**Bogdan Włodarczyk,**
**Krzysztof Kowalski**

**Technical University of Łódź**
**Department of Knitting Technology**
**and Structure of Knitted Products**
ul. Żeromskiego 116, 90-543 Łódź, Poland
E-mail: kjkowalski@p.lodz.pl

# A Discrete Probability Model of Forces in Yarns Transported Through the Drawing Zone

**Abstract**
*A simulation of forces acting in model-yarns transported through the drawing zone was carried out. The model is characterised by two random variables: the length of a yarn segment (link) and the yarn's drawing rigidity. The drawing zone consists of two pairs of rolls working with different tangential velocities. Computer simulations carried out indicate the causes of the stochastic character of tensions, which is connected with the irregularity of the yarn's mechanical properties. The simulations also determine the ranges of changes to the force dispersion in dependence on the length of the drawing zone, the coefficients of variation of the drawing rigidity, and the lengths of the component yarn segments which have various mechanical characteristics.*

**Key words:** *drawing zone, rigidity, elongation, tension, modelling, simulation, yarn, probability model, coefficient of variation.*

**Designations**:

*$F_0$*, in cN - initial yarn tension,
*F*, in cN - yarn tension in the drawing zone (instantaneous value of force in the zone),
ε - relative elongation (dimensionless quantity within the range of 0 – 1),
*$V_1$*, *$V_2$*, in m/s – input (feeding) and output (expenditure) velocities of the yarn into/out of the zone,
*CCE* – list with the $C_\varepsilon$ values, in cN, for a given ε, accepted at the moment of starting the program,
*LL0* - list with free lengths of the yarn segments $L_i$, in mm, accepted at the moment of starting the program,
*LLF0* – list *LL0* after stretching the yarn by the force *F0* (the initial tension before the zone),
*FF* – list of the subsequent calculation results of the force *F*, which draw the yarn in the zone,
*CEZ* – list of the subsequent calculation results of the equivalent relative drawing rigidity $C_\varepsilon$ , in cN, in the zone,
*L_str* – list with the free lengths $L_0$ of the segments, placed in the zone (zone composition),
*C_str* - list with the elasticity coefficients $C_{\Delta L}$ , in cN/mm, of segments in the zone (zone composition),
*CE_str* – list with the coefficients of the relative drawing rigidity $C_\varepsilon$ , in cN, of segments in the zone (zone composition),
The lists *L_str*, *C_str*, and *CE_str* define the mechanical properties of the yarn segment, which is positioned at the given moment in the zone,
*dl_zad_Rap* – list of the subsequent sampled report lengths (archive of the random modification of the yarn segment transported over the zone),
*Ls*, in mm, - length of the drawing zone (distance between the rolls),
*Lo*, in mm, - free length of the yarn, positioned in the zone (sum of the segments of list *L_str*),
$\Delta L = Ls - Lo$ – absolute elongation of a yarn segment in the zone,
Δ*x*, in mm, - length of the segment which determines the frequency of the calculations carried out after drawing the segment in the zone with the actual force,
*L*, in mm, - length of the last segment in the zone of given $C_\varepsilon$ , (it may occur that $L > x$, $L=x$, or $L < x$); this is the last element of the list *L_str* after drawing it in the zone by the actual force *F*,
*tk*, in s, - time period over which the segment of length Δ*x* leaves the drawing zone,
*xw*, in mm - free length of the segment which enters the zone over the time period tk,
*LLLA* – working list of the segments *xw*, which subsequently enter the zone,

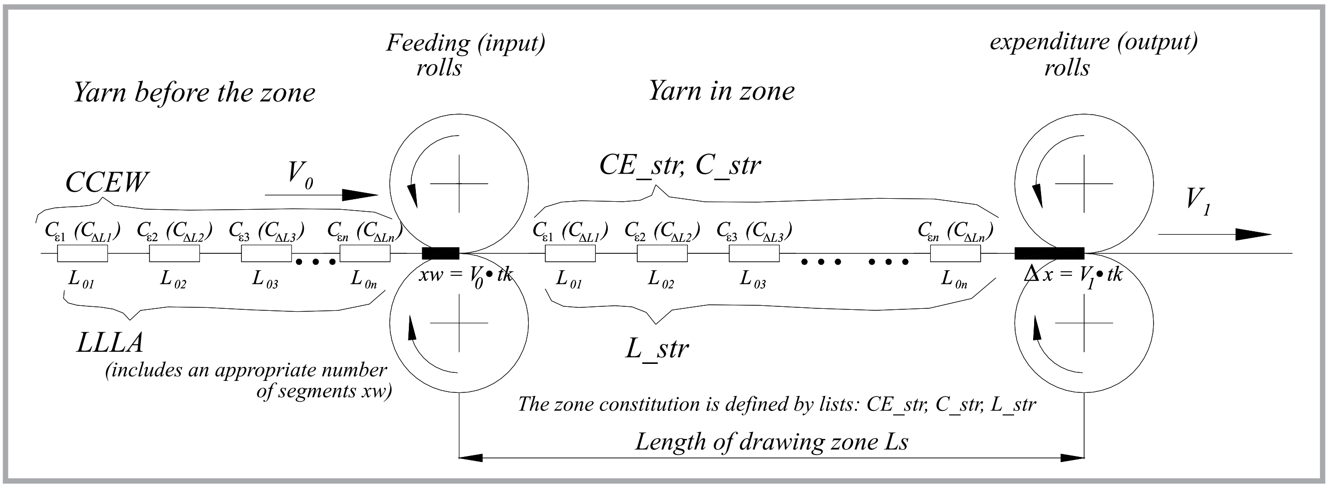

***Figure 1.*** *Model of the yarns' drawing zone accepted for our considerations; $V_0$ - feeding velocity of the yarn into the zone, $V_1$ - expenditure velocity of the yarn into the zone.*

*CCEW* – working list of the coefficients $C_\varepsilon$ which are related to the segments from the list *LLLA*,
*n_rap* – number of reports for transporting through the zone, number of drawings of the yarn segments, accepted at the starting moment,
*n_r* – number of the current report to be calculated of the subsequent sampling,
*n* – number of elements on the list *LLLA*, input into the zone.

## Introduction

While modelling the forces in yarns (as well as in fibres and threads), we may often neglect the aspect of the variability of the force values, which is connected with the internal yarn irregularities which arise. This is the reason that the results of modelling do not determine the parameters of the force value dispersion, which accompany the real textile processes [1 –9]. The dispersion of force values is explained by the occurrence of more ore less unidentified disturbances. The attempt to model the stochastic character of the yarn forces performed in this work is related to the drawing zone through which the yarn is transported and drawn. The yarn of an internal irregularity, as expressed by the variable drawing rigidity, is constantly drawn to a relative elongation of ε. Accepting the use of a yarn with irregular mechanical properties for modelling is entirely justified, as has been proved by considerations carried out in the research work [10], and confirmed by the results of the authors' own investigation into the relative drawing rigidity of yarns performed for short segments of 5 mm. The obtained results of the drawing rigidity $C_\varepsilon = \Delta F / \Delta\varepsilon$, which has been determined from the drawing curves carried out within the force range of $\Delta F = 10 - 150$ cN for three cotton knitting yarns, have been characterised by a coefficient of variation of V = 13 – 29%.

The article presented herewith aims to show the influence of changes to the relative drawing rigidity which occur at short segments on the character of the tension irregularities of a yarn transported through the drawing zone. In technological textile processes such as spinning, weaving, knitting, and texturing, zones of yarn drawing occur. Therefore the work we have carried out has not only a cognitive scientific character, but also clearly has practical importance.

## Construction of the yarn drawing zone

The variability of forces acting in yarns was modelled for a drawing zone composed of two pairs of rolls rotating synchronously, with various constant tangential velocities $V_0$ and $V_1$ (Figure 1).

### Theoretical basis of considerations

***Accepted assumptions:***

- The yarn transported is composed of short segments (links) with different values of relative drawing rigidity $C_\varepsilon$, in cN: $C_{\varepsilon 1}, C_{\varepsilon 2}, C_{\varepsilon 3}, \dots, C_{\varepsilon n}$.
- Every subsequent segment (link) has a different free length $L_0$: $L_{01}, L_{02}, L_{03}, \dots, L_{0n}$.

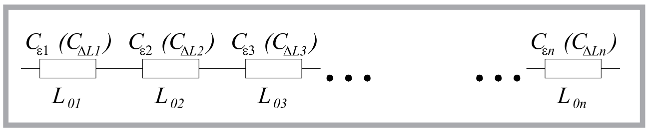

***Figure 2.*** *Yarn model according to the accepted assumptions.*

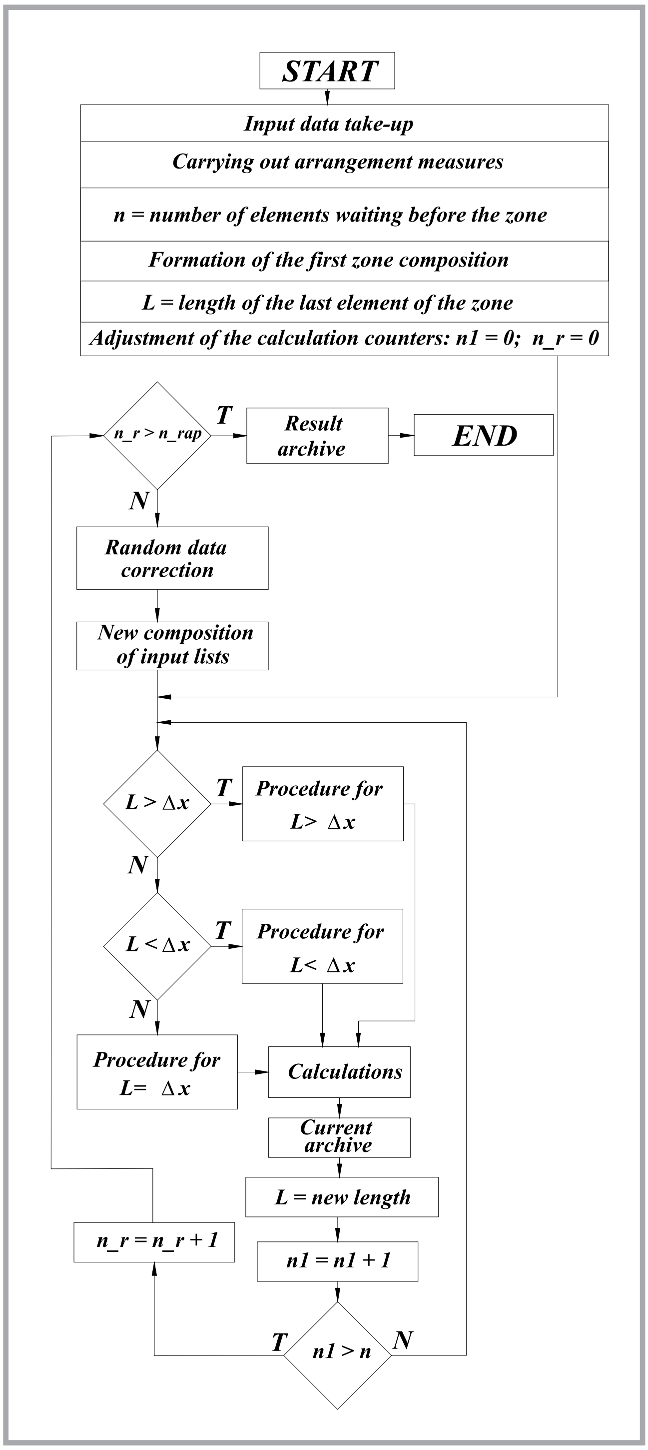

***Figure 3.*** *General block scheme of the calculation algorithm.*

- The values of the drawing rigidity coefficients change in a random way and are characterised by normal distribution.
- The relation between the relative elongation ε, and the drawing force has a linear character.
- The relative elongation in the zone is constant over the whole drawing process, ε=constant.
- The yarn transport was performed on the basis of a balance of the lengths which enter and leave the zone.
- The following input data should be accepted before starting the calculations:
  - average values of $C_\varepsilon$, in cN, and of $L_0$, in mm,
  - coefficient of variation for $C_\varepsilon$, in cN, and for $L_0$, in mm,
  - length of the drawing zone *Ls*, in mm,
  - expenditure velocity $V_1$, in m/s,
  - relative elongation $\varepsilon=(V_1 - V_0)/V_0$,
  - number of segments $L_0$ which are transported through the model drawing zone (number of sampling = *parameter n_rap*), and
  - calculation step, which means at how many millimetres along the yarn we repeat the calculation of the drawing force: $\Delta x$, in mm (elementary segment which leaves the zone).

### *Basic mathematical dependencies*

The relation between force *F* and the relative elongation ε is described by the dependencies (1):

$$F = C_\varepsilon \cdot \varepsilon \quad \text{or} \quad F = C_{\Delta L} \cdot \Delta L \tag{1}$$

The relative elongation ε is defined by:

$$\varepsilon = \frac{L_1 - L_0}{L_0} = \frac{\Delta L}{L_0} \tag{2}$$

The relation between $C_\varepsilon$ and $C_{\Delta L}$ may be calculated from (3):

$$F = C_\varepsilon \cdot \varepsilon = C_{\Delta L} \Delta L \cdot \frac{L_0}{L_0} = = C_{\Delta L} \cdot L_0 \cdot \frac{\Delta L}{L_0} = C_{\Delta L} \cdot L_0 \cdot \varepsilon \tag{3}$$

resulting in:

$$C_\varepsilon = C_{\Delta L} \cdot L_0 \tag{4}$$

The yarn model accepted is composed of many segments (links). Each segment fulfils the dependencies (1), and each of them has a different drawing rigidity $C_\varepsilon$ ($C_{\Delta L}$) and free length $L_0$ (Figure 2). The values have random character and normal distribution.

### General equivalent values of the coefficients characterising the mechanical properties of a yarn, which is composed from many component segments with different features

The free length of a part of the yarn composed of n segments with parameters as shown in Figure 2 are described by the following equation:

$$L_0 = L_{01} + L_{02} + L_{03} + \ldots + L_{0n} = \sum_{i=1}^{n} L_{0i} \tag{5}$$

If this kind of yarn fragment of the free length $L_0$ is drawn to the length $L_1$, then

$$F = C_{\Delta L\ ZAST} \cdot \Delta L = C_{\Delta L1} \cdot \Delta L_1 = = C_{\Delta L2} \cdot \Delta L_2 = C_{\Delta L3} \cdot \Delta L_3 = \ldots = C_{\Delta Ln} \cdot \Delta L_n$$

$$\Delta L = L_1 - L_0 = \Delta L_1 + \Delta L_2 + + \Delta L_3 + \ldots + \Delta L_n = \sum_{i=1}^{n} \Delta L_i \tag{6}$$

$$\Delta L = \frac{F}{C_{\Delta L\ ZAST}}; \quad \Delta L_1 = \frac{F}{C_{\Delta L1}}; \quad \Delta L_2 = \frac{F}{C_{\Delta L2}}; \quad \Delta L_3 = \frac{F}{C_{\Delta L3}}; \quad \ldots \quad \Delta L_n = \frac{F}{C_{\Delta Ln}} \tag{7}$$

After substituting the dependency (7) into (6), we obtain an equation which describes the equivalent elasticity coefficient (the equivalent drawing rigidity for the yarn fragment which is composed of n different component segments:

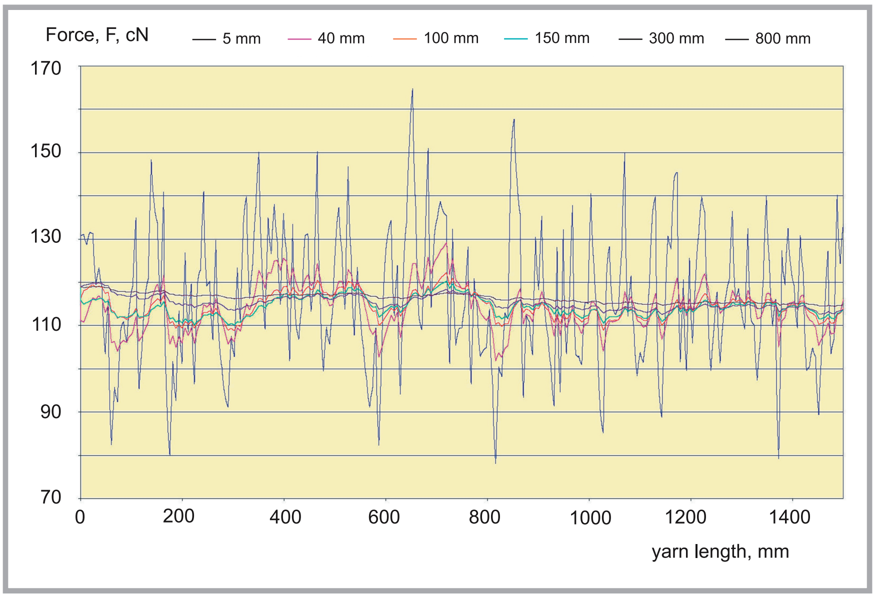

***Figure 4.*** *Stochastic changes of the model yarn tensions in the drawing zone, for zone lengths from 5 mm to 1000 mm, and for the average segment length of $L_0$=5 mm.*

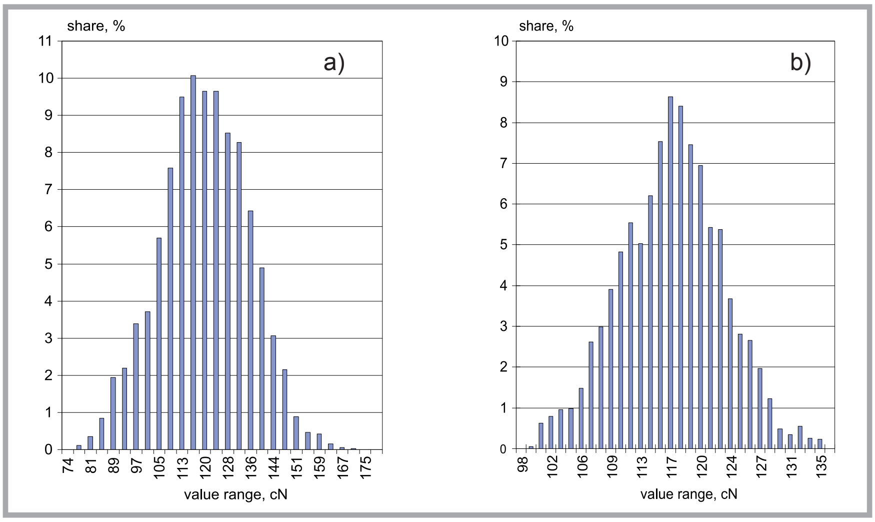

***Figure 5.*** *Yarn tension values distributions in the drawing zone, for two different zone lengths, and dispersion 19%; a) histogram of forces in the yarn for the zone of Ls=5 mm, b) histogram of forces in the yarn for the zone of Ls=40 mm.*

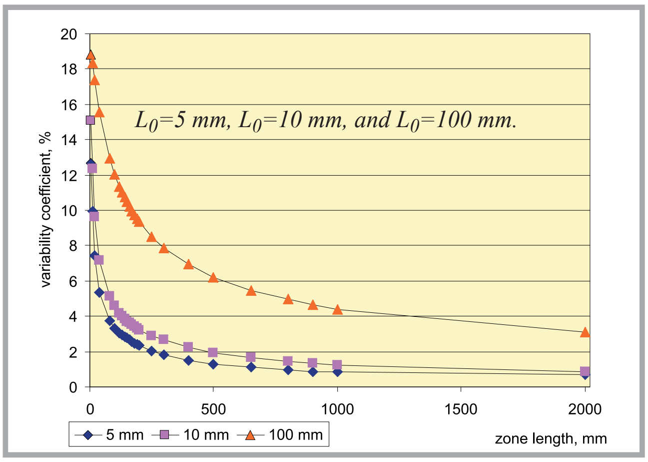

**Figure 6.** *Dependence between the drawing zone length and the variability coefficient of yarn tension in the drawing zone for three model yarns with different average lengths of the component segments.*

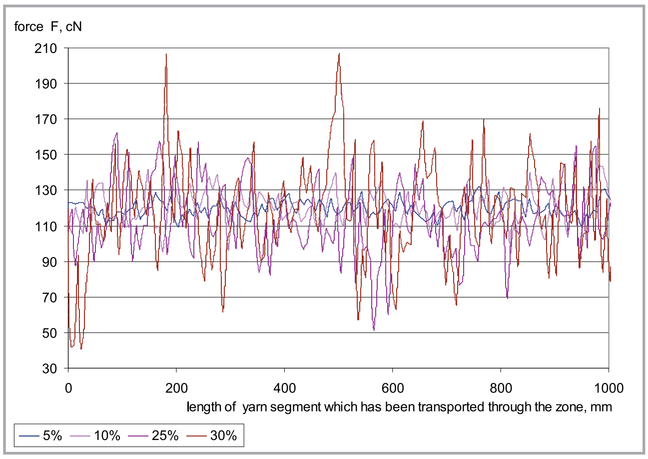

**Figure 7.** *Variability of the yarn tension in the drawing zone of 5 mm length for 4 model yarns with different variability coefficients (within the range of $V$=5-30%) of the relative drawing rigidity $C_\varepsilon$.*

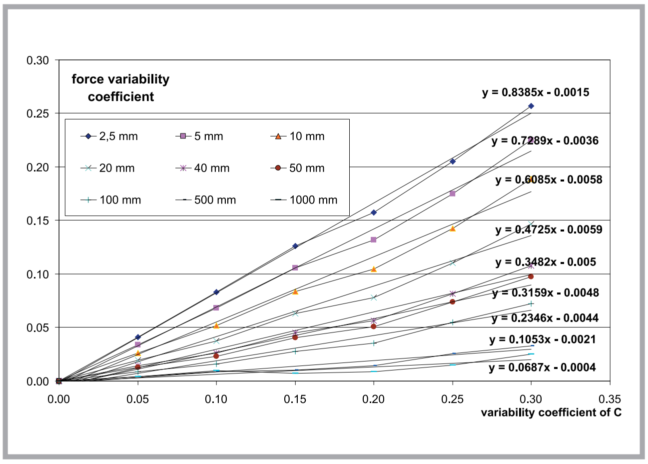

**Figure 8.** *Dependence between the variability coefficient of the yarn's drawing rigidity and the variability coefficient of force in the drawing zone, for different zone lengths.*

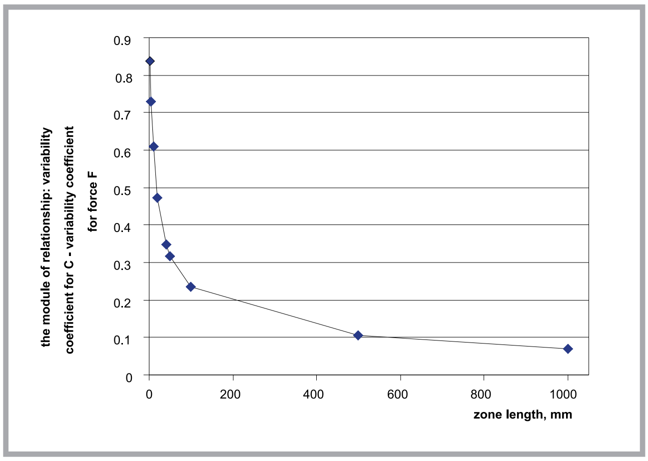

**Figure 9.** *Value of the angle coefficient of regression equations in dependence on the length of the drawing zone; M - modulus of the dependence: variability coefficient of C vs. variability coefficient of force F.*

$$\frac{1}{C_{\Delta L\ ZAST}} = \frac{1}{C_{\Delta L1}} + \frac{1}{C_{\Delta L2}} + \frac{1}{C_{\Delta L3}} + \ldots + \frac{1}{C_{\Delta Ln}} = \sum_{i=1}^{n} \frac{1}{C_{\Delta Li}} \qquad (8)$$

The coefficient $C_{\varepsilon ZAST}$ was calculated from the dependencies (4) and (5).

**An algorithm** for calculating the forces in a yarn in the drawing zone is presented in Figure 3.

## Analysis of the model

The basic investigations of the tensions in yarns in the drawing zone were carried out for model yarns with an average segment length of $L_0$=5 mm and an average drawing rigidity of $C_\varepsilon$=4000 cN; the range of the coefficients of variation of the above-mentioned parameters was $V$=(5 – 30)%. The length of the drawing zone was changed within the range of $Ls$=(2.5 – 2000) mm. Furthermore, an additional simulation was performed for the segments' average free lengths of $L_0$=10 mm and $L_0$=100 mm. The remaining input parameters of the simulation carried out are listed below:

$F_0$=10 cN, $V_1$=1.0 m/s, $\varepsilon$=0.03,
$\Delta x$=0.1 mm, and n_rap=1000.

The values of $C_\varepsilon$ and $L_0$ for the subsequent yarn segments transported through the drawing zone are modified by multiplication with the subsequent random number taken from the set of numbers with normal distribution, of an average value of 1.0, and of an accepted coefficient of variation.

### Influence of the drawing zone's length on the variability of forces

For the accepted model of a yarn with variable drawing rigidity along its length, the runs of forces in yarns which were generated (Figure 4) indicate the significant influence of the zone's length on the force values' dispersion parameters (Figure 5). The coefficients of variation of force decrease with the increase in the length of the drawing zone (Figure 6). The value drop of the coefficients of variation is all the greater as the average segments' lengths with random variable drawing rigidity are lower. Higher values of the coefficients of variation of forces for greater average lengths of the segments of the random variable drawing rigidity, especially at smaller drawing zone lengths, are caused by the simultaneous drawing of a smaller number of these segments. This is why the equivalent value of the drawing rigidity within the zone of subsequently drawn segments is characterised by a relatively high coefficient of variation, which is manifested by force oscillations. The results of computer simulation indicate that the variability of the forces in yarns in the drawing zone is mirrored

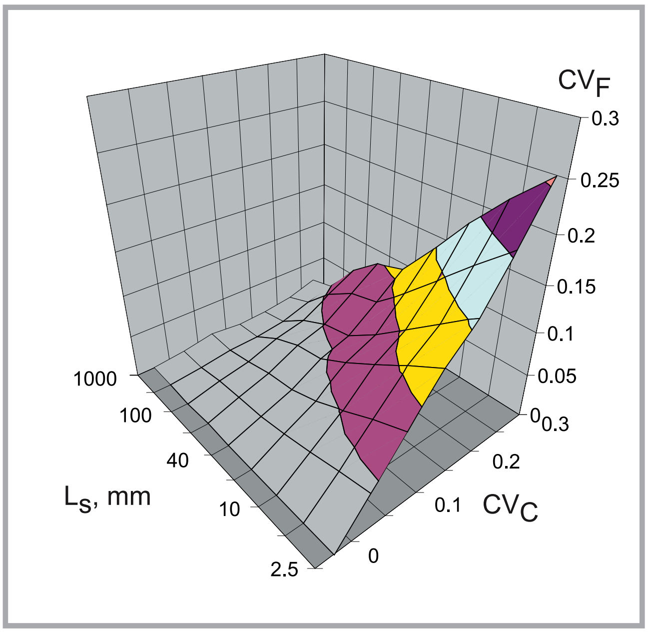

***Figure 10.*** *3-D dependence between the coefficient of variation of yarn drawing, the zone length, and the coefficient of variation of force in the drawing zone; $CV_F$ - coefficient of variation of force F, $L_s$ - zone length, $CV_C$ - coefficient of variation of the drawing rigidity coefficient C; boundary limits of the coefficient of variation of force F (from the bottom part to the upper part of the 3D-figure); ▨ - 0-0.05, 🟪 - 0.05-0.1, 🟨 - 0.1-0.15, 🟦 - 0.15-0.2, 🟪 - 0.2-0.25, 🟥 - 0.25-0.3.*

by the irregularity of the yarns' mechanical properties expressed by the variable value of the drawing rigidity. Applying a relatively short drawing zone obtains a high sensitiveness of the force-elongation relation on the irregularity of the internal yarn structure. It should be mentioned here that the yarn testers hitherto used [11, 12], the so-called 'strainometers' and 'tensomodules', which are based on the yarn elongation in the drawing zone, are characterised by long drawing zones (about $L$=300 mm), which is the reason for the averaging of the drawing rigidity values. Devices with such long drawing zones do not reflect the real variability of the rigidity on short segments by the force value.

### Influence of the variability coefficient of the relative drawing rigidity on force variability

From the analyses presented above, it results that the shorter the drawing zone is, the more intensively the influence of the variable drawing rigidity (related to short segments) is manifested on the value of the force oscillations. This is confirmed by the results of computer simulations for 5 model yarns with values of the coefficient of variation V of the relative drawing rigidity $C_\varepsilon$ within the range from 5% to 30%, presented in Figures 7 to 9. The calculated equations of linear regression and the high values of the correlation coefficients (within the range of $R$=98-0.99) between the values of drawing rigidity coefficients and the forces $F$, indicate close linear relations between the above-mentioned parameters.

The values of the angle coefficients of the regression equations indicate the ability to reflect the variations in the drawing rigidity $C_\varepsilon$ . They decrease more and more with the increase in the length of the drawing zone (Figure 8). It should be stressed that even for the shortest drawing zone's length of $Ls$=2.5 mm, the value of the linear regression coefficient equals 0.838 < 1, which indicates a decrease in the dispersion of forces in yarn in relation to the changes in the relative drawing rigidity. This is caused by the temporary occurrence in the drawing zone of two yarn segments with different values of the relative drawing rigidity $C_\varepsilon$. This phenomenon is the cause of the drop in the equivalent drawing rigidity in relation to the maximum value of the rigidity occurring in the zone composition. The longer the drawing zone is, the greater is the number of segments with random value of the drawing rigidity which are subjected to the drawing forces, and the value of the equivalent drawing rigidity approaches the average value. The 3D dependence between the coefficient of variation of yarn drawing, the zone length and the coefficient of variation of force in the drawing zone is presented in Figure 10.

## Conclusions

- The yarn model of random variable drawing rigidity along the length of the yarn allows us to generate tensions of stochastic character in a yarn transported through the drawing zone.
- The values of the coefficients of variation of forces decrease with the increase in the drawing zone's length, whereas the decrease in the values of the coefficients of variation is still greater, as the average segment lengths of the random variable drawing rigidity are shorter.
- The coefficient of the variation of tension in the drawing zone is lower than the coefficient of the variation of the yarn's drawing rigidity, independently of the length of the drawing zone; this is connected with the drop in the value of the equivalent drawing rigidity in relation to the maximum value of the rigidity occurring in the zone composition.

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