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3D Simulation of Warp Knitted Structures with a New Algorithm Based on NURBS

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Abstract

In order to obtain a three-dimensional computer simulation of warp knitted structures with more flexibility and realism, a new algorithm using Matlab was developed by NURBS based on empirical geometrical loop models. With the principles of NURBS curves, once the values of data points are known, the control points with two coincidence points at the start and end points can be uncomplicatedly calculated by Matlab. Then the NURBS curve of a single typical stitch can be simulated flexibly by Matlab. A new typical stitch selected from two stitches simulated directly by the new method is redefined to improve the joint of neighboring stitches, and it is found that there are two types of redefined typical warp knitted stitches judged by whether the two under lap on the same side or not. Based on the redefined typical warp knitted stitch, two warp knitted structures are simulated regardless of the loop offset, and all the joints of stitches are smooth.

Key words: *warp knitted stitch, three-dimensional simulation, NURBS, joint of stitches, Matlab.*

Introduction

For warp knitted fabrics, the general aim of computer simulation is to provide a fast, easy design and realistic simulation of structures to enable manufacturers to assess their design before the actual knitting. Thus three-dimensional computer simulation can predict the comfort, drape, handling characteristics and mechanical properties of fabric as well as the aesthetic aspects of the design. However, different mathematical descriptions of knitted loop geometry have existed for more than 50 years, but three-dimensional computer simulations of warp knitted structures have only existed for more than 10 years.

Goktepe [1 - 3] first simulated a true 3D solid representation of warp knit loops and structures on a computer based on empirical loop modelling by non-uniform rational B-spline (NURBS) surfaces. Based on Goktepe's 3D model, Cong Honglian etc. [4] also simulated the inlay of warp knitted structure besides stitch based on NURBS surfaces, and Li-zhe Zhang etc. [5] simulated warp knitted spacer fabric. And other researchers also simulated multi-axial warp knitted fabric [6], jacquard warp knitted towel fabric [7] in 3D based on experimental data of loop modelling by NURBS. The first time a professional three-dimensional CAD-program for general warp knitted structures was presented in 2007 by Kyosev and Renkens [8 - 10]. These works present a computer model which calculates the yarn axes in warp knitted structures. Then they simulate the warp knitted structures, which cannot be solved with pure geometrical algorithms, as it requires computational mechanics such as mechanical properties of the yarns [11]. Also Renkens and Kyosev exploited Quick 3D simulation, which gives new chances to teachers for training and testing the knowledge of students, and can be very useful especially for complex warp knitted structures with multiple guide bars and partial yarn threading [12].

Goktepe focused on simulation of details of the loop shape and empirical geometrical loop models, while Kyosev and Renkens concentrated on a more precise loop shape and more complex models based on computational mechanics. Whereas in these researches simulation of the details of the loop shape would be discussed further for 3D simulations of loops in warp knitted structures, they were still too precise to be close to a true loop.

The researchers mentioned above simulated warp knitted loops by NURBS, which was an accepted method to simulate a curve or surface, whereas they obtained the control polygon from the tangent of the surface's data point, by

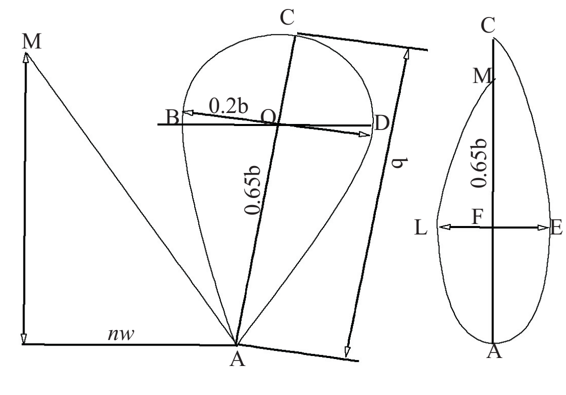

Figure 1. 3D loop model of Goktepe [2].

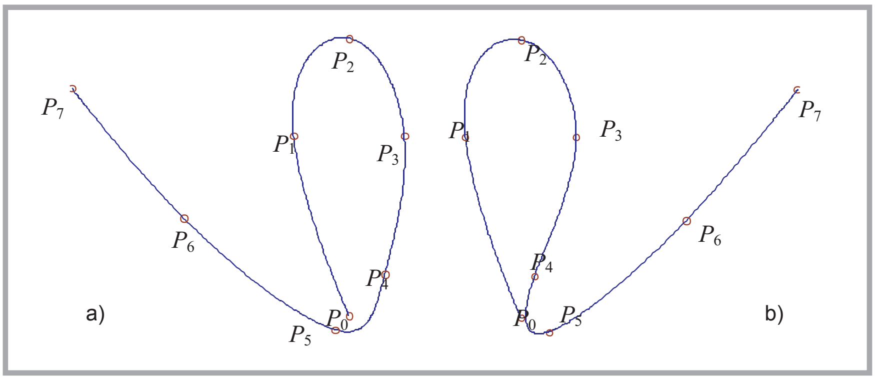

***Figure 2.** Value of the stitch's data point; a) closed stitch model, b) open stitch model.*

which the curve or surface gained can not been changed flexibly. Also the researchers mentioned above aimed for a single typical warp-knitted stitch while ignoring the link between the two stitches, hence the link between the stitches simulated was not smooth. Thus a new method of obtaining the control polygon based on NUBRS using empirical geometrical loop models is researched to get 3D warp knitted stitches, and also the link between the adjacent loops is researched.

Simulation of a single typical warp knitted stitch using NURBS

The value of the stitch's data point

A single typical warp knitted stitch, divided into a closed stitch and open stitch, concludes the loop and underlap. The stitch's 3D loop model, which was gained by experimental work is shown as ***Figure 1***. Furthermore, based on this model, Cong, Ge and Jiang [4] set eight data points and defined the knot sequence as $P_i(i = 0, 1, 2, \ldots, n)$. Based on the two references above, eight data points are set as ***Figure 2***.

The value of control points based on the stitch's data point

The NURBS curve cannot go through the stitch's data points except at the first and end points, hence the value of the control points should be calculated based on the stitch's data points. The cubic NURBS curve can be divided by n+2 control points into n-1 curves, one of which can be calculated from the control points (d_i), weights of the control points (w_i) and the node vector (u_i) according to the ***Equation 1*** [13]:

$$p_i(t) = \frac{T_3 M_i D_w}{T_3 M_i W_i} \quad t \in [0,1] \qquad (1)$$

where,

$U = [u_0, u_1, \cdots, u_{n+5}]$,

$u_0 = u_1 = u_2 = u_3 = 0$,

$u_{n+2} = u_{n+3} = u_{n+4} = u_{n+5} = 1$,

$u \in [u_3, u_{n+2}] = [0,1]$,

$u_3 < u_4 < \cdots < u_{n+1} < u_{n+2}$,

$T_3 = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix}$,

$D_w = \begin{bmatrix} w_i d_i & w_{i+1} d_{i+1} & w_{i+2} d_{i+2} & w_{i+3} d_{i+3} \end{bmatrix}^T$,

$W_i = \begin{bmatrix} w_i & w_{i+1} & w_{i+2} & w_{i+3} \end{bmatrix}^T$,

$$M_i = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix},$$

$$t = \frac{u - u_{i+3}}{u_{i+4} - u_{i+3}},$$

m_{ij} can be calculated from U [14].

Based on the interpolation condition:

$$p_i(0) = P_{i+1}(i = 1, 2, \cdots, n-2) \qquad (2)$$

thus,

$$P_{i+1} = \qquad (3)$$

$$= \frac{m_{11} w_i d_i + m_{12} w_{i+1} d_{i+1} + m_{13} w_{i+2} d_{i+2}}{m_{11} w_i + m_{12} w_{i+1} + m_{13} w_{i+2}}$$

If

$$\begin{cases} a_{i+1} = m_{11} w_i \\ b_{i+1} = m_{12} w_i \\ c_{i+1} = m_{13} w_i \end{cases} \qquad (4)$$

Equation 3 can be written as follows:

$$a_{i+1} d_i + b_{i+1} d_{i+1} + c_{i+1} d_{i+2} = \qquad (5)$$

$$= (a_{i+1} + b_{i+1} + c_{i+1}) P_{i+1} (i = 1, 2, \cdots, n-1)$$

There are $n + 4$ unknown values but only n formulas in the ***Equation 5***, hence another four formulas should be added.

Boundary conditions:

$$\begin{cases} p_1 = d_0 \\ p_n = d_{n+1} \end{cases} \qquad (6)$$

Tangent condition of the endpoints:

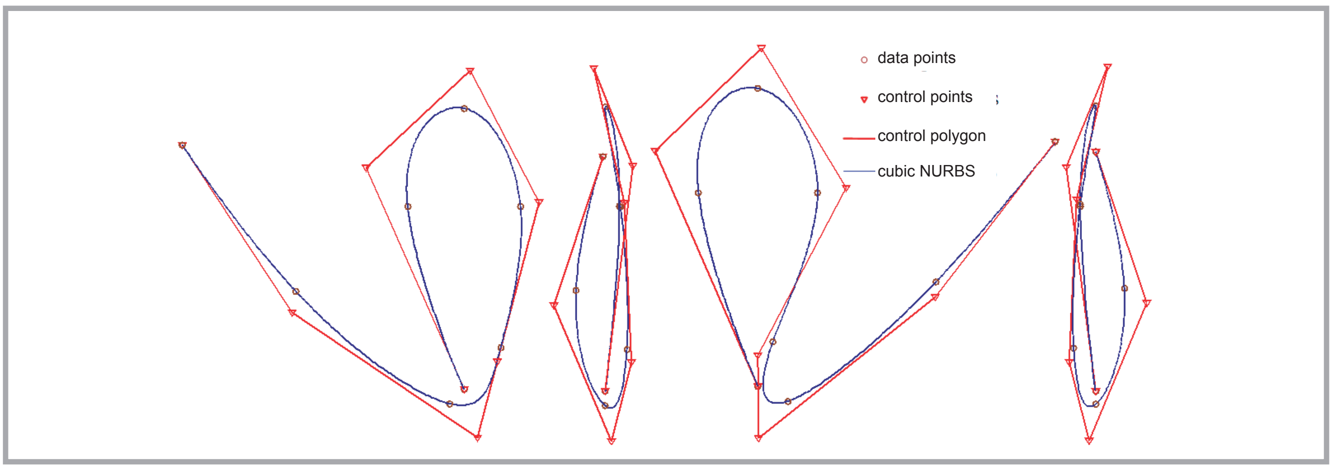

***Figure 3.** Single typical warp knitted stitch simulated by NURBS; a) closed stitch, b) open stitch.*

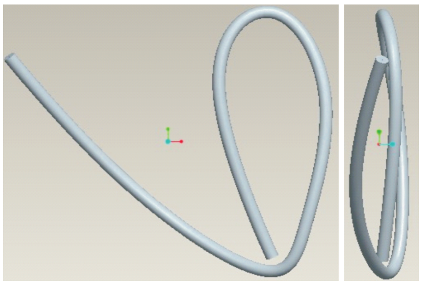

Figure 4. 3D solid representation of warp knitted loops by Pro-E.

$$\begin{cases} p_0^{\ *} = \dfrac{3w_1}{w_0(u_4-u_3)}(d_1-d_0) \\ p_{n+1}^{\ *} = \dfrac{3w_n}{w_{n+1}(u_{n+2}-u_{n+1})}(d_{n+1}-d_n) \end{cases} \quad (7)$$

Moreover, if $\begin{cases} d_1 = d_0 \\ d_{n+1} = d_n \end{cases}$, two unknown values can be calculated from ***Equation 7*** as follows:

$$\begin{cases} p_0^{\ *} = 0 \\ p_{n+1}^{\ *} = 0 \end{cases} \quad (8)$$

Set $a_0 = 0$, $b_0 = -\dfrac{3w_1}{w_0(u_4-u_3)}$,

$c_0 = \dfrac{3w_1}{w_0(u_4-u_3)}$, $Q_0 = p_0^{\ *} = 0$ and $Q_{n+1} = p_{n+1}^{\ *} = 0$, thus linear equations can be calculated from ***Equations 5, 6, 8*** as follows.

$$\begin{bmatrix} b_0 & c_0 & & & & \\ a_1 & b_1 & c_1 & & & \\ & a_2 & b_2 & c_2 & & \\ & & \cdots & \cdots & \cdots & \\ & & & a_n & b_n & c_n \\ & & & & a_{n+1} & b_{n+1} \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_n \\ d_{n+1} \end{bmatrix} = \begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \\ \vdots \\ Q_n \\ Q_{n+1} \end{bmatrix} \quad (9)$$

where $Q_i = (a_i + b_i + c_i)p_i^{\ *}$.

$$\begin{cases} [m_{11} \quad m_{12} \quad m_{13}][w_i \quad w_{i+1} \quad w_{i+2}]^T = h_{i+1} \\ h_1 = w_0,\ h_n = w_{n+1} \\ h_0 = \dfrac{3w_1}{w_0(u_4-u_3)}(h_2-h_1),\ h_{n+1} = \dfrac{3w_n}{w_{n+1}(u_{n+2}-u_{n+1})}(d_{n+1}-d_n) \end{cases} \quad (10)$$

Equations 10.

Thus, control points $d_i (i = 1, 2, \ldots, n)$ with two coincidence points at the start and end points can be calculated according to linear ***Equations 9***.

The weights of control points (w_i), similarly available as control points (d_i), should be calculated before control points (d_i). The weights of control points (w_i) can be calculated from data points (P_i), the weights of data points (h_i) and the node vector (u_i) according to the ***Equations 10***.

Simulation of a single typical warp knitted stitch based on data points and control points

Based on the control points, the curve of a single typical warp knitted stitch can be calculated by Matlab according to ***Equation 1***. As shown in ***Figure 3***, there are ten control points, as two points at the start and end of the curve are coincidental. Also the curve of a single typical warp knitted stitch goes through all the data points and the curve is smooth. Values of the NURBS curve simulated by Matlab, such as ***Figure 3,*** can be inputed to Pro-E software to get a 3D solid computer representation of the warp knitted stitch, shown as ***Figure 4***. At the same time, the cross section of the stitch can be elliptical or other, as well as circular.

Improving simulation of the start and end curves of the stitch

Taking the closed stitch as an example to simulate the warp knitted structure using a single typical warp knitted stitch, it is found that the link between the two neighboring stitches is not smooth (shown in ***Figure 5***), since every single stitch is stimulated regardless of the interaction between neighboring ones. Otherwise, because of the different topologies of the yarn path, shown in ***Figures 5.a*** and ***5.b***, the shapes of the joints are different.

In considering the interaction between neighboring stitches, two stitches including fifteen data points are simulated by the same method mentioned above to redefine another typical warp knitted stitch, shown in ***Figure 6***. It can be seen that the link between the two neighboring stitches is smooth, and also the start and end curve of the single stitch shows a slight change. Thus the curve between data points P_5 and P_{12} is selected to redefine a single typical stitch. To get the shape of a typical stitch as above, the curve between P_7 and P_{12} is moved to the same position of that between P_0 and P_5. Thus a new single typical stitch which can link smoothly with another one is redefined, shown in ***Figure 7.a***. The open stitch can be also simulated by the same method, shown in ***Figure 7.b***.

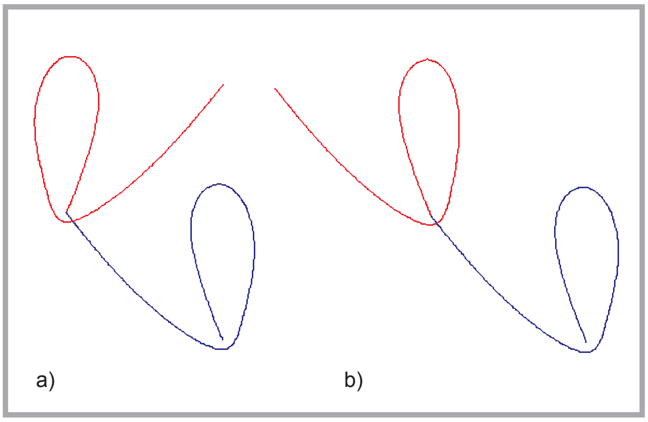

Figure 5. Link of a single typical warp knitted stitch; a) 1-0/1-2, b) 1-0/2-1.

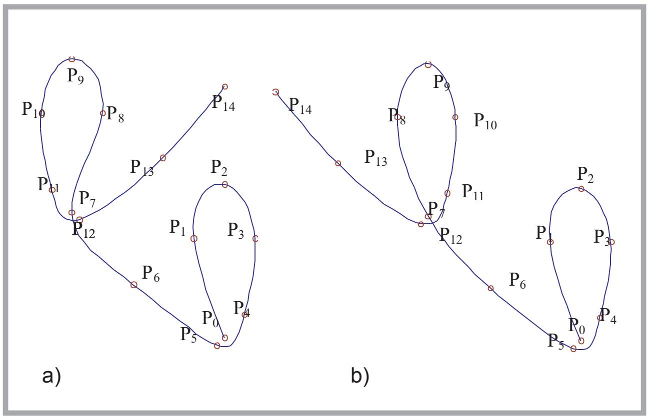

Figure 6. Two neighboring stitches simulated by NURBS; a) 1-0/1-2, b) 1-0/2-1.

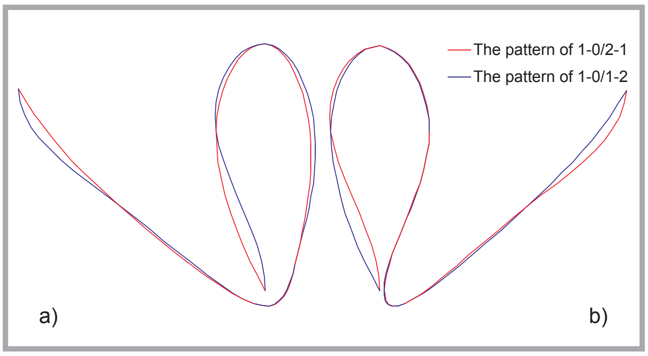

***Figure 7.** Redefined single typical stitch in different patterns; a) closed stitich, b) open stitch.*

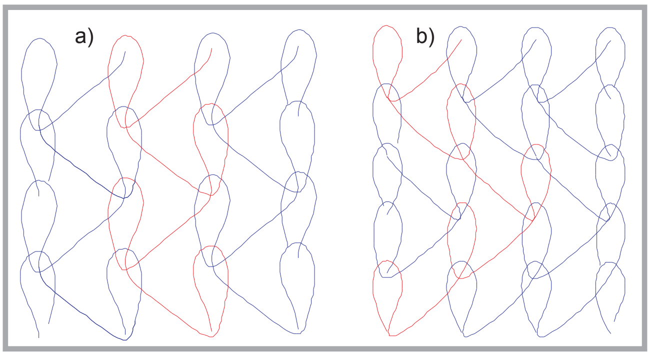

***Figure 8.** Simulation of the warp knitted structure; a) tricot stitch, b) atlas stitch.*

Furthermore, it can be seen that the different topologies of the warp knitted structures lead to a different shape of the curve of a single typical stitch, especially at the start of the loop because of the different orientation of the neighboring stitch's underlaps. If the stitch's two underlaps (the curve between P_5 and P_7 and that between P_{12} and P_{14} , shown in ***Figure 6***) are on the same side, as shown in ***Figure 6.a***, the stitch must be the curve in blue in ***Figure** 7*; otherwise, if the stitch's two underlaps are not on the same side, as shown in ***Figure 6.b***, the stitch must be the curve in red in ***Figure** 7*.

Simulation of the warp knitted structure using the redefined single typical stitch

Based on the redefined single typical stitch, different warp knitted structures can be simulated regardless of the loop offset. Taking the tricot stitch and atlas stitch as examples, as shown in ***Figure 8.a***, all the stitches' two underlaps are on the same side, hence the curve in colour blue, shown in ***Figure 7.a***, is selected to form a tricot stitch. As shown in ***Figure 8.b***, all the open stitches' two underlaps are on the same side, hence the curve in colour blue, shown in ***Figure 7.b***, is selected to form an atlas stitch; all the closed stitches' two underlaps are not on the same side, hence the curve in colour red, shown in ***Figure 7.a***, is selected to form an atlas stitch. And from the warp knitted structures simulated, it can be seen that all joints between stitches are smooth, thus the redefined single typical stitch can be applied to simulate any warp knitted structure.

Conclusions

Based on empirical geometrical loop models, a new method of simulating a warp knitted structure by NURBS was researched with using Matlab. Once the values of data points are known, control points with two coincidental points at the start and end points can be calculated uncomplicatedly by the principle of NURBS. Then based on the control points, the NURBS curve of a single typical stitch can be simulated, which can be achieved easily with changes of the numbers or the location of data points; hence the method of simulation is flexible. Furthermore a 3D solid computer representation of the stitch can be achieved by inputting values of the NURBS curve simulated by Matlab to Pro-E software

In further consideration of the link between neighboring stitches, it is found that the joint of two typical stitches is not smooth. Then two stitches including fifteen data points are simulated by the same method mentioned above, and a section of the curve to redefine another typical warp knitted stitch is selected. Otherwise, for different topologies of the yarn path, the open or closed redefined typical warp knitted stitches have two types judged by whether their underlaps on the same side or not.

Based on the redefined typical warp knitted stitch, two warp knitted structures are simulated regardless of the loop offset, and all the links of stitches are smooth.

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