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Energy Stored in the Electric Field Produced by a Charged Fabric with a Conductive Mesh

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Abstract
The aim of the research was to set up a simplified analytical fabric-grounded object model for estimation of the energy stored in the electric field occurring in the space surrounding a charged fabric. A standard spherical electrode was used as the grounded object. Synthetic fabric with regularly spread conductive yarns (in the form of a grid with regular cells) was used as a sample. The model allowed to combine the energy stored in the space of the cell with the geometry of the fabric-object system and the surface charge density. The model led to a power type relation between the energy W and the cell, with a diameter of a, in the form $W = K \cdot a^n$, $n \approx 3$. These results were verified by those obtained from numerical modeling using the COMSOL Multiphysics program. It was found that for a cell with a diameter of 10 to 50 mm, the difference in results was lower than 38%. Based on the results obtained, it can be stated that the model proposed can be used for the design of fabrics (used in ESD protection) with the maximum acceptable stored energy.

Key words: *ESD-protection, antistatic fabric, conductive yarns.*

Introduction

Fabrics which contain synthetic fibres have the ability to store electric charge for a long time. The life-time of the electric charge of modern synthetic fibre fabrics is usually in the range of several seconds to several hours, but it may also reach a couple of years. The electric charge, located on the fabric or in its structure, produces an electric field in the direct surrounding space, which allows fabrics to accumulate energy in the electric field. The energy stored in the electrostatic field can be released in an uncontrolled way, e.g. due to the electric discharge (ESD – from ElectroStatic Discharge), which can cause dangerous accidents [1]. The main hazards listed in the literature are the ignition risks from ESD and damage to electronic devices. To assess the possibility of a risk, it is necessary to compare the energy released during the ESD with the threshold energy, the exceeding of which leads to the occurrence of hazard. In the case of an ignition hazard, the Minimum Ignition Energy (MIE) is taken as the threshold energy. For the most common inflammable media, the MIE value is usually in the range of 1-20 μJ (gaseous fuels, hydrogen-acetylene) up to 200-500 μJ (liquid fuels fumes, powders) [2]. The level of energy released and tolerated by electronic devices is significantly lower [3-6]. For such devices, energy as low as 1 μJ can lead to permanent damage to a device (for the newest technologies and the most sensitive MOS devices, the acceptable energy released during ESD can be around 0.005 μJ!). The maximum energy released from a dielectric fabric can be reduced by introducing conductive paths into its structure, e.g. in the form of a regular mesh made of conductive fibres (metal, carbon, etc.), as shown in ***Figure 1***.

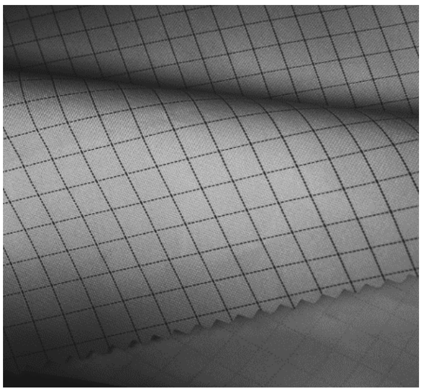

Figure 1. ESD fabric with square conductive grid [7].

However, synthetic fabrics with a conductive fibre inserted can also store an electric charge, and consequently electric energy in the electrostatic field. The maximum value of energy accumulated in such fabrics depends on the density of fibre insertion. However, there are no clear requirements for choosing these densities in the literature. The estimation of energy that can be released from fabric during ESD discharge has great practical importance. It allows the assessment of possible hazards (both fire and explosive, as well as damage to electronics) which may arise when an object covered with a fabric (work clothes, containers) is in an environment where such hazards can occur.

Energy stored in electric field

The amount of energy W, stored in an electrostatic field of intensity E can generally be written as:

$$W = \frac{1}{2}\varepsilon_0 \int_V \varepsilon_r \left|\vec{E}\right|^2 \mathrm{d}V, \tag{1}$$

where: $\varepsilon_0 = 8.85\ 10^{-12}$ F/m – is the permittivity of free space, and ε_r is the relative permittivity of the medium filling the element of volume dV.

In the case of fabrics, the source of the electric field is the electric charge accumulated on them and in their structure. Thus the value and distribution of the electric field E can be determined from the electric charge using the equation:

$$div\,\vec{E} = \frac{q_v}{\varepsilon_0 \varepsilon_r}, \tag{2}$$

where: q_v is the spatial charge density.

From these equations two conclusions result: 1) the greater the volume of space with the charge (field), the higher the total energy accumulated in it ***Equation (1)***; 2) the lack of electric charge leads to a lack of an electric field, and, as a consequence, no hazard is possible. The practical usability of ***Equations (1)*** and ***(2)*** to determine the energy stored in the space V is limited, because it is necessary to know the distribution of permittivity e_r and the electric field E or the spatial distribution of the charge density q_v

in this space. Due to the above-mentioned difficulties, approximate methods based on simplified models or computer modeling using specialised programs are used to estimate the energy.

Model of the sphere-charged fabric system

A scheme of the model system is shown in ***Figure 2***. It includes a system that comprises a fabric with an inserted grid of conductive fibres forming regular "cells" and a grounded object placed near the fabric. The following assumptions were made:

- the fabric was modelled with a dielectric layer of thickness d and relative permittivity ε_r;
- a single cell of conductive mesh inserted into the fabric structure in both directions with a conductive fibre distance (CFD) equal to a was modeled with a ring of radius $R = 0.5a$ (this assumption was made because in the first approximation of the problem it was assumed that the corners of the square mesh, i.e. region where conductive fibres cross each other, could be excluded from the analysis as "low energy" regions – small or very small local potential), made of a conductive wire of circular cross-section with a diameter equal to the thickness of fabric d. The ring was at the ground potential;
- the grounded object closest to the fabric surface was modeled with a spherical electrode of radius R_0, in accordance with the standard [8]. The grounded electrode was placed on the axis of the ring, which represents a single cell;
- an electric charge (in the form of a surface charge) applied to the fabric surface from the spherical electrode side, with a density $q_s = const$, was the source of the electric field. The density q_s was constant over the entire surface of the sample.

Considering the grounded object near the fabric in the system model was important from the point of view of the maximum energy that can be accumulated in the whole system, as the electrically charged fabric was considered as an object with a constant charge [9, 10].

Energy stored in the electric field is considered as energy accumulated only in the cell. It has been assumed that ESD discharges, which are brush discharges,

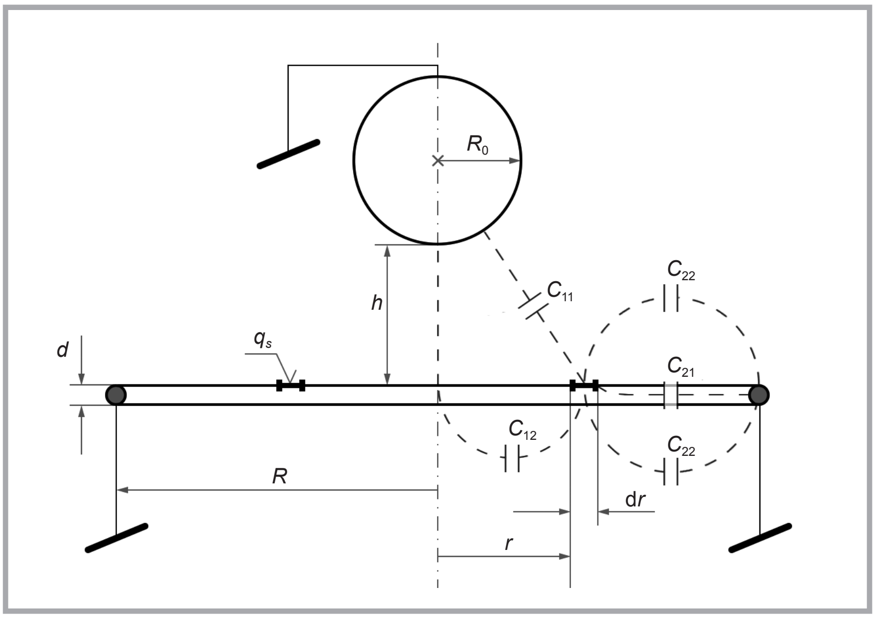

Figure 2. *Model of sphere-charged fabric system; a detailed description of the model is given in the text.*

could release charge (and energy) accumulated only in the area of one cell.

The energy stored in the system (fabric-spherical electrode-grounded ring of the cell) can be determined from the equation:

$$W = \frac{1}{2}\mathrm{CU}^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\mathrm{QU}, \quad (3)$$

Where, C is the equivalent capacitance of the fabric-grounded space system, Q the charge stored on the fabric in the cell area, and U is the equivalent voltage in the cell area. A practical definition of the "equivalent voltage" phrase is given in section 5.2.

Due to the non-uniform distribution of voltage U on the surface of the fabric (in the area of the cell), the energy stored in the system can be determined from the integral:

$$W = \int_S \mathrm{d}W, \quad (4)$$

Where, S is the surface of the cell within the conductive mesh, and $\mathrm{d}W$ is the energy stored in the electric field produced by the charge $\mathrm{d}Q$, given by the equation:

$$dW = UdQ, \quad (5)$$

where $\mathrm{d}Q$ is the charge collected on the elementary surface $\mathrm{d}S$ with a constant U-potential, which is determined by:

$$dQ = q_s \mathrm{d}S. \quad (6)$$

Assuming the cylindrical geometry of the model as well as the distribution of the potential U, the elementary charge $\mathrm{d}Q$ is uniformly distributed ($q_s = const$) on the surface in the form of a ring with radius r and width $\mathrm{d}r$. Thus for $\mathrm{d}Q$, it can be written:

$$\mathrm{d}Q = 2\pi r q_s \mathrm{d}r. \quad (7)$$

The local potential value on an element of $\mathrm{d}S$ area can be determined from the equation:

$$U(r) = \frac{dQ}{C_T(r)}, \quad (8)$$

Where, the capacitance $C_T(r)$ is the sum of two capacitances; $C_1(r)$ is the capacitance between the spherical electrode and $\mathrm{d}S$ element ($\mathrm{d}S = 2\pi r\ \mathrm{d}r$), and $C_2(r)$ the capacitance between the ring (conductive grid) and $\mathrm{d}S$ element.:

$$C_T(r) = C_1(r) + C_2(r). \quad (9)$$

The potential distribution on the fabric is not constant (because of the presence of a spherical electrode), and the fabric is open to the electric field; hence all capacitances specified in the model were proposed on the basis of analysis of the electric field distribution in the vicinity of the cell analyzed.

Capacitances $C_1(r)$ and their components $C_{11}(r)$, $C_{12}(r)$ can be estimated from the following equations:

$$C_1(r) = C_{11}(r) + C_{12}(r), \quad (10)$$

Where, C_{11} is the capacitance between the grounded spherical electrode and $\mathrm{d}S$ element, associated with electric field lines in the air above the fabric:

$$C_{11}(r) = \frac{2\pi\varepsilon_0 r}{\sqrt{r^2 + (R_0+h)^2} - R_0}\mathrm{d}r, \quad (11)$$

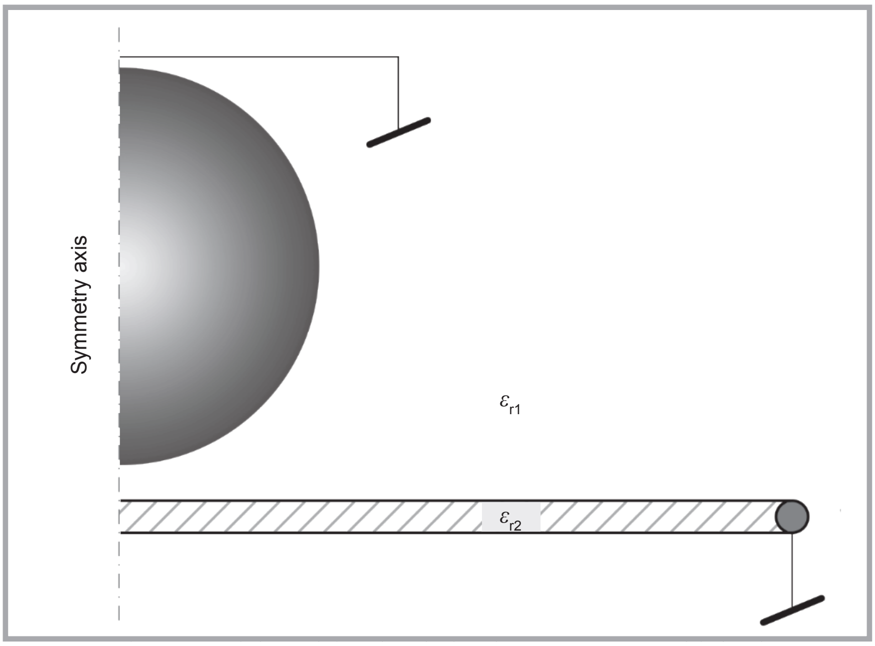

Figure 3. *Model used for simulation in COMSOL Multiphysics.*

C_{12} is the capacitance between the grounded spherical electrode and dS element, associated with electric field lines spread under and above the fabric in the central part of the cell:

$$C_{12}(r) = \frac{2\pi\varepsilon_0 r}{h+\pi r}\mathrm{d}r. \qquad (12)$$

Capacitances $C_2(r)$ and their components $C_{21}(r)$ and $C_{22}(r)$ can be estimated from the following equations:

$$C_2(r) = C_{21}(r) + 2C_{22}(r), \qquad (13)$$

Where, C_{21} is the capacitance between the grounded conductive mesh and grounded dS element, associated with electric field lines in the fabric:

$$C_{21}(r) = \frac{2\pi\varepsilon_0\varepsilon_r r}{R-r}\mathrm{d}r, \qquad (14)$$

C_{22} capacitances are coplanar and appear between the grounded conductive mesh and dS element:

$$C_{22}(r) = \frac{4\varepsilon_0 r}{R-r}\mathrm{d}r. \qquad (15)$$

Finally, the total capacitance $C_T(r)$ can be written as:

$$C_T(r) = 2\varepsilon_0 r\left[A(r) + \frac{\pi}{h+\pi r} + \frac{\pi\varepsilon_r+4}{R-r}\right]\mathrm{d}r. \qquad (16)$$

Substituting ***Equations (16)*** and ***(7)*** into ***Equation (8)*** and then into ***Equation (5)***, and next integrating over the whole surface of the cell (circle), it is possible to estimate the energy W:

$$W = \int_S \mathrm{d}W = \frac{2\pi^2 q_s^2}{\varepsilon_0} \int_0^R \frac{r}{A(r) + \frac{\pi}{h+\pi r} + \frac{\pi\varepsilon_r + 4}{R-r}}\mathrm{d}r, \qquad (17)$$

where:

$$A(r) = \frac{\pi}{\sqrt{r^2+(R_0+h)^2}-R_0}. \qquad (18)$$

Equation (17) allows to determine the dependence of $W = \mathrm{f}(R)$ on the basis of numerical integration for given values of R, R_0, h, ε_r & q_s.

Simulations in COMSOL Multiphysics

In order to verify the analytical model, the energy stored in the system, shown in ***Figure 2***, was estimated by numerical simulations carried out with COMSOL Multiphysics software using an electrostatic module. It was assumed that the electric charge was uniformly distributed on the flat, dielectric disc surrounded by a conductive grounded ring. Above the disc there was a grounded spherical electrode – ***Figure 3***.

The simulations were carried out with the following assumptions:

- the whole system was placed in air with permittivity $\varepsilon_{r1} = 1$;
- the fabric was an ideal dielectric with permittivity $\varepsilon_{r2} = 1.5$;
- the electrode (sphere) and ring were the only grounded elements in the system;
- the radius of the spherical electrode was $R_0 = 7.5$ mm;
- fabric thickness was $d = 0.6$ mm;
- the distance between the sphere and fabric surfaces was $h = d = 0.6$ mm;
- the cell diameter (conductive fibre distance) was assumed to be in the range $a = 10$-50 mm;
- the cross-section of the ring was circular and had a diameter equal to the thickness of the sample;
- the “working area” where the electric field was calculated was a cylinder with a radius equal to $20a$ and height equal to $10a$, where a is the conductive fibre distance (cell diameter), i.e. $a = 2R$;
- there was no normal field component (tangential only) on the external boundaries of the “working area”;
- an electric charge was placed on the fabric surface (from the spherical electrode side), whose distribution is given by the profile shown in ***Figure 4.***

The radius of the area covered by the charge was assumed to be equal to $r_q = R - d/2$. In the area of radius $r_{qc} = R - d$, the charge density was constant.

In the remaining area, in order to minimise numerical errors, a sinusoidal transition from the maximum value to zero was assumed. The shape of the distribution in the area close to the fibres does not have a significant impact on the total energy value. In reality, the distribution of charge near conductive fibres is difficult to describe due to the occurrence of partial discharges which can appear in this region. The charge density distribution assumed in the analysis is described as follows:

$$q_s = \begin{cases} const & \text{for } r < (R-d) \\ cos\dfrac{r}{r_0} & \text{for } (R-d) < r < (R-d/2) \\ 0 & \text{for } r > (R-d/2) \end{cases}$$

Where, r_0 is the unit distance.

The total energy stored in the system was calculated numerically. The normalised energy density distribution W_e, determined by

$$W_e = \frac{dW}{dV}, \qquad (19)$$

in the sample (from its center to the edge) and in its surroundings is shown in ***Figures 5*** and ***6*** for two ring diameters.

These results indicate that the energy of the system was mainly accumulated in the electric field in the air. For small grids, i.e. complying with condition $R \le R_0$, the energy density in the space under the spherical electrode and that near the conductive ring are comparable. In the case of grids with larger diame-

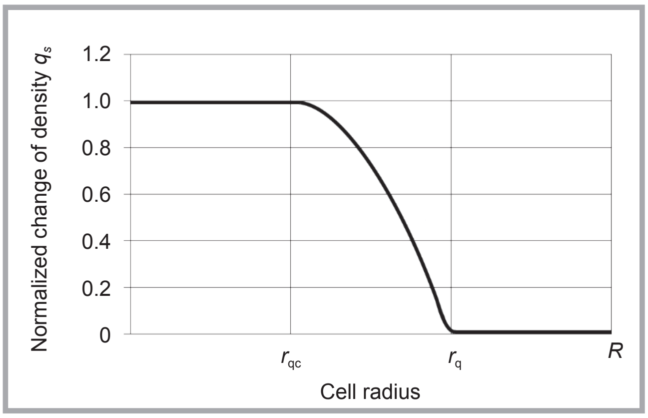

Figure 4. Charge distribution profile near the conductive fibre.

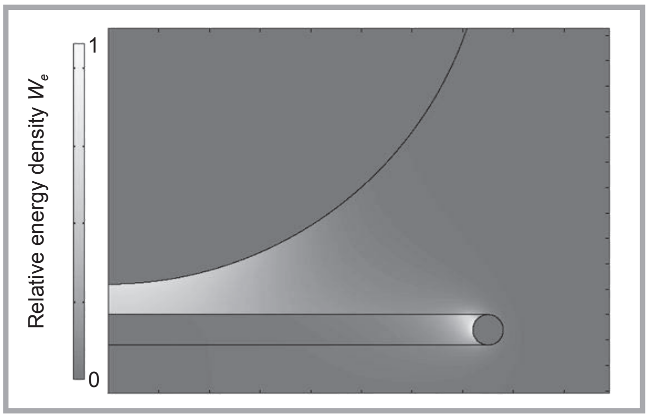

Figure 5. Distribution of energy density W_e (relative to the maximum value) for a system of fabric containing a ring with a diameter of 15 mm.

ters ($R > R_0$), the energy density is much higher in the surrounding ring than under the spherical electrode.

Measurements

Experimental tests were carried out on samples of PP (polypropylene) fabric with a thickness of ca. 0.6 mm, containing metallic conductive fibres in a plane of the square. The conductive fibres' pitch/distance (CFD) was changed in the range of 10-50 mm in the sequence of 10, 15, 20, 30 and 50 mm (length of the side of the square). Electrification of the fabric samples was carried out by the corona method. The experiments included:

- measurements of potential distribution on the fabric surface,
- determination of the maximum value of the surface charge density on the sample in the area of a single cell,
- measurements of the charge decay rate,
- examination of the possibility of discharges from the fabric surface within the cell area.

Sample electrification

The samples were charged with a corona discharge, shown schematically in ***Figure 7***. Fabric electrification was carried out in the following conditions: corona/electrification voltage $U_c = -10 \pm 1$ kV, electrification time $t_c = 30 \pm 1$ s, air temperature $T = 24 \pm 2$ °C, air humidity $RH = 45 \pm 5\%$.

Potential distribution measurement

Equivalent voltage distribution measurements were made in the system shown in ***Figure 8***. Equivalent voltage can be defined as probe voltage compensating the electric field in the probe-charged sample area, in conditions where the sample is placed on a grounded surface. A TREK Model 347 electrostatic voltmeter operating in a compensation setup was used. To obtain the distribution of equivalent voltage U_z (x, y), the samples' surfaces were scanned with a voltmeter probe. Areas larger than the cell area were analysed.

The electric charge density was determined with the maximum equivalent voltage measured in the cell area, using the equation:

$$q_s = \frac{\varepsilon_0 \varepsilon_r U_{z\,max}}{d}, \qquad (20)$$

Where, $U_{z\,max}$ is the maximum value of the equivalent voltage measured over

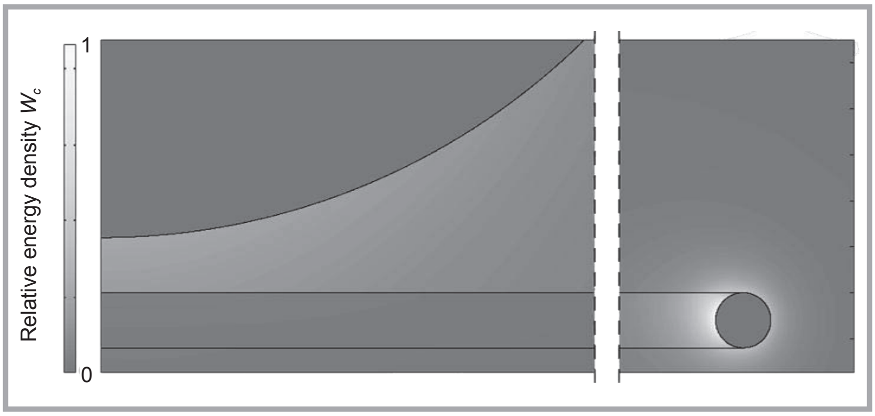

Figure 6. Distribution of energy density W_e (relative to the maximum value) for a system of fabric containing a ring with a diameter of 50 mm.

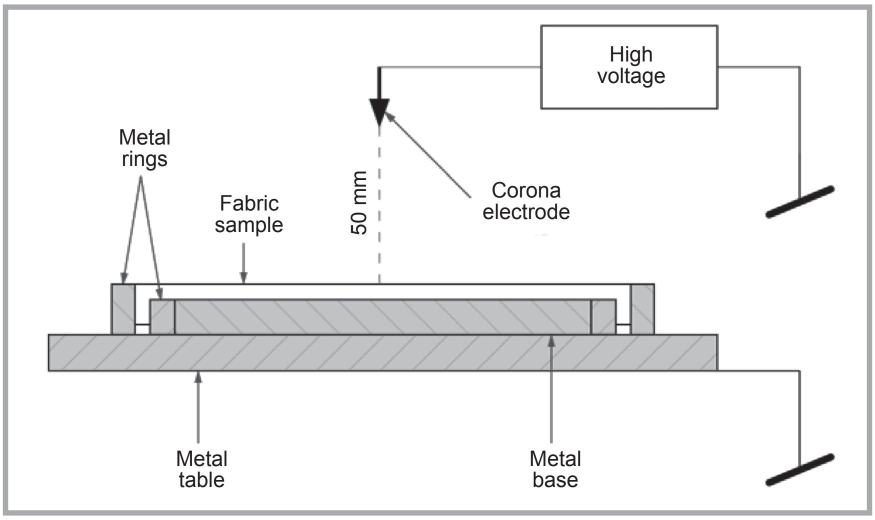

Figure 7. Schematic of the electrification system.

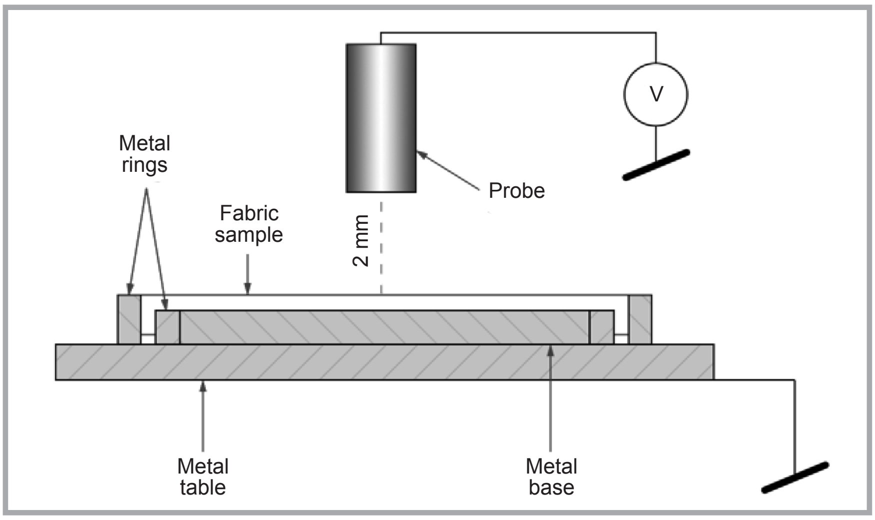

Figure 8. *Surface voltage measurement method.*

the whole cell surface, and d is the fabric thickness.

Results and discussion

Results of the measurements of equivalent voltage obtained for a sample of CFD = 20 mm are shown in ***Figure 9***. For samples with a different CFD, similar equivalent voltage distributions (in the form of a "pillow") were observed.

Values of the surface charge density q_s, determined from ***Equation (20)***, for fabric samples with various CFD are presented in ***Table 1***.

Due to small differences in the values of the charge densities determined, it was assumed the constant value q_s of the surface charge density is equal to 40 μC/m^2. The approaching of the spherical electrode to the fabric surface leads to the occurrence of ESD and energy release. The discharge causes the flow of charge, leading to a change in its surface distribution and, consequently, to a change in the distribution of equivalent voltage. Typical changes in the equivalent voltage distribution observed after the discharge are shown in ***Figure 10***. The visible "crater" of the potential illustrates the area in which a significant reduction in the surface charge density occurred. In the case of electrostatic discharges for fibres with CFD $\geq 20 \times 20$ mm, similar distributions were observed.

The charge accumulated on the fabric tested (in the cell area) has a relatively long life-time. A typical equivalent voltage decay characteristic is shown in ***Figure 11***. The estimated value of the half-decay time (i.e. the time after which the equivalent voltage decreases to 50% of the initial value) was equal to $t_{1/2} \approx 4.3 \times 10^4$ s. Because no regular dependence of the half-decay time for samples with different CFD was found, the results seem to confirm the small influence of the CFD on the charge decay characteristic.

The results of calculations of stored energy based on the model described in Section 3 and simulations from Section 4 are collected in ***Table 2***. Calculations were carried out with the following assumptions: surface charge density $q_s = 40$ μC/m^2, sphere-fabric distance $h = d$, relative permittivity of the fabric investigated $\varepsilon_r = 1.5$.

The energy values W_c and W_s estimated differ each other by no more than 38% for CFD in the range of 10-50 mm, which confirms the validity of the relation (17) proposed for determination of the stored energy for fabric with a particular CFD (distance a).

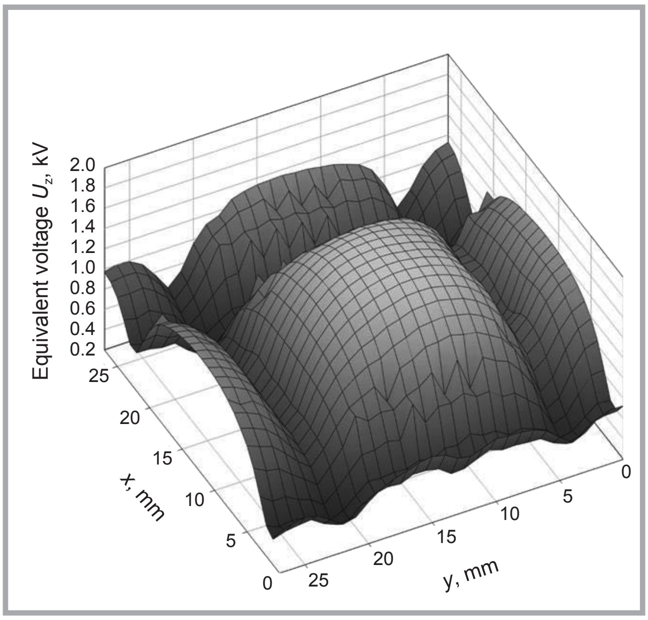

Figure 9. *Distribution of the equivalent voltage for a fabric with CFD 20×20 mm and subject to corona charging. Charging in air (T = 24 ± 2 °C, RH = 45 ± 5%), corona voltage U_c = -10 ± 1 kV, electrification time t_c = 30 ± 1 s.*

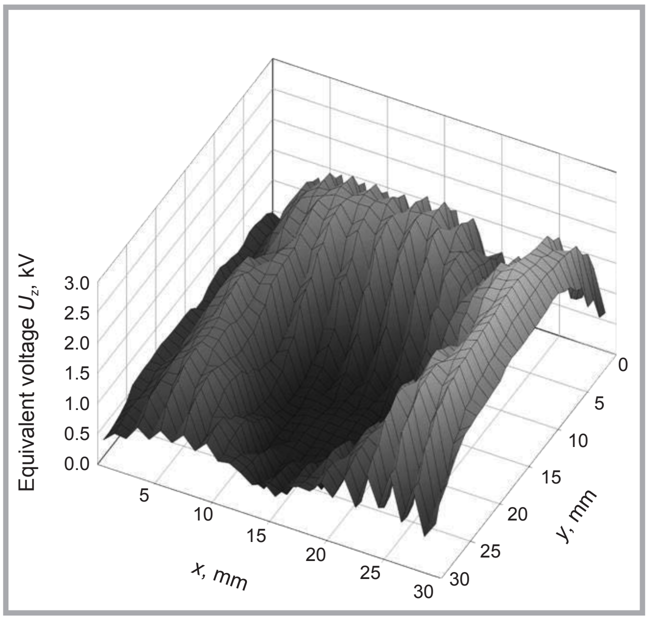

Figure 10. *Voltage "crater" observed for a fabric sample after brush discharge in its central part. Fabric CFD: 30×30 mm, submitted corona charging (conditions like in the Figure 9), and the following ESD due to sphere electrode approach.*

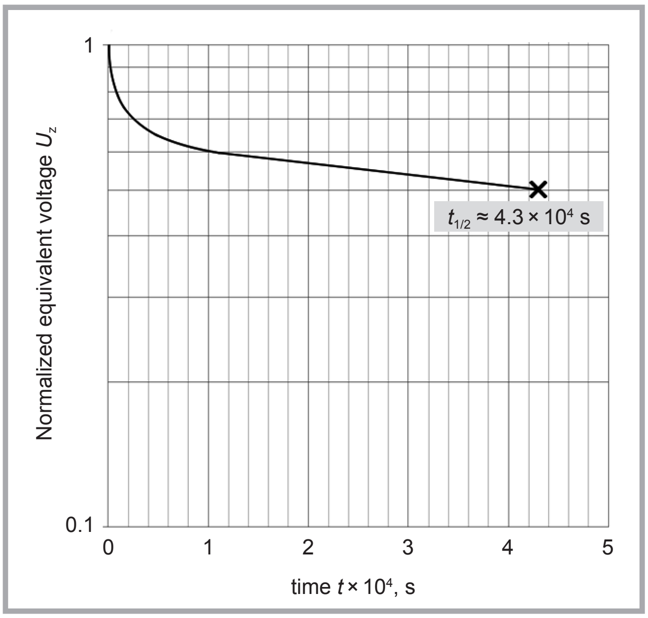

***Figure 11.** Equivalent voltage – charge decay characteristic obtained for the fabric with 20×20 mm CFD. Corona charging and measurement in air ($T = 24 \pm 2$ °C, $RH = 45 \pm 5\%$), corona voltage $U_c = -10 \pm 1$ kV, electrification time $t_c = 30 \pm 1$ s.*

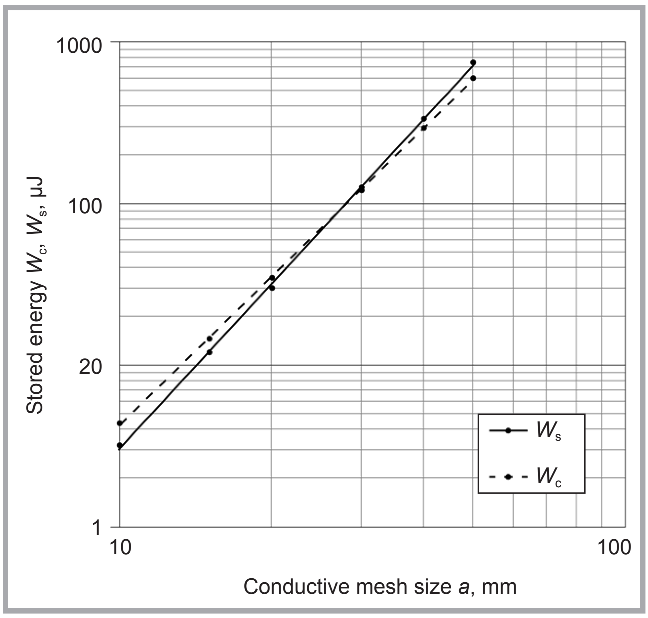

***Figure 12.** W(a) characteristics obtained from Equation (17) and from the COMSOL Multiphysics simulation for the system shown in Figure 2. Calculations made with the following conditions: $q_s = 40$ µC/m², $h = d = 0.6$ mm, $\varepsilon_r = 1.5$, $R_0 = 7.5$ mm.*

It should be noted that the stored energy is much smaller (in comparison to the energy calculated for the system shown in ***Figure 2***) when the same fabric with the same charge density q_s is placed on the grounded surface, especially for higher a values. W = f(a) characteristics for the case analysed above are shown in ***Figure 12***.

The W = f(a) characteristics can be approximated by the power-type function (R^2 better than 0.9999):

$$W(a) = Ka^n, \qquad (21)$$

Where, $W(a)$ – the total energy [µJ] stored in the system shown in ***Figure 2***, K – a constant in the range of 1.0-4.0 × 10^{-3}, a – CFD [mm], and n – the power factor, whose value ranges 3.0-3.5.

At this stage of the work, the dependencies of constants K and n on parameters h, and ε_r were not determined. It should be noted that such dependencies must exist, as is visible from ***Equations (17)*** and ***(18)***.

Visible differences in W(a) between the analytical model (***Equation (17)***) and numerical calculation are related to the fact that for small CFD (small a) most of the energy was concentrated in the sphere-fabric area. For large a values, energy was stored mainly in the surroundings of conductive fibres.

Conslusions

Analysis of the results presented in the paper allows to formulate the following conclusions:

- Synthetic fabrics or fabrics with synthetic fibres have the ability to store electric charge for a long time ($t_{1/2} = 10^3 - 10^5$ s) even if conductive fibres are incorporated into their structure;
- The analytical model proposed allows to determine the energy stored in the electric field in a single grid cell surrounding for the charged fabric (with conductive mesh) from the geometry of the cell and from the charge stored on the fabric (charge with density q_s);
- The spatial distribution of the density of the energy stored in the grounded object-fabric system changes with the distance between it and the CFD value;
- The value of energy that can be stored in the electric field in a single grid cell surrounding estimated for the charge density q_s = 40 µC/m² (introduced according to the standard [8]) can significantly exceed 10 µJ for a cell with a = 10-20 mm. This means that if only a part of that energy is released during ESD, it may be sufficient to damage some electronic devices [11];
- Results of energy estimations based on the analytical model show high compliance with the those obtained from numerical modeling. These differences did not exceed 38%, which confirms the validity of the model proposed and assumptions made;
- The dependence linking the energy stored in the field produced by

***Table 1.** Surface charge density for corona charged fabric samples. **Note:** * – conductive fibre – distance in the fabric structure.*

CFD a*, mm	Charge density q_s, µC/m²
10	28
20	39
30	43
50	47

***Table 2.** Energy stored in the sphere-charged fabric system. **Note:** * – the relative error dW was determined from relation $(W_c - W_s)/W_s$.*

CFD a, mm	Energy (calculations) W_c, µJ	Energy (simulations) W_s, µJ	Relative error dW*, –
10	4.4	3.2	0.38
15	15	12	0.22
20	35	30	0.16
30	121	122	0.01
40	296	336	0.12
50	593	743	0.20

the charge q_s in the fabric volume and the cell's nearest surroundings can be approximately given by the power type function $K \cdot a^n$, where $K = 1.0 - 4.0 \times 10^{-3}$, n = 3.0-3.5, and a (mesh size) is CFD [mm] in the range of 10-50 mm;
- The simplified expression (21) allows to determine (in the first approximation) the value of the total energy stored in the field (in and around the fabric) for practical use.

Acknowledgements

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