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Modelling of Transient Heat Transfer within Bounded Seams

Abstract
The heat transfer within seams bounded by a thermowelding machine is analysed as a transient problem, i.e. time- and space-dependent. The physical model determines a multilayer textile composite made of connected fabrics and also it presents the material homogenisation. The mathematical model introduces a state differential equation accompanied by a set of boundary and initial conditions. The problem can be solved numerically and visualised by means of the graphical modulus of the ADINA program. Temperature maps are shown for the diversified temperatures of the machine head on the upper surface of the seam as well as the different time steps.

Key words: *transient heat transfer, thermo-welding machine, temperature distribution.*

The problem of transient heat transfer within bounded seams

The most convenient process of seam creation is using a thermo-welding machine to form a continuous structure by means of concentrated heat flux and head pressure. The main advantage is the lack of dimension altering during the tension of the fabric. We should define the temperature as a state variable on the surface between the thermo-welding head and the material and within the bounded seam.

Physically speaking, we create a composite structure made of clothing layers, with plastic tape as an additional element. The composite structure of seams is determined by a combination of two phenomena: (i) the concentrated heat flux, (ii) the thermo-welding process, i.e. the sum of the chemical activation of polyurethane tapes and the creation of adhesive film on the seam. First, the heat flux introduced into the bounded materials activates the glue. Next appropriate pressure should be applied to the joint. The minimal pressure does not create the adhesion required, and the maximal pressure can destroy the bounded seam. The crucial fact is to introduce a special machine to determine the prescribed value of temperature as well as initiate the chemical processes and adhesion. Every such machine is designed for the specified parameters of the tape, the speed of the bounding process and the weight of the thermo-welding head. The weight of the heads applied is within the wide range 2 - 40 kg, and it is important to apply an appropriate pressure force. Of course, the pressure force is not only a simple function of the head weight.

The most important technical factor is the heat flux applied, which depends on the following: (i) technological parameters of the thermo-welding machine, i.e. the appropriate heat flux density, (ii) the environmental conditions, i.e. the temperature, air speed, the dissipation of heat etc., (iii) the velocity of the seam creation. The state variable (the temperature) changes over time; the problem is also a transient one for normal working conditions. In fact, the technological conditions are controlled before and during seam creation at the different points selected on the working surface.

The heat transfer can be described by means of a physical model, i.e. the composite structure of the bounded seams. The mathematical model is a state equation (i.e. a second-order differential equation with respect to the state variable and a first-order equation with respect to time), accompanied by the boundary and initial conditions, cf. Korycki [3 - 5].The present paper is an extension of a previous work concerning the steady heat transfer problem during seam creation, cf. Korycki [4]. The types of heat transfer typical for human clothing have been discussed by different authors, cf. Li [6], Pan and Gibson [8], Tomeczek [10].

The modelling of the heat transfer needs the homogenisation of each structural layer. There are a few homogenisation methods, for example the rule of mixture and the hydrostatic analogy by Golanski, Terada & Kikuchi [1], the homogeneous structure made of fibres situated regularly within the filling by Tomeczek [10], and the particular methods for composite materials created by Rocha and Cruz [9]. The main goal of the present paper is to discuss heat transport during seam bounding by a thermo-welding machine as well as to define the temperature distribution within the structure. We will introduce a physical model of the heat transport and next a mathematical model to formulate a second-order differential equation with respect to temperature, accompanied by a set of boundary and initial conditions. The simplest steady problems can be solved analytically. The other problems are too complicated and are solved numerically. The results can be visualised by means of any graphical program, for example the graphical modulus of the ADINA program. This class of problem has not yet been considered in the literature analysed concerning heat transport within seams bounded by thermo-welding machines. The distribu-

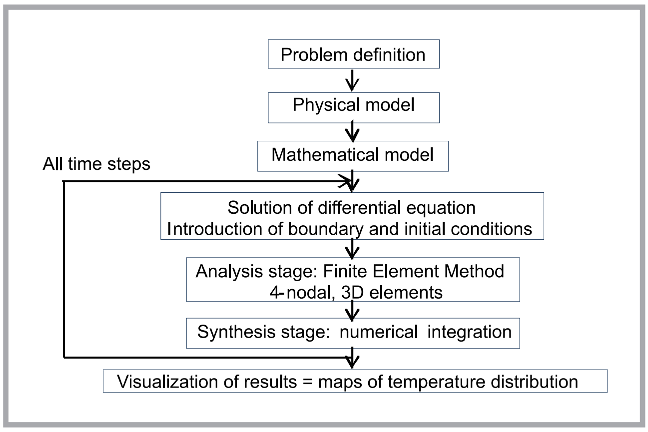

Figure 1. *Solution strategy for transient problems.*

tion of the temperature field is important for composite materials and can help to optimise the warming-up time as well as shape the material of the bounded seams.

Solution strategy for the heat transfer problem - a physical model

A solution strategy for the transient problems is shown in ***Figure 1***. The state variable within the bounded seam is the temperature T. Let us first introduce a physical model which introduces necessary information concerning the nature of the heat transfer as well as the homogenisation of the irregular textiles to the homogeneous structure. A bounded seam is in fact a composite structure made of (i) bounded textile materials (fabrics), (ii) an external membrane on the material surfaces, (iii) glue activated from the tape and diffusing during the adhesion process into the material. The structure is supported by a special reference plane from the bottom side, see ***Figure 2***. The physical properties of the seam structure are different from those of other fabric because of the activated glue, the adhesion process and the concentrated heat flux.

The homogenisation of the connected fabrics is necessary to describe heat transfer within the structure. Irregular random approaches are transferred to a regular homogeneous material of the same heat transfer properties within each point. Homogenisation can be considered as different physical and chemical processes. The method applied here is the rule of mixture acc. Golanski, Terada, Kikuchi [1], in which the equivalent heat conductivity coefficient has the form

$$\lambda_z = \lambda_m \xi_m + \lambda_f \xi_f ;$$

$$\xi_m = \frac{V_m}{V_m + V_f}; \; \xi_f = \frac{V_f}{V_m + V_f} \qquad (1)$$

where λ_z is the equivalent heat conductivity coefficient, ξ_m, ξ_f - the volume coefficients of the textile material of volume V_m and interfibre spaces of volume V_f; index f denotes the filling, and index m - the material. The material is homogenised by means of this method.

The second homogenisation method discussed by the above authors is Turner's model, which introduces the hydrostatic analogy, cf. [1]. Tomeczek [10] homogenised a composite material made of cylindrical fibres of regular shape situated within the filling.

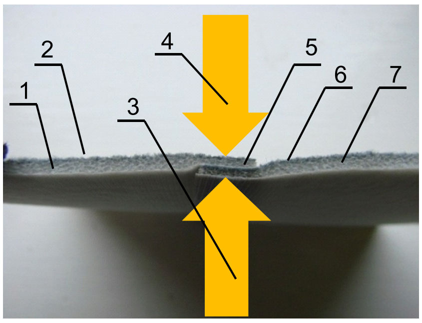

Figure 2. *Physical conditions within seams bounded by a thermo-welding machine. 1 – fabric #1, 2 – semi-permeable waterproof layer, 3 – reference plane, seam support, prescribed temperature, 4 –head of a thermo-welding machine, prescribed temperature, 5 – glue activated from polyurethane tape + adhesion, 6 – fabric #2, 7 – semi-permeable waterproof layer.*

The most important parameter of the homogenisation is the surface filling factor, which describes surface irregularities. It is evident that these investigations should be introduced by means of experimental methods, particularly by different methods of surface analysis. Mikołajczyk [7] discusses the most effective method of image processing, which analyses the brightness and contrast of particular cells of the homogenised material to obtain both volume coefficients.

The transient heat transfer during seam creation is a typical 3D space problem because the temperature and other parameters are time- and space-dependent. Assuming the same shape and physical parameters of the heat transfer within the material, the problem can be reduced to a 2D plane problem, which simplifies the calculations.

Mathematical model

The next step of the solution procedure introduces a mathematical model. The state equation has a typical form, as presented, for example, by Korycki [3, 4],in which there is a second-order correlation with respect to the design variable and first-order equation with respect to time. Bounded textile structures do not contain internal heat sources. It is also evident that boundary conditions depend on the problem configuration and material applied, cf. ***Figure 3***.

The optimal technology needs a prescribed value of the temperature of the thermo-welding head, and consequently the Dirichlet condition exists on the boundary portions, Γ_T. These portions are located on the upper part of the seam (the variable and prescribed temperature distribution of the head) and lower part of the seam (the temperature of the reference plane).

The structure is subjected to Neumann conditions, describing the heat flux densities on the boundaries Γ_q. The problem is symmetric, the heat flux is transported unidirectionally, and we can introduce second kind boundary conditions on the surfaces, Γ_q. The heat flux density is negligible, $q_n = 0$, on the side surfaces. Additionally we assume that the bounded fabrics have a special non-permeable layer on the upper surface, i.e. a special non-permeable membrane against water

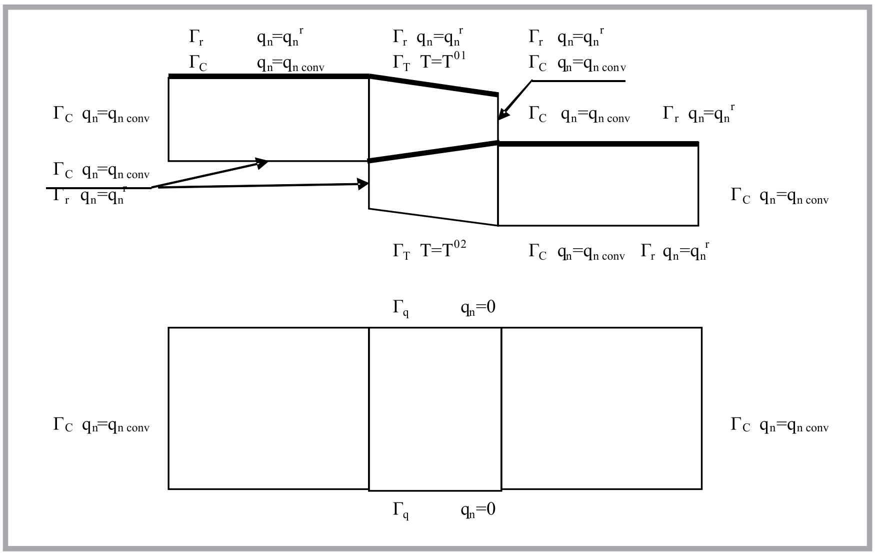

Figure 3. *Boundary conditions for bounded seam.*

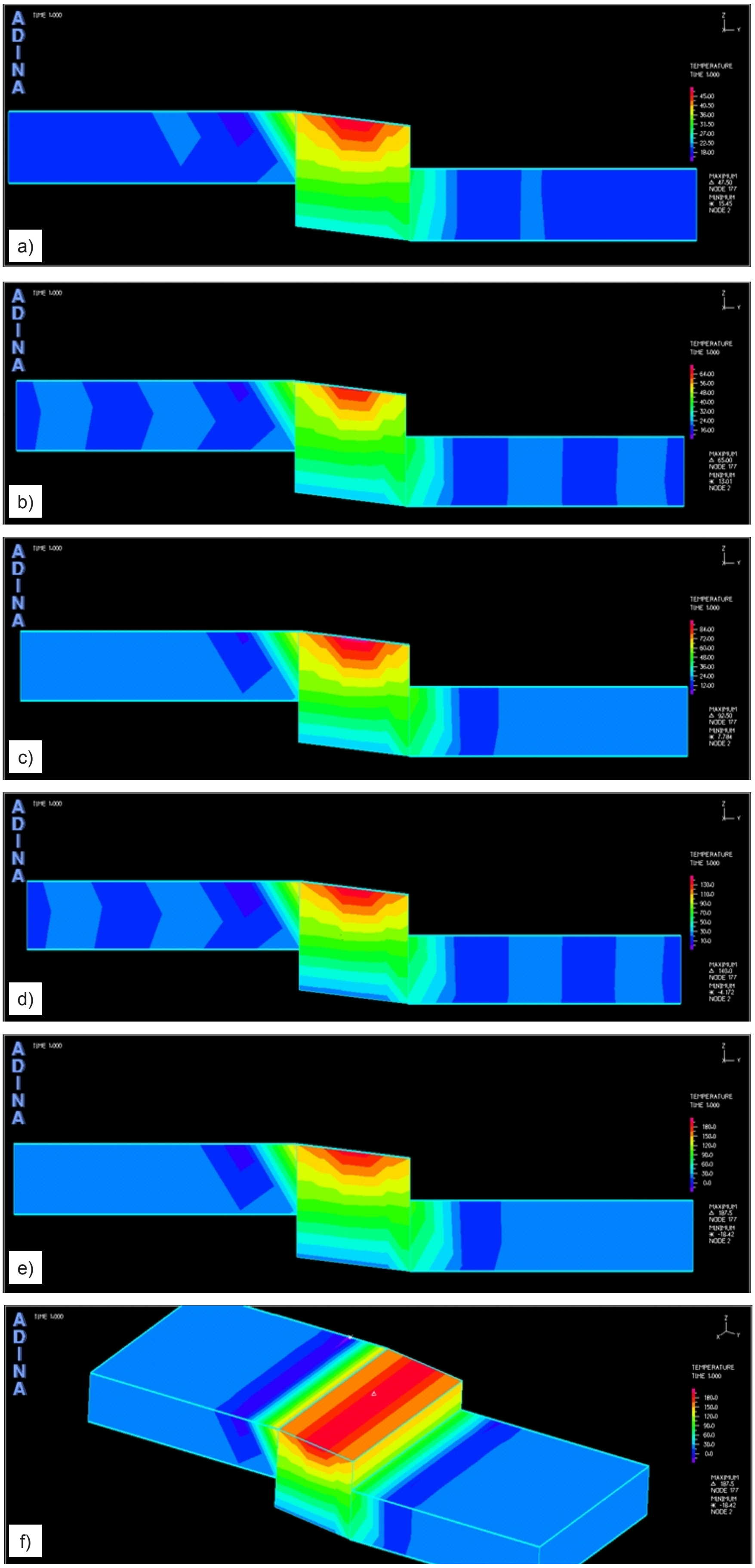

Figure 4. Distribution of temperatures within the bounded seams for different time steps: a) t = 1 s, lower heat temperatures, b) t = 2 s, c) t = 3 s, d) t = 4 s, e) t → ∞ cross section, and f) t → ∞ perspective view.

and other weather conditions. It immediately follows that these surfaces, Γ_q, are also subjected to second kind boundary conditions, and that the heat flux density is negligible, $q_n = 0$.

The other surfaces are not protected against heat lost and are subjected to the prescribed convectional heat flux $q_n = q_{nconv}$ from the structure to the environment. Thus, boundary portion Γ_C is subjected to the third-kind boundary condition.

The bounded seam contains the internal boundary/boundaries Γ_N characterised by the same heat flux densities in the common parts. It immediately follows that fourth-kind conditions are given here.

The initial condition describes the distribution of the state variable T within the structure area Ω bounded by the external boundary Γ.

The state equation as well as the set of boundary and initial conditions are as ***Equation 2***, where i is the number of layers, **q** - the vector of the heat flux density, $\mathbf{q}^*$ - the vector of the initial heat flux density, $\mathbf{q}^r$ - the vector of the radiation heat flux density, **A** - the matrix of the heat conduction coefficients within the material, c - the heat capacity, T - the temperature, t - the real time, T^0 - the prescribed value of temperature, h - the surface film conductance, and T_∞ is the surrounding temperature. The transient problem is too complicated to solve analytically, therefore numerical methods are introduced.
The problem can be simplified for steady heat transfer, i.e. the same temperature distribution in time. The time derivative in Eq.(2) is now equal to zero, and the problem can be denoted in ***Equation 3***.

The steady problem is relatively simple and in some elementary cases can be determined analytically. The problem mentioned was solved numerically by Korycki [4] and the results visualised by the special graphical modulus of the ADINA program.

Solution procedure for transient heat transfer

Let us consider that the material applied is a typical textile, isotropic, and homogenised according to the rule of mixture, cf. ***Equation 1***. The bounded materials are subjected to high heat flux density and have different thermal properties from the other approaches, the reason for which is the change in properties due to the concentrated heat flux and additional chemical and adhesive processes, i.e. glue penetration into the connected materials. Additionally we assume that the glue penetrates the membrane and diffuses into the fabrics.

Both materials are of the same cotton fabric. The thermal conductivity of isotropic materials is characterised by a single-component matrix of heat conductivity coefficients. Thus, we have in fact **A** = |l|.

Let us first define the material parameters for the fabric portions not subjected to concentrated heat flux density. The thermal conductivity and volumetric heat capacity of cotton fabrics can be defined, according to Haghi [2], as ***Equation 4***: where C_f is the water vapour concentration in fibres within the fabric, and ρ = 1300 kg/m^3 is the density of the fibres. Let us assume that the water vapour concentration within the fibres is constant in time and equal to C_f = 130 kg/m^3. The thermal conductivity of cotton fabric is equal to **A** = 0.0504 W/(mK) and the volumetric heat capacity c = 1175 J/(m^{3}K). Let us introduce the surface film conductance, acc. Li [6] h = 0.15 W/(m^{2}K), for surfaces coated by a material non-permeable to water and h = 0.15 W/(m^{2}K) for other surfaces, with the surrounding temperature T_∞ = 18 °C.

The thin layer of waterproof coating on the surface of both fabrics has a thermal conductivity coefficient **A** = 0.02 W/(mK). The thickness of the the nonpermeable layer is negligible in relation to the thickness of the fabrics.

The thermal conductivity coefficient of the central part of both fabrics (i.e. subjected to heat transport) is **A** = 0.0404 J/(mK) and the volumetric heat capacity c = 1025 J/(m^{3}K).

The radiation properties are defined by the constant emissivity coefficient, equal to 0.9, for all surfaces subjected to this process.

The temperature applied on the upper part of the seam surface should be varied due to the influence of the surrounding conditions. At the beginning of the process, the fabrics are subjected

$$\begin{cases} \mathrm{div}\mathbf{q}^{(i)} = c^{(i)} \dfrac{\partial T^{(i)}}{\partial t} \\ \mathbf{q}^{(i)} = \mathbf{A}^{(i)} \cdot \nabla T^{(i)} + \mathbf{q}^{*(i)} \end{cases} \quad \text{within}\,\Omega; \tag{2}$$

$$T^{(i)}(\mathbf{x},t) = T^{0(i)}(\mathbf{x},t) \;\; \mathbf{x} \in \Gamma_T; \;\; q_n(\mathbf{x},t) = 0 \;\; \mathbf{x} \in \Gamma_q;$$

$$q_n^{(i)}(\mathbf{x},t) = h\left[T(\mathbf{x},t) - T_\infty(\mathbf{x},t)\right] \;\; \mathbf{x} \in \Gamma_C; \quad \mathbf{n} \cdot \mathbf{q}^r(\mathbf{x},t) = q_n^r(\mathbf{x},t) \;\; \mathbf{x} \in \Gamma_r;$$

$$q_n^{(1)}(\mathbf{x},t) = q_n^{(2)}(\mathbf{x},t) \;\; \mathbf{x} \in \Gamma_N; \quad T^{(i)}(\mathbf{x},0) = T_0^{(i)} \;\; \mathbf{x} \in (\Omega \cup \Gamma); \;\; i = 1,2.$$

$$\begin{cases} \mathrm{div}\mathbf{q}^{(i)} = 0 \\ \mathbf{q}^{(i)} = \mathbf{A}^{(i)} \cdot \nabla T^{(i)} + \mathbf{q}^{*(i)} \end{cases} \quad \text{within}\,\Omega; \tag{3}$$

$$T^{(i)}(\mathbf{x}) = T^{0(i)}(\mathbf{x}) \;\; \mathbf{x} \in \Gamma_T; \;\; q_n^{(i)}(\mathbf{x}) = 0 \;\; \mathbf{x} \in \Gamma_q; \;\; \mathbf{n} \cdot \mathbf{q}^r(\mathbf{x}) = q_n^r(\mathbf{x}) \;\; \mathbf{x} \in \Gamma_r;$$

$$q_n^{(i)}(\mathbf{x}) = h\left[T(\mathbf{x}) - T_\infty(\mathbf{x})\right] \;\; \mathbf{x} \in \Gamma_C; \;\; q_n^{(1)}(\mathbf{x}) = q_n^{(2)}(\mathbf{x}) \;\; \mathbf{x} \in \Gamma_N; \qquad i = 1,2.$$

$$\mathbf{A} = |\lambda| = \left(44.1 + 63.0\, C_f \rho^{-1}\right) \cdot 10^{-3}\, \mathrm{W/(mK)};$$

$$c = \frac{\left(1663.0 + 4184.0\, C_f \rho^{-1}\right)}{1610.9\left(1 + C_f \rho^{-1}\right)} \cdot 10^3\, \mathrm{W/(m^3K)}; \tag{4}$$

Equations: 2, 3 and 4.

to a constant surrounding temperature T_∞ = 20 °C for t = 0, and for a time t = 1 s they were exposed to temperatures of the corresponding values: T^{01} = 150 °C (the central zone), T^{02} = 125 °C (the intermediate zone), and T^{03} = 100 °C (the external zone), which is the influence of the heat loss. For a time within the ranges t ≥ 1.5 s and t ≤ 4 s, the state variables increase to the maximal values, and we have three zones of temperature: the central T^{01} = 200 °C, the intermediate T^{02} = 175 °C, and the external T^{03} = 150 °C. The temperature applied on the lower part of the seams on the reference plane is constant and equal to T^0 = 25 °C. The warming-up time of the seam material by the thermo-welding machines most used should not be greater than t = 4 s. To compare the results obtained, ***Figure 4*** contains the temperature distribution for a time t → ∞, which in fact determines the temperature equalisation and the steady problem of heat transfer.

The finite element net applied is shown in ***Figure 4*** as a fas3D space problem within the graphical modulus of the ADINA program. Convective heat transfer can be described as a space phenomenon on the external surface, therefore we introduce a space model of the structure. The finite element net is made of 4-nodal3D-space elements. The results obtained is a distribution of the temperature within the seam, which can be visualised by means of any graphical modulus, cf. for example the ADINA program.

The distribution of the state variable is shown in ***Figure 4.a - 4.f***. The problem is time-dependent. The relatively low temperatures on the upper surface at the beginning of the process (***Figure 4.a***, t = 1 s) do not have a fundamental influence on the distribution within the whole structure. The fabric warms up, and the differences are insignificant during a short time. Normal warming is a time-consuming process, described in ***Figure 4.b - 4.d*** for the time steps prescribed by the different temperature distributions (i.e. the temperature maps obtained after visualisation). The last case t→∞ in fact describes the temperature equalisation and, consequently, the steady problem. The existing temperature gradient on the common surface changes along the seam and is maximally equal to 35 °C, the main reasons for which are the nonpermeable layers on both materials as well as the heat convection on the external surfaces, minimising the heat flux.

The temperature distributions obtained for the different operation steps allow to optimise the operation time. The longer the time, the higher the temperature within the seam; however, the technology determines the correct temperature, being no higher than the level assumed. Practically and technologically speaking the maximal time of the warming up should be no longer than 2 s.

The temperature distribution is irregular within the cross-section, which can influence the seam structure and its stability. The central part of the upper fabric within the seam is always subjected to the maximum temperature, whereas the side and lower parts of the structure transport the heat to the surroundings. Thus, the shapes of the isotherms within the upper part of the seam are comparable during the whole process, but the values are quite different. The maximal temperature values are distributed deeply in the material surrounding the phase change surface. The internal surface between the connected materials is also the interface of the temperature distribution. The internal surface is coated by the special nonpermeable membrane to prevent the penetration of the water from the surrounding. This layer influences the temperature distribution within the seam, but the difference is not the fundamental one. The nonpermeable coating on the internal boundary ensures that the heat transported by the reference plane determines the more regular temperature distribution within the lower fabrics.

Irregularities caused by accuracy errors within the elements are not significant, which is shown in the space temperature map for a steady case in ***Figure 4.f***. This situation is also similar to that of steady heat transfer, indirectly confirming the correctness of the calculations performed, cf. Korycki [4].

Conclusions

It is impossible to obtain a precise distribution of the temperature field within the connected seams of fabrics for transient problems by means of mathematical function and simple analytical methods. Thus, we have to introduce numerical procedures, and only approximate results are available. The temperature distributions obtained allow to optimise the duration of the heat impulse within the seam because the temperature should be determined from a clearly defined range. Lower values are unacceptable because the glue is not activated from the tape and adhesion between the layers is not initiated. Higher temperatures damage the delicate material,destroy the tape and disturb the correct adhesion process within the connected materials.

Visible irregularities in the temperature can influence the stability of the seam sides subjected to lower heat transport. Thus maximal heat flux density and durability exist in the central part of the seam structure, although the differences obtained are not significant. The difference between some points within the connected seam arouses considerable attention. Thus irregularities of temperature influence structural durability and the correctness of the seam structure.

The model of heat transfer introduced is defined by means of three classical methods: the conduction within the material, the convection, and the radiation on the external surfaces of the connected structure. The influence of the convection and radiation heat transfer is significant because heat is always transferred more effectively from the side parts of the connected fabrics, and the temperature decreases rapidly within the material. Additionally there is radiation interaction between the machine head and upper surface of the seam because both are subjected to radiation heat transfer. Practically speaking, it is difficult to describe precisely the radiation interaction within theoretical engineering models. Some interesting advice has been formulated by different authors, cf. the works of Siegel and Howell. Physically speaking, radiation heat transfer increases on the upper seam surface, which can be described, for example, by means of the increase in the demissivity coefficient on the surface.

The temperature distributions obtained are always similar irrespective of the temperature on the upper surface. Of course, the values obtained are different depending on the heating time. The bigger the temperature, the longer the equalisation zone within the side parts of the fabrics because more heat is transported within the material. The differences obtained are relatively small for different temperatures, and the distributions are comparable, which means the temperature decreases rapidly in the material, i.e. the temperature gradients are greater at increased temperatures on the upper surface. The temperature distributions obtained have some irregularities caused by the accuracy errors, cf. ***Figure 4.b*** & ***4.d***. The different material properties of the central and side parts of the fabrics also cause irregularities in the temperature distributions.

It is evident that theoretical and numerical modeling can be a promising tool for generating temperature distributions within composite textile dressings. The seams are subjected to concentrated heat flux, and it is important to create a correct structure. This class of problem can be developed in the future for different materials and temperature imperfections. Heat transfer problems can be applied for the shape optimisation and shape identification of different textile structures bounded by a thermo-welding machine. We can apply a multilayer seam structure as well as different cases of fabric materials. Of course, the problem should be verified experimentally. A detailed analysis of such verifications, implementations, their efficiency and accuracy is beyond the assumed scope of this paper.

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