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Modelling the Stress-strain Curve of Textile Products. Part 2 – Simulation of Real Stress-strain Curves

Abstract
This paper presents the second part of our study describing the modelling of the yarn stretching phenomena recently published in F&T in EE [1]. The continuation of this approach involves the identification of unknown parameters in the model, as well as proper stress-strain curve division, in order to avoid problems of identification. In order to avoid the effects of parametric compensation, which has been very often observed, when the number of model parameters becomes significant we use a new strategy of identification. The division of the model into submodels contributes to the correct operation of the identification, because we use a method which partially identifies with submodels which have a small number of parameters. The parameters of the proposed model were identified using MATLAB software. Various types of yarns were tested according to whether their origin was natural and man-made. Finally, the superposition of the practical and simulated curves was carried out, with the aim of verifying the correctness of the proposed model. The results show that the model was adapted perfectly in almost all types of the yarns used.

Key words: *stress-strain curve, yarn, modelling, simulation.*

Introduction

In [1] we presented several models of the stress-strain curve, together with our new proposition. According to the system proposed, we observed the stress-strain curve as a signal over time. This makes it possible to study the sensitivity of single parameters influencing the curve's shape. We analysed each single parameter with the other parameters constant for each model of the stress-strain curve, as well as for the models established by other researchers which described the yarn stretching phenomena. We established an initial hypothesis to divide the time period into three distinct zones (see Figure 1, [1]). The division of the model into submodels contributes to correct identification, because we used a method of identification by parts, with submodels having a small number of parameters (see Equations 1 to 3, [1]). After checking that

the parameters of our model contribute to a good representation of the curve, by acting in a similar way as for parameters of other models, we wrote a simulation program using the MATLAB software in order to facilitate the identification of the parameters of the model proposed.

Procedure and algorithm of identification

The proposed procedure of identification is given in Figure 2. The model of the stress-strain curve was judiciously selected in order to avoid problems of identification. The division of the model into submodels is strategic, because it enables the identification of vector parameters as a whole by assembling the parameters of the submodels. In the case of a single model, which integrates a great number of parameters, complex methods of identification must be implemented in order to avoid parametric compensations caused by the large number of parameters [8].

In our case, the identification was carried out in several stages. The first consisted of identifying the parameters of submod-
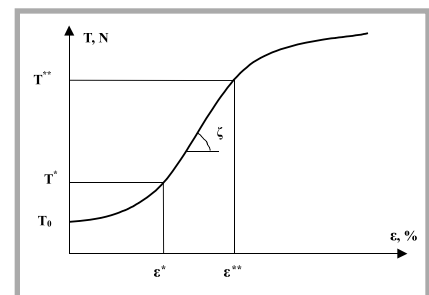

***Figure 1.** Model of the stress-strain curve [1].*

els relating to zones 1 and 2 of the stress-strain curve. The second corresponded to the identification of the parameters of the submodel relating to zone 3. The flow-chart of Figure 2 presents the methodology described by the following stages:

- Analysis of a given stress-strain curve in order to locate the linear zone (2), and to obtain the approximate values of the parameters ε^*, T^*, ε^{**}, T^{**}.
- Introduction of the initial vector parameters necessary to start the program of identification. Only submodels 1 and 2 are used.
- Identification of the parameters ε^*, T^*, ε^{**} starting from zones 1 and 2

$$\begin{cases} T = \zeta\left[\left(1 - \dfrac{T^*}{\zeta\varepsilon^*}\right)\varepsilon + \dfrac{2T^*}{\zeta} - \varepsilon^*\right]\left(\dfrac{\varepsilon}{\varepsilon^*}\right) + T_0 & \varepsilon \leq \varepsilon^* \quad (1) \\[3mm] T = \zeta\left(\varepsilon - \varepsilon^*\right) + T^* & \varepsilon^* \leq \varepsilon \leq \varepsilon^{**} \quad (2) \\[3mm] T = \zeta\left[1 - \left(a + d\exp\left(-\dfrac{\varepsilon}{c}\right)\right)\right]\times\left(\varepsilon - \varepsilon^{**}\right) + T^{**} & \varepsilon \geq \varepsilon^{**} \quad (3) \end{cases}$$

Equations 1, 2, and 3.

of the stress-strain curve (nonlinear method of least squares). The results correspond to a first approximation of these data. The aim of this stage is to facilitate the convergence of the identification algorithm.

- Introduction of the initial vector parameters necessary to the identification of submodels 1, 2 and 3. The results from the earlier identification are included in this vector.
- Identification of the parameters ε^*, T^*, ε^{**}, T^{**}, a, b, c, by accounting all the points of the stress-strain curve (zones 1, 2 and 3). We always use the nonlinear least square method.
- Presentation of the values obtained by superposition of the given real curve and the simulated curve obtained.
- Acceptance or rejection of the results. If they are rejected, it is necessary to take the procedure again, as in the first point.

Results: Comparison of the given real and obtained simulated stress-strain curves

Verification of the proposed model

In the following chapter, we will validate our model for various types of yarn differentiated by their origin: cotton, cellulose (Tencel), and polyester. It is already known that the yarn stress-strain curves may have various shapes. In the future, we also want to test the influence of the yarn origin more precisely, in order to check the impact of its parameters on the curve's shape, because in clothing applications, the yarns are often classified according to the fibre origin. Two basic categories of fibres appear: natural, and man-made, divided into fibres made of natural polymers, and those of synthetic polymers.

The whole process of identifying the model parameters was carried out using the MATLAB software, as shown by the algorithm in Figure 2. In order to start our program, it was necessary to determine approximately the initial vector parameters after an analysis of the shape of the stress-strain curve, and an identification of the break points related to the three zones of the model proposed. The following steps of the program were carried out automatically until the vector parameters relating to zones 1, 2 and 3 were obtained. The procedure of identification also enables us to notice the changes of the vector parameter values during the analysis of the two first zones (1 and 2). Below we can see the changes of the
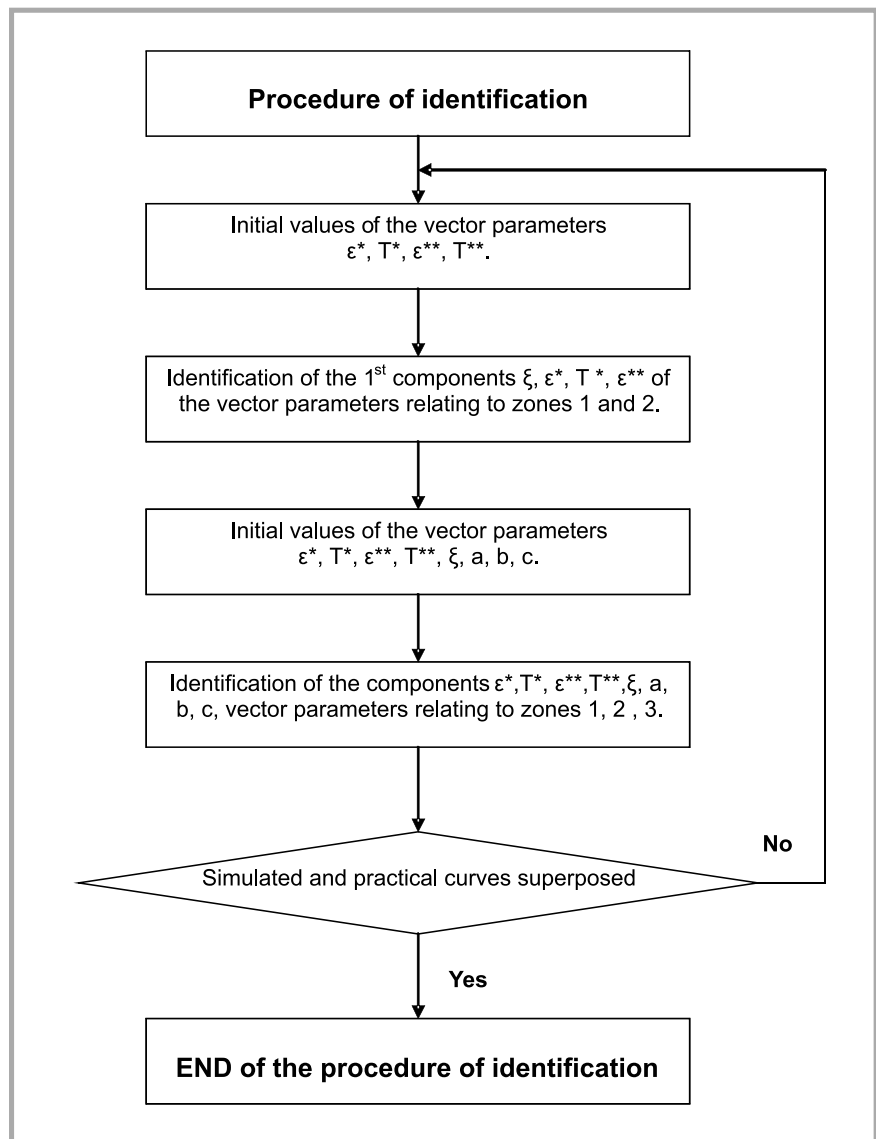

Figure 2. *Procedure of identification.*

vector parameter values throughout the whole process of identification [2]:

- before identification – the initial vector parameters,
- during identification - the results from phase 1 of the identification (zone 1 and 2),
- after identification - the results from phase 2 of the identification (zone 1, 2 and 3).

a) Natural yarn - cotton

Initial vector parameters: $\xi=1.746$, $T^*=0.6038$, $\varepsilon^*=0.3323$, $\varepsilon^{**}=0.84$, $T^{**}=0.76$, a=0.1388, b=1.22, c=3

Intermediate results: $\xi=1.9211$, $T^*=0.7986$, $\varepsilon^*=0.4557$, $\varepsilon^{**}=1.0000$

Final results: $\xi=1.9247$, $T^*=0.8062$, $\varepsilon^*=0.4600$, $\varepsilon^{**}=0.9700$, $T^{**}=1.7607$, a=0.1718, b=1.5703, c=1.7612

b) Man-made yarn from natural polymer (Tencel)

Initial vector parameters: $\xi=1.746$, $T^*=0.4$, $\varepsilon^*=0.2$, $\varepsilon^{**}=0.8$, $T^{**}=1.4$, a=0.1388, b=1.22, c=3

Intermediate results: $\xi=1.7032$, $T^*=0.3913$, $\varepsilon^*=0.1474$, $\varepsilon^{**}=0.8000$

Final results: $\xi=1.7187$, $T^*=0.2230$, $\varepsilon^*=0.0510$, $\varepsilon^{**}=0.7696$, $T^{**}=1.4187$, a=-0.0368, b=1.0494, c=2.8014

c) Man-made yarn from synthetic polymer – polyester

Initial vector parameters: $\xi=1.746$, $T^*=1.3$, $\varepsilon^*=2.5$, $\varepsilon^{**}=3.7$, $T^{**}=1.6$, a=0.1388, b=1.22, c=3

Intermediate results: $\xi=0.2342$, $T^*=1.1863$, $\varepsilon^*=2.0708$, $\varepsilon^{**}=3.7000$

Final results: $\xi=0.2334$, $T^*=1.1874$, $\varepsilon^*=2.0745$, $\varepsilon^{**}=3.8602$, $T^{**}=1.6038$, a=2.1467, b=-3.3471, c=3.898
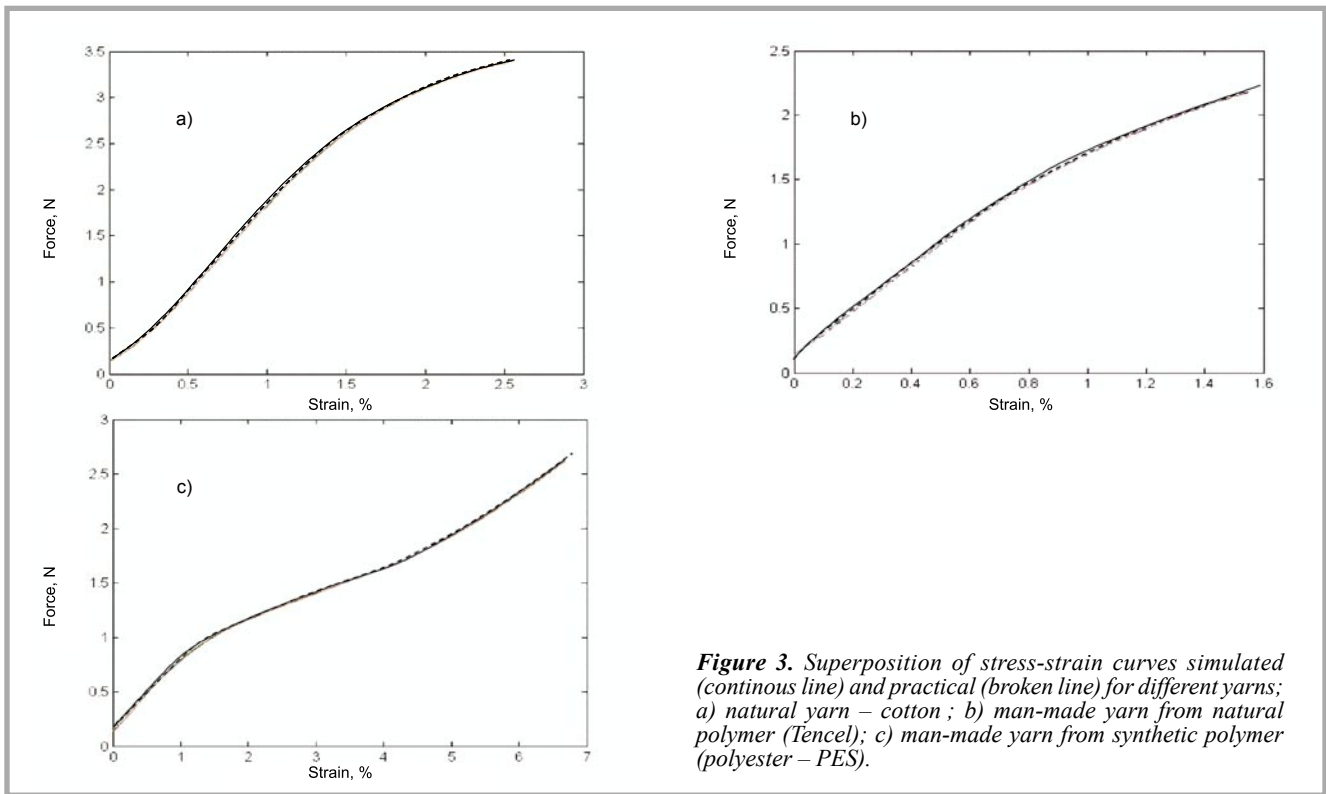
Figure 3. *Superposition of stress-strain curves simulated (continous line) and practical (broken line) for different yarns; a) natural yarn – cotton ; b) man-made yarn from natural polymer (Tencel); c) man-made yarn from synthetic polymer (polyester – PES).*

Table 1. *The complete list of values of the parameters for polyester, Tencel and cotton yarns obtained for the subsequent iterations (from 1 to 5) beginning with the initial and up to the final, where: T - express the tension, ε – presents the proportional strain, and coefficients ζ, a, b, c are the variables of the proposed model, described in part 1, and presented in Figure 1, and Equations 1 – 3.*

Yarn	Iterations from the initial to the final	ζ	T*	ε *	ε **	T**	a	b	c
PES	1	0.2150	1.7426	4.8997	6.0153	1.9896	2.8106	-6.4603	4.0443
	2	0.2334	1.1874	2.0745	3.8602	1.6038	2.1467	-3.3471	3.8982
	3	0.2827	0.8511	1.3591	5.8951	2.2076	1.1444	0.2192	0.0868
	4	0.3158	0.8746	1.1311	2.9671	1.4329	1.7069	0.2192	0.0868
	5	0.2747	0.9092	1.5093	3.1111	1.2882	1.1784	0.2192	0.0868
TENCEL	1	1.7030	0.7714	0.3687	0.7618	1.4055	0.3150	0.8729	1.2883
	2	1.2855	1.2657	0.7391	0.8000	1.3508	0.5098	0.5870	1.3114
	3	2.0875	0.5075	0.1848	0.6700	1.4865	0.1748	1.2296	1.1495
	4	2.0221	0.3931	0.1379	0.6717	1.4484	0.3769	0.7267	1.3386
	5	1.9609	1.4561	0.6454	0.6680	1.4708	0.3274	0.7604	1.3788
COTTON	1	1.7460	0.5976	0.3287	0.9874	1.7226	0.1765	1.3811	1.6450
	2	1.9214	0.8061	0.4598	0.9817	1.7888	0.0912	1.5405	2.0498
	3	1.7053	0.5839	0.3022	0.9900	1.7373	0.1790	1.4690	1.5070
	4	1.6629	0.7504	0.4638	1.0600	1.7222	0.1828	1.6374	1.4467
	5	1.9247	0.8062	0.4600	0.9700	1.7607	0.1718	1.5703	1.7612

The complete list of values of the parameters for polyester, Tencel and cotton yarns is shown in Table 1.

Finally, we stated that the superposition of the simulated and the practical stress-strain curves presented in Figure 3 shows that the model is adapted well (but not perfectly) for most types of the yarns used by us for checking the model.

■ Conclusions

We proposed a model for predicting the shape of yarn stress-strain curves. In order to verify the correct performance of our model, three types of yarns of differ-ent origin were tested. We selected natural and man-made yarn from natural and synthetic polymers (cotton, Tencel, polyester). The curves presented show that the model was adapted. By superpositioning the simulated and real curves of stretched synthetic yarns, a small divergence in the first and second zone of the proposed model was observed. Nevertheless, this model presents the same defect as that of other studied models: the failure of not integrating the phenomena of hysteresis. Moreover, the majority of these models have a disadvantage caused by the re-lationship between the curve shape and the speed of the practical test carried out; and thus it cannot be regarded as a ge-neric model for all the tested yarns. This is why further complementary studies and the development of a more suitable model are necessary.

References

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