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Impact of an Object on a Layer of Fibres Submerged in a Fluid

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Abstract
In this paper the phenomena occurring during the impact of a body falling from a height on a layer of fibres submerged in a fluid are studied. The properties of the layer are assumed to be determined by both the bending elasticity of the fibres and the resistance to fluid flow which is squeezed of the layer. It was found that filling the layer with fluid considerably decreases the impact force.

Key words: *impact energy, energy absorber, fibre layer submerged in fluid, fibre compression.*

Introduction

A study of the problem of the impact of elastic bodies on beams and plates with respect to transverse deformations can be found in paper [1]. In the present paper a mathematical model of an object impacting on a layer of fibres is formulated. Account is taken of both the fluid flow squeezed out of the layer and the gradual locking of fibres resulting from their increased mutual contact, causing an increase in the layer stiffness. Mathematical descriptions and experimental results of textiles subjected to compressive forces can be found in papers [2-14].

Theory

Consider an object that is falling from a height u on a layer of fibres submerged in a fluid. The object is composed of two masses, m_1 and m_2,connected by a spring of stiffness k_2 and damping coefficient c_2.

The scheme of a model describing this configurations is presented in ***Figure 1***.

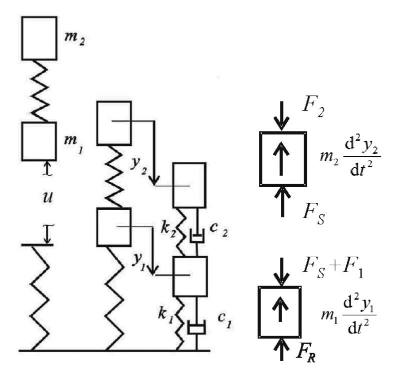


Figure 1. *Model of a body composed of two masses and a spring (m_1, m_2, k_2, c_2) falling from a height u on a layer of fibres (k_1, c_1).*

The motions of the masses are described by equations (1).

$$m_1 \frac{\mathrm{d}^2 y_1}{\mathrm{d}t^2} + F_R - F_S - F_1 = 0, \qquad m_2 \frac{\mathrm{d}^2 y_2}{\mathrm{d}t^2} + F_S - F_2 = 0. \tag{1}$$

The reaction of the spring of stiffness k_2 and damping coefficient c_2 to the relative motion of the two masses m_1 and m_2 is expressed in the form (2).

$$F_S = c_2\left(\frac{\mathrm{d}y_2}{\mathrm{d}t} - \frac{\mathrm{d}y_1}{\mathrm{d}t}\right) + k_2(y_2 - y_1). \tag{2}$$

In order to determine the reaction of the layer to the impacting object, the following three assumptions for the fibre layer properties are made:

1. When a mass pressing the layer moves down by y_1, then the thickness H of the layer decreases (H-y_1), and as a consequence the area of its free side $C(H$-$y_1)$ decreases. Let us assume that the volume of the fluid pushed down in the layer by a pressing plate of area A is equal to the volume of the fluid (3), which is released to the free sides of the layer in direction x, being perpendicular to the direction of the pressing plate's movement y .

$$A\mathrm{d}y_1 = C(H - y_1)\mathrm{d}x, \quad \frac{\mathrm{d}x}{\mathrm{d}y_1} = \frac{A}{C}\frac{1}{(H - y_1)}, \quad \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}y_1}\frac{\mathrm{d}y_1}{\mathrm{d}t} = \frac{A}{C}\frac{1}{(H - y_1)}\frac{\mathrm{d}y_1}{\mathrm{d}t}. \tag{3}$$

2. Since the fluid escapes due to the decreasing area of the side , its velocity in relation to the that of the pressing plate must increase. The component of the compressive force F_y, which overcomes the fluid flow resistance, is assumed to be proportional to the second power of the fluid velocity (as can be seen from equation 4).

3. It is assumed that the fibres in the layer simply lie one on the other without any other binding. The influence of the torsion of the fibres and friction forces are not considered in this paper. Besides the resistance to fluid flow, only the bending stiffness is considered here.

Let us assume that the properties of each free segment of a fibre can be approximately found using beam theory. The transverse deflection y of a beam of length l made of a material of Young's

$$F_c \mathrm{d}y = c_0 \,\mathrm{sgn}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 \mathrm{d}x,$$

$$F_c = c_0 \,\mathrm{sgn}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 \frac{\mathrm{d}x}{dy} = c_0 \,\mathrm{sgn}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\left(\frac{A}{C}\frac{1}{H - y_1}\frac{\mathrm{d}y_1}{\mathrm{d}t}\right)^2 \frac{A}{C}\frac{1}{H - y_1}$$

$$= c_0 \,\mathrm{sgn}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\left(\frac{A}{C}\right)^3 \frac{1}{(H - y_1)^3}\left(\frac{\mathrm{d}y_1}{\mathrm{d}t}\right)^2 = c_0\left(\frac{A}{CH}\right)^3 \mathrm{sgn}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)\frac{1}{\left(1 - \frac{y_1}{H}\right)^3}\left(\frac{\mathrm{d}y_1}{\mathrm{d}t}\right)^2, \tag{4}$$

$$F_c = c_1 \,\mathrm{sgn}\left(\frac{\mathrm{d}y_1}{\mathrm{d}t}\right)\frac{\left(\frac{\mathrm{d}y_1}{\mathrm{d}t}\right)^2}{\left(1 - \frac{y_1}{H}\right)^3}, \qquad c_1 = c_0\left(\frac{A}{CH}\right)^3.$$

Equations 4

$$Y = F\,\frac{\left(L_0 - aY\right)^3}{k_0}. \tag{5a}$$

$$y_1 = nY = nF\,\frac{\left(L_0 - aY\right)^3}{k_0} = F\,\frac{\left(nL_0 - naY\right)^3}{n^2 k_0} = F\,\frac{\left(nL_0 - ay_1\right)^3}{n^2 k_0},$$

$$F_k = Nn^2 k_0\,\frac{y_1}{\left(nL_0 - ay_1\right)^3} = N\,\frac{n^2 k_0}{\left(nL_0\right)^3}\,\frac{y_1}{\left(1 - \frac{ay_1}{nL_0}\right)^3} = N\,\frac{k_0}{nL_0^{\,3}}\,\frac{y_1}{\left(1 - \frac{ay_1}{nL_0}\right)^3} = k_1\,\frac{y_1}{\left(1 - \frac{y_1}{L}\right)^3},$$

$$k_1 = \frac{Nk_0}{nL_0^{\,3}}, \quad L = \frac{nL_0}{a}. \tag{5b}$$

$$F_r = \frac{k_1 y_1}{\left(1 - \frac{y_1}{L}\right)^3} + \frac{c_1 \operatorname{sgn}\left(\frac{\mathrm{d}y_1}{\mathrm{d}t}\right)\left(\frac{\mathrm{d}y_1}{\mathrm{d}t}\right)^2}{\left(1 - \frac{y_1}{H}\right)^3}, \quad y_1 < H, \quad y_1 < L \tag{6}$$

where: $F_R = F_r \;\; for \;\; F_r \geq 0 \qquad F_R = 0 \;\; for \;\; F_r < 0$

Equations 5 and 6

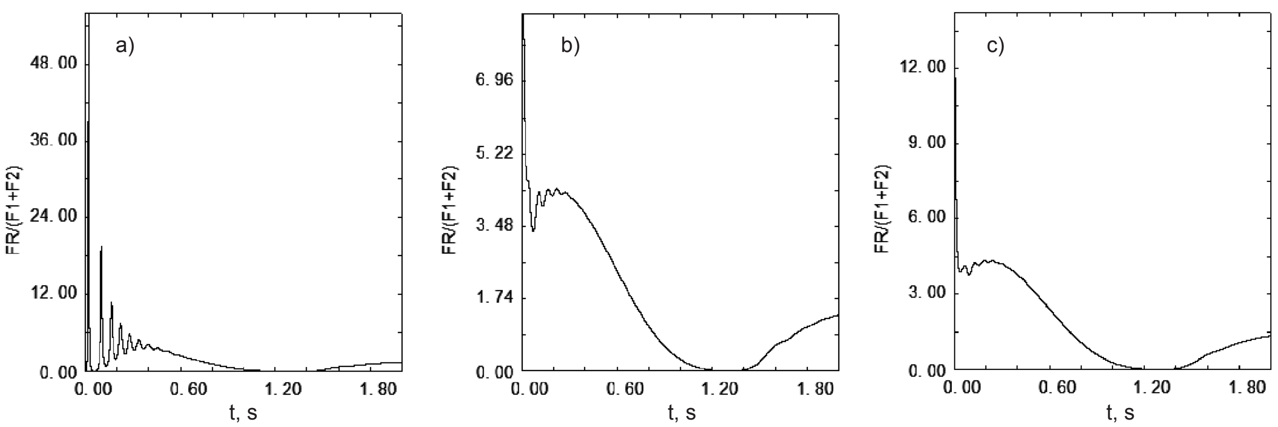
modulus E with the moment of inertia of a cross-section I under the action of a concentrated force F is $Y=FL_0^3/k_0$, where stiffness $k_0=bEI$, and b depends on the boundary conditions. The compression of a layer of fibres, which have some curvature, results in their flattening due to the bending or unbending of each fibre. With an increase in the force, the fibres gradually come into mutual contact, as a result of which the free length of a fibre that is undergoing unbending gets shorter, and thus the fibre becomes stiffer. It is assumed that the not supported span length of the fibre L_0 decreases by a value proportional to its deflection Y (5a).

By summing the deflection for n fibres of average properties lying one on the other, and then summing the resistance forces of N pieces of fibre lying horizontally, one gets relationships (5b).

Finally, from assumptions (3,4,5b) the dependence between the compressive force F_R and the compression magnitude y_1 of a collection of fibres is found to have form (6). Constants k_1, c_1, L, H have to be determined experimentally.

Numerical results

The set of differential equations (1) together with relationships (2) and (6) was solved numerically for masses m_1 = 5 kg & m_2 = 40 kg, a gravity acceleration g = 9,81 m/s^2, gravity forces $F_1 = m_1 g$ & $F_2 = m_2 g$, a stiffness coefficient of the layer k_1 = 500 N/m, a spring damping coefficient c_2 = 100 Ns/m, a thickness of the layer H = 0.3 m, $L = H$, a height of falling u = 10 m and



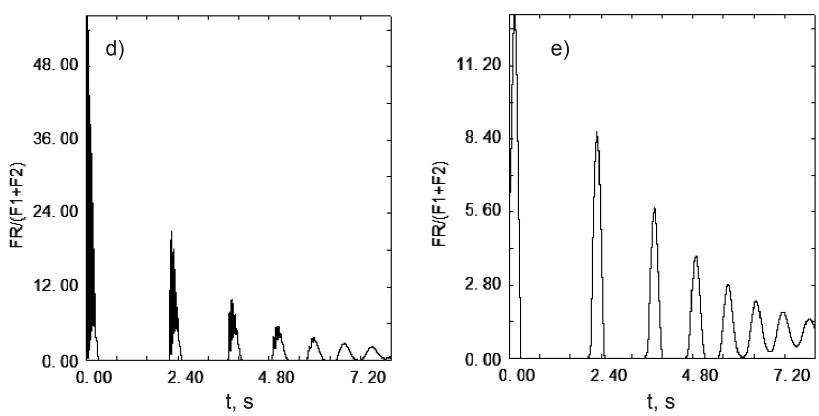


Figure 2. *Reaction of layer F_R versus the time t[s] for a soft spring (k_2 = 500 N/m) connecting the masses – for a layer without fluid c_1 = 0 (a), and for a layer submerged in fluid c_1 = 16 Ns²/m² (b) & c_1 = 32 Ns²/m² (c), and for a hard spring (k_2 = 5000 N/m) connecting the masses, a layer without fluid c_1 = 0 (d), and a layer submerged in fluid c_1 = 16 Ns²/m² (e).*

for initial conditions $y_1(0) = y_2(0) = 0$, $dy_1/dt(0) = dy_2/dt(0) = (2gu)^{0.5}$. The results of calculations for the stiffness of the spring connecting masses k_2 = (500, 5000) N/m and fluid parameter associated with its viscosity c_1 = (0, 16, 32) Ns^2/m^2 are shown in ***Figure 2***.

The reaction force F_R of the layer relative to the gravity of both masses just after the impact is presented in ***Figure 2*** for a layer without fluid (a,d), a layer submerged in fluid (b,e,c), for a soft spring (a,b,c) connecting the masses and for a hard spring (d,e). In the figures two components of vibration at two different frequencies can be observed. The quickly varying component of vibration is associated with the smaller mass m_1, which is in contact with the layer, where fibres come into mutual contact, locking one to another. The slowly varying component of the reaction force results from the delayed pressing action of the bigger mass m_2, which has no direct contact with the layer. Comparing the maximum reaction force for a layer without fluid and with fluid, ***Figures 2a-b*** & ***2d-e***, one can see that when the layer is submerged in the fluid, the force is significantly decreased. Comparing the maximum forces in ***Figures 2a-c***, one can see that the force is smallest for an intermediate value of the parameter associated with fluid viscosity.

Concluding remarks

1. The mathematical model of a layer of fibres formulated takes the following into account: (I) the gradual locking of fibres when coming into mutual contact, (II) the energy absorption due to squeezing a fluid out of the layer, and (III) the increasing resistance to fluid flow caused by the decreasing distances between fibres and the decreasing side area of the layer.

2. When fluid is absent in the layer, the fibres quickly come into mutual contact, making the layer stiff and the reaction force acting on a falling object high.

3. When fluid is present in the layer, the fibres are not locked so tightly, and the reaction force is smaller.

4. It is possible to choose a value of a fluid parameter associated with its viscosity in order to minimise the reaction force acting on a falling object.

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