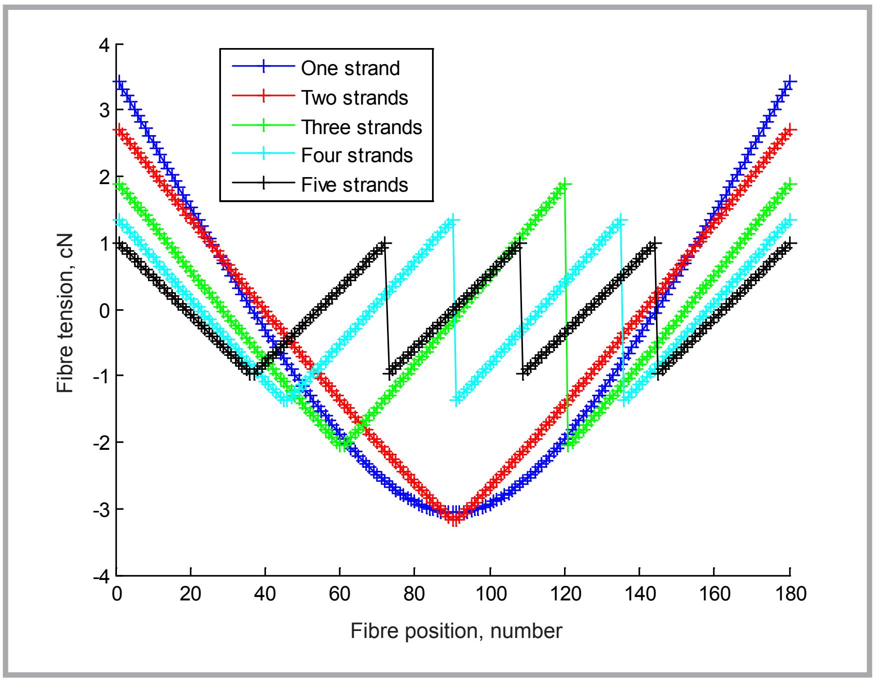

Figure 10. *Fibre tension in spinning triangles with different strands when the height of the spinning triangle is changed proportionally to the width.* ***Attention****: The full lines does not present functions but only trends of the values.*

simulation results of fibre tension distributions in spinning triangles with different strands and the corresponding average fibre tension in each layer were obtained using Matlab software, where the height of the spinning triangles was assumed to be identical. Simulation results show that the magnitude of fibre tension in the spinning triangle decreases significantly when the strand number is increased from two to five, which leads to corresponding changes in yarn torque. Meanwhile it is shown that with an increase in w_2^0, yarn torque with two feeding strands constantly decreases, whereas with an increase in w_3^0, w_4^0 or w_5^0, yarn torque with three,

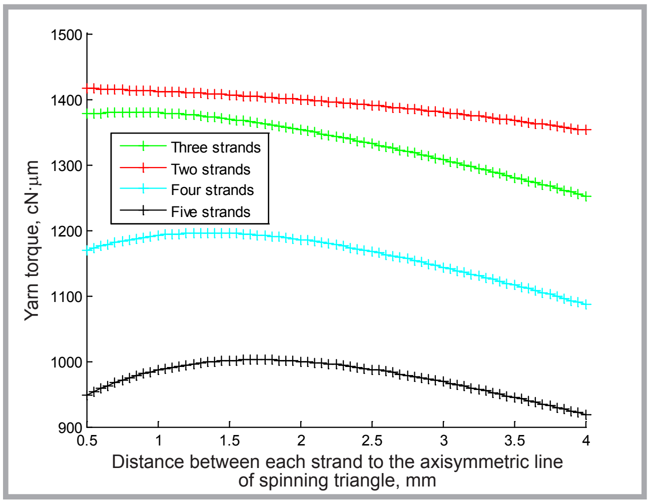

Figure 9. *Yarn torque with different distances between each strand to the axisymmetric line of the spinning triangle when the height of the spinning triangle is identical.*

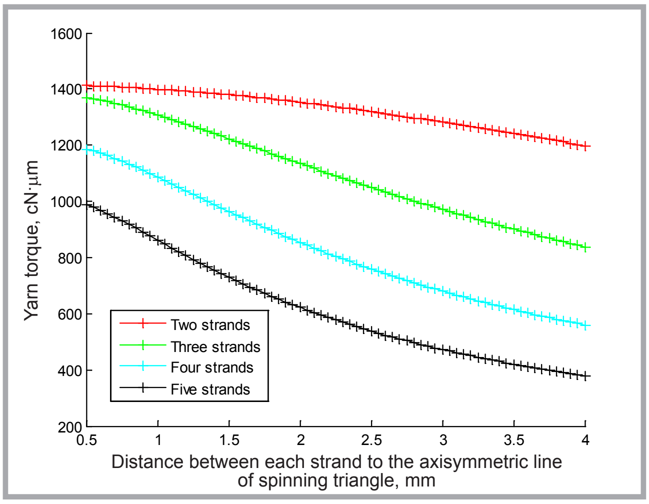

Figure 11. *Yarn torque for different distances between each strand to the axisymmetric line of the spinning triangle when the height of the spinning triangle is changed proportionally to the width.*

four or five feeding strands increases at first and then decreases, i.e. showing a parabola shape. When the height of the spinning triangle h_j is assumed to be changed proportionally to the width w_1^j, the simulation results show that with an increasing in w_j^0, yarn torque with corresponding feeding strands constantly decreases for j = 2, 3, 4, 5.

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