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Thickness Optimisation of Sealed Seams in Respect of Insulating Properties

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Abstract
The main goal is to determine the optimal thickness of material layers in seams in respect of their insulating properties. The impact of structure on heat insulation and moisture resistance is also described. The insulation was analysed for different types of sealed seams which are typically used to connect the different parts of clothing. The coupled heat and mass transport problem is defined by the transport equations and a set of boundary and initial conditions. Based on the variational approach, we introduce the typical objective functionals and the optimality conditions. The problem was solved using the finite element method at the analysis stage and sensitivity approach at the synthesis stage.

Key words: *thickness optimization, sealed seams, insulating properties.*

Nomenclature

- $\mathbf{A}$ — *matrix of heat conduction coefficients, $Wm^{-1}K^{-1}$*
- $\mathbf{b}$ — *vector of design parameters, m*
- c — *thermal heat capacity, $J\,kg^{-1}K^{-1}$*
- c_v — *volumetric heat capacity of textile material, $J\,m^{-3}K^{-1}$*
- $\mathbf{D}$ — *matrix of water vapour diffusion coefficients within fibers, m^2s^{-1}*
- $\mathrm{div}_\Gamma \mathbf{v}^p$ — *tangent divergence of the vector $\mathbf{v}^p$ on the external boundary Γ, –*
- F — *objective functional, –*
- F' — *Lagrange functional (the auxiliary function), –*
- $g_p = \frac{\mathrm{D}g}{\mathrm{D}b_p}$ — *global (material) derivative of g in respect of design parameter b_p, –*
- $g^p = \frac{\partial g}{\partial b_p}$ — *partial (local) derivative of g in respect of design parameter b_p, –*
- H — *mean curvature of the external boundary Γ, m^{-1}*
- h — *surface film conductance, $Wm^{-2}K^{-1}$*
- h_w — *mass transport coefficient of water vapour in air, $m\,s^{-1}$*
- N — *number of functionals during sensitivity analysis, –*
- $\mathbf{n}$ — *unit vector normal to external boundary Γ, directed outwards to domain Ω bounded by this boundary, –*
- P — *number of design parameters during the sensitivity analysis, –*
- p — *proportion between sorption rates, –*
- $\mathbf{q}$ — *vector of heat flux density, Wm^{-1}*
- $\mathbf{q_w}$ — *vector of mass flux density, $kg\,m^{-2}s^{-1}$*
- $q_n = \mathbf{n}\cdot\mathbf{q}$ — *heat flux density normal to external boundary, Wm^{-1}*
- $q_n = \mathbf{n}\cdot\mathbf{q_w}$ — *mass flux density normal to external boundary, $kg\,m^{-2}s^{-1}$*
- T — *temperature, K/°C*
- t — *real time in primary and additional structures, s*
- t_{eq} — *time to reach quasi-equilibrium during the sorption process, s*
- u — *unit cost of structure, –*
- $\mathbf{v}^p(\mathbf{x},\mathbf{b},t)$ — *transformation velocity field associated with design parameter b_p, –*
- $v_n^p = \mathbf{n}\cdot\mathbf{v}^p$ — *transformation velocity normal to external boundary Γ, –*
- w_a — *water vapour concentration in air filling interfiber void space, $kg\,m^{-3}$*
- w_f — *water vapour concentration within fibers, $kg\,m^{-3}$*
- $W_C = \frac{w_f}{\rho}$ — *fractional water content on fiber surface, –*
- Γ — *external boundary of structure, –*
- β — *approximation coefficient of sorption / desorption on boundary of fibers, –*
- γ — *boundary integrand of the objective functional, –*
- ε — *effective porosity of the textile material, –*
- η — *absorption coefficient, –*
- λ_w — *cross coefficient described as the heat sorption of water vapour by fibers, $J\,kg^{-1}$*
- ξ — *slack variable of Lagrange functional for inequality problems, –*
- Σ — *discontinuity line between adjacent parts of piecewise smooth boundary Γ, –*
- ρ — *density of fibers, $kg\,m^{-3}$*
- τ — *transformed time in adjoint structure, s*
- χ — *Lagrange multiplier, –*
- Ω — *domain of structure, m^2*
- ∇ — *gradient operator, –*

Introduction

Seams are applied to connect the various parts of textiles. The total external surface of seams is insignificant in relation to the area of clothing, hence the surface fraction is small. However, seams are essential in respect of the thermal protection and moisture resistance of a protective suit. A combination of different materials and threads inside needle channels reduces significantly the insulating properties in these places. Thus the seam structure is additionally sealed by means of a tape fastened by polymer film. The appropriate components are multilayer textile laminates equipped with semi-permeable membranes. The total structure is also a complex multilayer laminate consisting of several material layers, several layers of polymer, membrane and sealing tape.

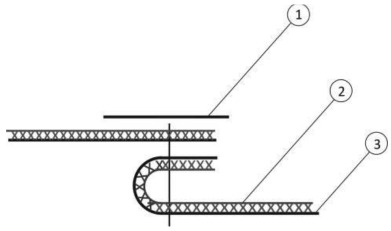
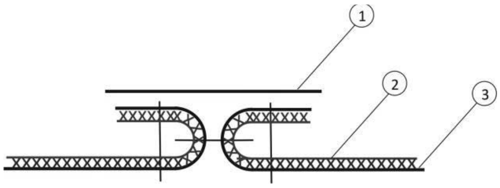
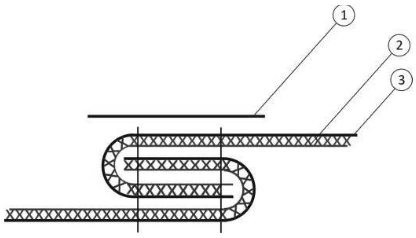
Breathable waterproof fabric transports the water vapour, but prevents the passage of water through itself [1]. The problem can also be found in the seam pack. As is presented in [1, 2], hydrophilic membranes are highly resistant to water penetration and bi-component membranes have enhanced strength and elongation properties. During the manufacturing process of clothing, we can introduce different joints cf. seams, glue joints and welding joints [3]. In order to ensure the good quality of waterproof clothing, it is important to apply an appropriate stitch structure irrespective of the selection of adequate textile materials. Some studies have shown that the mechanical properties of the seam affects the mutual arrangement of materials, the number of layers in fabric, the type of thread, the stitch density, and the type of seam [4-6]. Many studies [2-5] have shown that the seam performance (seam strength, elongation, seam efficiency) depends on the interrelationship of fabrics, threads, stitch and seam type selection and sewing conditions, which include needle size, stitch density etc.

The general process used to manufacture a garment from these fabric pieces is sewing, but it is possible for this fabric to leak water at stitch holes, which causes a fatal problem to the functionality of breathable waterproof fabric. Therefore it is necessary to seal the seam line using waterproofing tape. Study [7] presents the influence of the sealing process on the seam characteristics of breathable waterproof fabrics made by various finishing methods, in which the authors state that the sealing of seams improves their strength properties compared to sewn ones. It is evident that the quality of bonded seams depends on bonding parameters. In [8] the authors investigated the performance of bonded seams of some thermoplastic materials with an additional layer of silicone paper. Although the results of the investigation showed the delamination of the seams created, it was determined that the bonding strength of textile depends on fabrics structure characteristics as well.

The Authors of works [7, 9, 10] analysed the strength of the seam after the sealing process for two directions (along and across the seam). The result of this test confirms that the seam strength for the material tested is lower across than in the longitudinal direction. Comparing the test results presented in [10, 11], we determine that quality of sealing seams depends on process parameters such as the temperature, pressure intensity and sealing speed as well as the quality of joined layers defined by the parameters of the bonding process (i.e. temperature, pressure intensity and pressing duration). The problem can be consistently solved by means of a global approach i.e. the sensitivity of heat and mass transport within clothing [12]. The alternative can be a local approach and analysis of some transport problems within the structural component [13-15]. Some problems concerning laminates in respect of technological procedures are analysed in [16].

The main goal of the paper is to determine the optimal thickness of material layers in respect of insulating properties of the seam. Additionally it is possible to determine the impact of the structure on the heat insulation and moisture resistance of sealed seams. The insulation was analysed for different types of seams which are typically used to connect the different parts of clothing. To improve thermal and water vapour protection, every seam is additionally secured by means of sealing tape.

__Table 1.__ Analyzed seams according to ISO 4915:1991 [17]; 1 – sealing tape; 2 – membrane; 3 – PES fabric.

No	Seam type	Stitch type	Seam scheme
1	2.02.02, LSb-1	301	
2	4.03.03, SSz-3	301	
3	2.04.03, LSc-2	301	

The novelty elements are as follows: (i) analysis of the seams which are typically used in the manufacturing process, (ii) optimisation of material thickness to obtain the optimal structure in respect of different criteria, (iii) choice and characteristics of the most sensitive elements of the seam.

Physical and mathematical model of coupled heat and moisture transport

Let us first define a physical model of the coupled heat and mass transfer within a typical seam. The objects are the seams types according to ISO 4915:1991 [17] 2.02.02, LSb-1; 4.03.03, SSz-3; 2.04.03, LSc-2; cf. ***Table 1***. The two-layer laminate is made of PES fabric and membrane connected by polymer. The sealing tape is also connected by polymer to the complete seam to create a seam pack. Thus there are a few material layers of different material characteristics. The main goal of the seam is to secure the comfortable heat and moisture level on the skin, drain sweat outside and prevent the penetration of water vapour inwards. Let us assume a dynamic process of the short moisturizing time and relatively insignificant moisture flux density from the skin. Thus volume changes in PES fabric caused by moisture diffusion are negligible, Li [18]. The crucial problem is now to optimise the material thickness of some layers to secure optimal behavior of the seam.

The standard seam is a space structure of the same internal configuration repeatable in the subsequent cross-sections. However, the physical conditions are similar along the seam, with the only difference being the position on the user. To simplify the calculations and save time, the spatial problem can be reduced to an optional cross-section of the seam i.e. we analyse the plane problem. The finite element formulation is considerably simplified and the calculations are not time-consuming.

The typical coupled problem of heat and moisture transport is characterised in textile structures by Li [18], Li, Luo [19].The coupled transport indicates that moisture transmits a portion of heat, whereas heat transports moisture at the level of molecules. Textiles have a repeatable structure made of fibrous material and void spaces between textiles.

Therefore the material is defined by an instantaneous thermodynamical equilibrium on the contact surfaces. Heat is transported in fibers by conduction whereas from the outer fiber surfaces it is by radiation and convection to the voids. The only method of moisture transport is diffusion within textiles and free spaces. The diffusion process can be determined additionally from the experimental relationship according to Li [18]. The equilibrium time t_{eq} is defined empirically for some textiles, cf. Li [18]; Haghi [20]. Let us assume that the analysis time is shorter than the equilibrium time of fabric $t_{eq} = 540s$. The fibers are near-cylindrical and the moisture concentration is negligible at the beginning of the coupled transport. Thus the first stage of sorption in a dry textile according to Fick is a diffusion of moisture into the relaxed fibrous material.

Of course, the inhomogeneous material of each textile layer should be first homogenised. Fibrous material consists of yarns and free spaces between material. Therefore the first step is homogenisation at the micro level, that is determination of homogeneous fibers within the particular layer. Thus the entire material layer is homogenised as a result of homogenization at the macro level. It is evident that there is a complex two-stage homogenization procedure. The corresponding parameters depend on the fractional water content on the fiber surface, cf. for example Haghi [20].

The physical behaviour of a material system subjected to coupled transport is defined by the set of state variables. The variables are representative for the system and depend on its physical formulation. Typical parameters are temperature, moisture level, thermal entropy, enthalpy, pressure etc. The approach proposed uses a balance formulation of heat and moisture transport determined according to [18, 19] as second-order differential equations in respect of temperature T as well as moisture concentrations in fibers w_f and void spaces w_a. Therefore these three parameters are assumed as the state variables.

The mathematical model of coupled heat and mass transport finally contains the following: (i) heat and mass balances, (ii) constitutive material equations, (iii) relationships between state variables, and (iv) physical and chemical correlations in each material phase. The seam does not include the internal heat and mass sources ($f = f_w = 0$). The initial heat and mass flux densities are $\boldsymbol{q}^* = \boldsymbol{q}_w^* = 0$.

The first phase of sorption is determined by the heat and mass transport equations and the third experimental equation according to David, Nordon [21]. Let us introduce coefficient β, cf. Crank [22], Li [18] $w_f = \rho\beta w_a$ to simplify the description of moisture sorption on the fiber surface. Finally the transport equations in the *(i)-th* layer have the form given by Korycki in [23], see ***Equation (1)***.

To solve the equations, it is necessary to introduce the boundary conditions, ***Figure 1***. The lower portion of the seam pack contacts skin of specific levels of temperature and water vapour concentration. The model is characterised by first kind conditions on portions Γ_T and Γ_1. Side parts of the structure have insignificant dimensions and are secured by the sealing tape. The model is defined by second kind conditions of heat and water vapour transport on the adequate boundary portions Γ_q and Γ_2 of the side boundaries. Heat and water vapour are transported from the upper portion of the seam pack (i.e. the sealing tape) to the surroundings. Heat is lost by convection (the third kind condition, portion Γ_C) and radiation (portion Γ_r), while for moisture it is by convection (third kind condition, portion Γ_3). The common contact surfaces between materials of the adjacent layers *(i)*, *(i+1)* are described by the fourth kind conditions and the corresponding boundaries are Γ_i, Γ_4. The initial conditions determine the distribution of temperature and water vapour concentrations at the beginning of the process i.e. the same as in the environment for the relaxed material. All conditions have the form of ***Equation (2)***.

First-order approach of sensitivity analysis

The thickness of material layers is optimised using the sensitivity approach. The material derivative concept $DFDb_p = F_p$ is implemented as the first-order sensitivity of the objective functional in respect of design parameters. Let us assume that the objective functional is generally defined using parameters w_f; q_w; $w_{f\infty}$ on the external boundary.

$$F = \int_0^{t_f} \left[\int_{\Gamma(b)} \gamma_2 (w_f, q_w, w_{f\infty}) d\Gamma \right] dt; \quad (3)$$

Integrand γ is the continuous and differentiable function of the listed arguments. The direct approach [14, 23] is advantageous for the structures defined by the insignificant number of design variables, for example the thicknesses of textile layers. Introducing P design variables,

$$\begin{cases} \eta^{(i)}\left(1-\varepsilon^{(i)}+\dfrac{\varepsilon^{(i)}}{\beta^{(i)}}\right)\dfrac{dw_f}{dt} = -\mathrm{div}\mathbf{q}_w^{(i)}; \quad \mathbf{q}_w^{(i)} = \mathbf{D}^{(i)} \cdot \nabla w_f^{(i)}; \\ \rho^{(i)} c^{(i)} \dfrac{dT^{(i)}}{dt} + \lambda_w^{(i)}\left(1-\varepsilon^{(i)}\right)\dfrac{dw_f^{(i)}}{dt} = -\mathrm{div}\mathbf{q}^{(i)}; \quad \mathbf{q}^{(i)} = \mathbf{A}^{(i)} \cdot \nabla T^{(i)}. \end{cases} \quad (1)$$

Lower boundary: $T(\mathbf{x},t) = 33 + \dfrac{t}{60} \quad t \in \langle 0;60 \rangle s; \quad \mathbf{x} \in \Gamma_T;$

$w_f(\mathbf{x},t) = 0{,}08\left[1+\sin(0{,}003\Pi t)\right] \quad t \in \langle 0;60 \rangle s; \quad \mathbf{x} \in \Gamma_1;$

Side boundaries: $q_n(\mathbf{x},t) = 0; \ \mathbf{x} \in \Gamma_q; \qquad q_w(\mathbf{x},t) = 0; \ \mathbf{x} \in \Gamma_2;$

Upper boundary: $q_n(\mathbf{x},t) = h\left[T(\mathbf{x},t) - T_\infty\right] \ \mathbf{x} \in \Gamma_c; \quad q_n^r(\mathbf{x},t) = \sigma T^4 \quad \mathbf{x} \in \Gamma_r$ (2)

$q_w(\mathbf{x},t) = h_w\left[w_f(\mathbf{x},t) - w_{f\infty}\right] \ \mathbf{x} \in \Gamma_3;$

Internal boundaries: $T^{(i)}(\mathbf{x},t) = T^{(i+1)}(\mathbf{x},t) \quad \mathbf{x} \in \Gamma_i; \quad w_f^{(i)}(\mathbf{x},t) = w_f^{(i+1)}(\mathbf{x},t) \quad \mathbf{x} \in \Gamma_4;$

Initial conditions: $T(\mathbf{x},0) = T_\infty \quad \mathbf{x} \in (\Omega \cup \Gamma); \qquad w_f(\mathbf{x},0) = w_{f\infty} \quad \mathbf{x} \in (\Omega \cup \Gamma).$

Equations (1) *and* (***2***).

we solve *(P+1)* problems i.e. *P* additional problems of heat and mass transfer associated with each variable and the *1* primary problem. The state variables are the temperature $T^p = \partial T/\partial b_p$ and water vapor concentrations $w_f^p = \partial w_f/\partial b_p$, $w_a^p = \partial w_a/\partial b_p$. Each of the additional problems has the same geometric shape and transport conditions inside the structure and on the external boundary as the primary one. Some fields within the domain and on the specified portions of the outer boundary are different in both problems [14, 23]. Moreover the direct and additional problems are analysed in real time. The necessary correlations are derived by the differentiation of the equations appropriated for the primary problem ***Eqations (1), (2)***. Some components are independent of the design parameters, thus the material derivatives have the form [14] $\frac{DT^0}{Db_p} = 0; \mathbf{x} \in \Gamma_T$; $\frac{Dw_f^0}{Db_p} = 0$; $\mathbf{x} \in \Gamma_1$; $\frac{Dq_n^0}{Db_p} = 0; \mathbf{x} \in \Gamma_q$; $\frac{Dq_w^0}{Db_p} = 0; \mathbf{x} \in \Gamma_2$; $\frac{DT_0}{Db_p} = 0$; $\frac{Dw_{f0}}{Db_p} = 0; \mathbf{x} \in (\Omega \cup \Gamma)$. The final transport equations and boundary conditions are presented as ***Equation (4)***.

The first-order sensitivity expression can be simplified to the form [14, 23] of ***Equation (5)***.

The first-order sensitivity can be alternatively analysed using the adjoint approach and system of adjoint problems associated with each objective functional. We now solve *(N+1)* problems because the

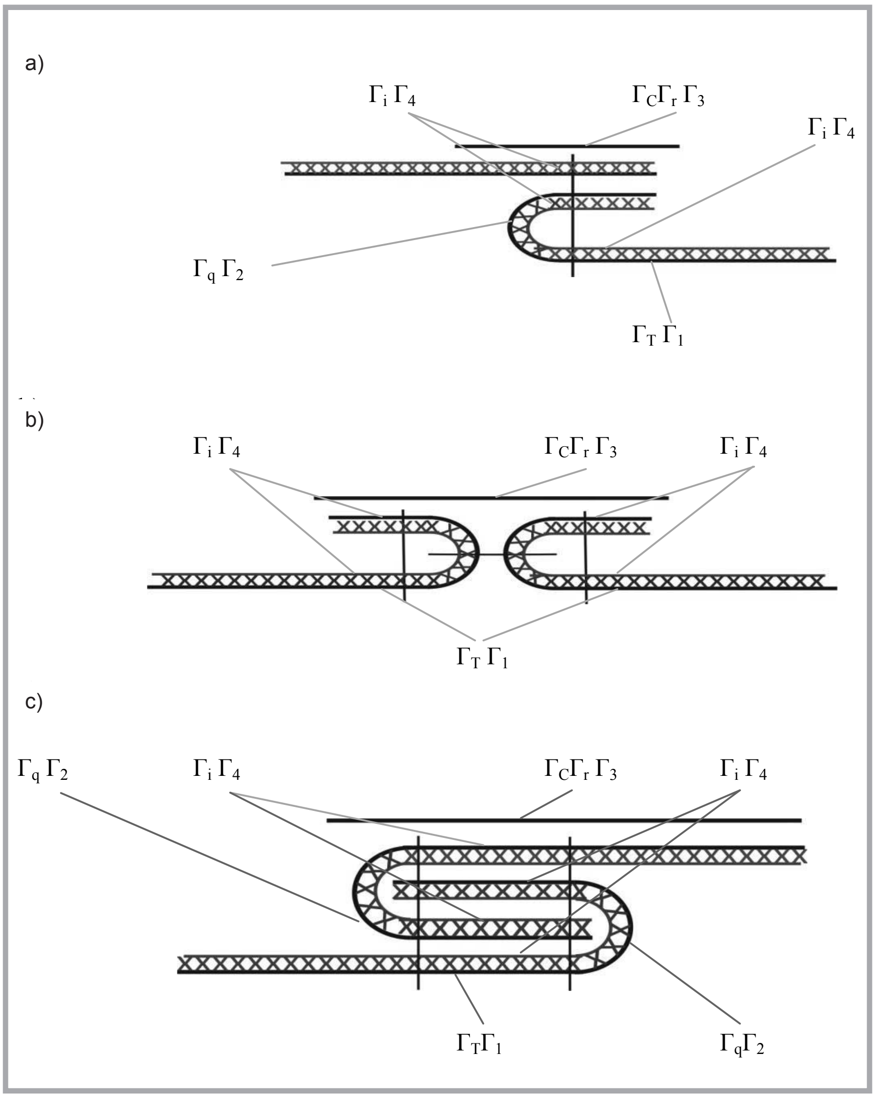


Figure 1. *Boundary conditions of the seams analysed, a) 2.02.02, LSb-1; b) 4.03.03, SSz-3; c) 2.04.03, LSc-2 (acc. to ISO 4916:1991).*

$$\begin{cases} \eta^{(i)}\left(1-\varepsilon^{(i)}+\frac{\varepsilon^{(i)}}{\beta^{(i)}}\right)\frac{dw_f^p}{dt} = -\mathrm{div}\,\mathbf{q}_w^{(i)p}; \quad \mathbf{q}_w^{(i)p} = \mathbf{D}^{(i)}\cdot\nabla w_f^{(i)p}; \\ \rho^{(i)}c^{(i)}\frac{dT^{(i)}}{dt} + \lambda_w^{(i)}\left(1-\varepsilon^{(i)}\right)\frac{dw_f^{(i)p}}{dt} = -\mathrm{div}\,\mathbf{q}^{(i)p}; \quad \mathbf{q}^{(i)p} = \mathbf{A}^{(i)}\cdot\nabla T^{(i)p}. \end{cases}$$

$$\begin{aligned} &\text{Lower boundary:} && T^p(\mathbf{x},t) = -\nabla T^0\cdot\mathbf{v}^p \quad \mathbf{x}\in\Gamma_T; \quad w_f^p(\mathbf{x},t) = -\nabla w_f^0\cdot\mathbf{v}^p \quad \mathbf{x}\in\Gamma_1; \\ &\text{Side boundaries:} && q_n^p(\mathbf{x},t) = \mathbf{q}_\Gamma^0\cdot\nabla_\Gamma v_n^p - \nabla_\Gamma q_n^0\cdot\mathbf{v}_\Gamma^p - q_{n,n}^0 v_n^p \quad \mathbf{x}\in\Gamma_q; \\ &&& q_w^p(\mathbf{x},t) = \mathbf{q}_{w\Gamma}^0\cdot\nabla_\Gamma v_n^p - \nabla_\Gamma q_w^0\cdot\mathbf{v}_\Gamma^p - q_{w,n}^0 v_n^p \quad \mathbf{x}\in\Gamma_2; \\ &\text{Upper boundary:} && q_n^p(\mathbf{x},t) = h\left(T^p - T_\infty^p\right) + \mathbf{q}_\Gamma\cdot\nabla_\Gamma v_n^p \quad \mathbf{x}\in\Gamma_c; \quad q_n^{rp} = 4\sigma T^3 T^p \quad \mathbf{x}\in\Gamma_r; \\ &&& q_w^p(\mathbf{x},t) = h_w\left(w_f^p - w_{f\infty}^p\right) + q_{w\Gamma}\cdot\nabla_\Gamma v_n^p \quad \mathbf{x}\in\Gamma_3; \\ &\text{Internal boundaries:} && T^{p(i)}(\mathbf{x},t) = T^{p(i)}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_i; \quad w_f^{p(i)}(\mathbf{x},t) = w_f^{p(i)}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_4; \\ &\text{Initial conditions:} && T_0^p(\mathbf{x},0) = -\nabla T_0\cdot v^p \quad \mathbf{x}\in(\Omega\cup\Gamma); \quad w_{f0}^p(\mathbf{x},0) = -\nabla w_{f0}\cdot v^p \quad \mathbf{x}\in(\Omega\cup\Gamma). \end{aligned} \tag{4}$$

$$\begin{aligned} F_P = \int_0^{t_f}\Bigg\{ &\int_{\Gamma_1}\left[-\gamma_{,w_f}\left(\nabla_\Gamma w_f^0\cdot\mathbf{v}_\Gamma^p + w_{f,n}^0 v_n^p\right) + \gamma_{,q_w}\left(q_w^p - \mathbf{q}_{w\Gamma}\cdot\nabla_\Gamma v_n^p\right)\right]d\Gamma_1 + \int_{\Gamma_2}\left\{\gamma_{,w_f} w_f^p - \gamma_{,q_w}\left(\nabla_\Gamma q_w^0\cdot\mathbf{v}_\Gamma^p + q_{w,n}^0\cdot v_n^p\right)\right\}d\Gamma_2 + \\ &+ \int_{\Gamma_3}\left[\gamma_{,w_f} w_f^p + \gamma_{,q_w} h_w\left(w_f^p - w_{f\infty}^p\right)\right]d\Gamma_3 + \int_\Gamma\left(\gamma_{,n} - 2H\gamma\right)v_n^p\, d\Gamma + \int_\Gamma\left(\gamma_{,w_{f\infty}}\, w_{f\infty}^p\right)d\Gamma + \int_\Sigma \left]\gamma\mathbf{v}^p\cdot\boldsymbol{\upsilon}\right[\Bigg\}dt \qquad p = 1, 2...P. \end{aligned} \tag{5}$$

Equations (4) *and* (***5).***

number of functionals N is the same as the that of the problems, and the primary problem is also included. The method is convenient for a small number of objective functionals. Geometry and material characteristics are the same in the primary and each adjoint system, but some fields of state variables should be redefined. Logical is the application of complex shapes which are a synthesis of simple bodies defined by a large number of design variables, because the optimisation introduces the only or a few objective functionals. A large majority of engineering problems in the textile industry comprise the single functional and only adjoint problem. The state variables are now the temperature T^a and water vapor concentrations w_f^a, w_a^a. The transport equations and set of conditions has an analogical form like the equations in the primary structure [14, 23]. The necessary correlations are determined by the analysis of corresponding terms within the mathematical equations [14, 23]. The transport equations and set of conditions are expressed as ***Equation (6)***.

Disadvantageous and mathematically inconvenient is the transformation of time between systems. The time τ in the adjoint approach is measured reversely to the time t for the primary system. It is troublesome to store the information of time because the final time $t = t_f$ in the primary problem is predefined as the starting time $\tau = 0$ according to the rule $\tau = t_f - t$. The sensitivity expression has a simplified form [14, 23] of ***Equation (7)***.

Optimal design of material thickness within seam

The optimal design of material thickness is defined as the search for the minimum of objective functional F with the imposed constraint of the structural cost C. Assuming a homogeneous structure of each material layer, the structural cost is proportional to the area of domain Ω. Introducing the Lagrangian functional in the form $F' = F + \chi(C - C_0 + \xi^2)$ and its stationarity, we formulate the optimality conditions [15].

$$\begin{cases} \dfrac{\mathrm{DF}}{\mathrm{Db}_p} = -\chi \int_\Omega \mathrm{u}\, \mathrm{v}_n^p\, \mathrm{d}\Gamma \\ \int_\Omega \mathrm{u}\, \mathrm{d}\Omega - C_0 + \xi^2 = 0. \end{cases} \tag{8}$$

The first order sensitivity vector of the objective functional DF/Db_p can be obtained by adopting the general sensitivity expressions for the direct and adjoint approaches. The shape is optimised using the variational formulation of the finite element method. Thus the objecitve functional describes the physical model of the real problem and the optimisation concerns the differential description of the mathematical model. The variational formulation should always be accompanied by a clear physical interpretation.

The important criterion is always the minimal moisture content on the external boundary of the sealing tape. Therefore the moisture flux density should be maximised on the external boundary portion of tape to secure effectively an adequate level of moisture on the skin.

$$F = \int_0^{t_f} \left[\int_{\Gamma_{\mathrm{ext}}} q_w d\Gamma_{\mathrm{ext}} \right] \mathrm{dt} \rightarrow \mathrm{min}. \tag{9}$$

$$\begin{cases} \eta^{(i)}\left(1-\varepsilon^{(i)}+\dfrac{\varepsilon^{(i)}}{\beta^{(i)}}\right)\dfrac{dw_f^a}{dt} = -\mathrm{div}\mathbf{q}_w^{(i)a}; \quad \mathbf{q}_w^{(i)a} = \mathbf{D}^{(i)}\cdot\nabla w_f^{(i)a}; \\ \rho^{(i)}c^{(i)}\dfrac{dT^{(i)}}{dt} + \lambda_w^{(i)}\left(1-\varepsilon^{(i)}\right)\dfrac{dw_f^{(i)a}}{dt} = -\mathrm{div}\mathbf{q}^{(i)a}; \quad \mathbf{q}^{(i)a} = \mathbf{A}^{(i)}\cdot\nabla T^{(i)a}. \end{cases}$$
$$\begin{aligned} & w_f^a(\mathbf{x},\tau=0)=0 \quad \mathbf{x}\in(\Omega\cup\Gamma); \qquad w_f^{0a}(\mathbf{x},\tau)=\gamma,_{q_w}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_1; \\ & q_w^{*a}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Omega; \qquad q_w^{0a}(\mathbf{x},\tau)=-\gamma,_{w_f}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_2; \\ & w_{f\infty}^a(\mathbf{x},\tau)=\frac{1}{h_w}\gamma,_{w_f}(\mathbf{x},t)+\gamma,_{q_w}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_3. \quad T^a(\mathbf{x},\tau=0)=0 \quad \mathbf{x}\in(\Omega\cup\Gamma); \\ & f^a(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Omega \qquad \mathbf{q}^{*\mathrm{a}}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Omega; \qquad T^{0a}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Gamma_T; \\ & q_n^{0a}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Gamma_q; \qquad q_n^{ar}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Gamma_r; \qquad T_\infty^a(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Gamma_c; \end{aligned} \tag{6}$$

$$\begin{aligned} F_p = -\int_0^{t_f}\Bigg\{ & \int_{\Gamma_1}\left[\left(\gamma,_{w_f}+q_{nw}^a\right)\left(\nabla_\Gamma w_f^0\cdot\mathbf{v}_\Gamma^p + w_f^0,_n v_n^p\right)+\gamma,_{q_w}\mathbf{q}_{w\Gamma}\cdot\nabla_\Gamma v_n^p\right]d\Gamma_1 + \\ & -\int_{\Gamma_2}\left[\left(\gamma,_{q_w}-w_f^a\right)\left(\nabla_\Gamma q_w^0\cdot\mathbf{v}_\Gamma^p - q_w^0,_n v_n^p\right)-w_f^a\mathbf{q}_{w\Gamma}\cdot\nabla_\Gamma v_n^p\right]d\Gamma_2 + \\ & \int_{\Gamma_3}\left[\left(w_f^a-\gamma,_{q_w}\right)h_w w_{f\infty}^p - w_f^a\mathbf{q}_{w\Gamma}\cdot\nabla_\Gamma v_n^p\right]d\Gamma_3 + \int_\Gamma\left(\gamma,_n-2H\gamma\right)v_n^p\,d\Gamma + \int_\Gamma \gamma,_{w_{f\infty}} w_{f\infty}^p\,d\Gamma + \int_\Sigma \left]\gamma\mathbf{v}^p\cdot\boldsymbol{\upsilon}\right[\Bigg\}dt \end{aligned} \tag{7}$$

***Equations (6)** and **(7)**.*

The comfort within clothing during activity requires equalised moisture distribution on the external boundary portion. The functional is a global measure of moisture concentration within fibers w_f related to the assumed level w_{f0}. The optimisation criterion ensures the minimal global measure which minimises the local maxima of state variables. The locations of these maxima can change because the maximal concentrations are time- and place-dependent.

$$\begin{aligned} F &= \int_0^{t_f} (F_W)^{\frac{1}{n}}\, \mathrm{dt} = \\ &= \int_0^{t_f} \left\{ \left[\int_{\Gamma_{\mathrm{ext}}} \left(\frac{w_f}{w_{f0}}\right)^n \mathrm{d}\Gamma_{\mathrm{ext}} \right]^{\frac{1}{n}} \right\} \mathrm{dt} \rightarrow \mathrm{min}\,; \\ & \mathrm{n} \rightarrow \infty. \end{aligned} \tag{10}$$

The algorithm of thickness optimisation of material layers within seams is shown in ***Figure 2***.

Thickness optimisation of seams

Let us introduce the two-layer laminate R2511A/150 Granat 690/T+ME1. The components are the fabric 20020/160/690L, made of 100% PES of initial thickness $0.40 \cdot 10^{-3}$ m, and a membrane made of 100% PES of initial thickness $0.015 \cdot 10^{-3}$ m. External protection is two-layer sealing tape T020/2/8 STO–NOR, which is $0.12 \cdot 10^{-3}$ m thick. The seams are sewn up by means of PES threads made of staple fibers – Coats Astra (thickness 30 tex, number tkt in metric system M110). Design variables are the material thickness g_1 of the laminate and g_2 of the sealing tape. Let us additionally assume that moisture is transported as water vapor from the skin to the surroundings.

The material parameters of fabric and thread are a function of the fractional water content on the fiber surface described by the water vapour concentration in fibers and material density. The material parameters of both the membrane and tape are moisture independent. The orthotropic matrix of diffusion coefficients of water vapour in material can be determined by means of [20]. Let us code the particular *i-th* layer by *i = 1* PES fabric, *i = 2* membrane, *i = 3* sealing tape, and *i = 4* PES threads. The local directional description depends on the material analysed. Thus, the x axis is always tangential to the fabric, membrane and sealing tape (i.e. directed along the structure), whereas the y axis is normal to these structures (i.e. orthogonal to the surface). A different system is introduced for PES threads. The x axis is directed along the cross-section and the y axis along the thread i.e. normal to the cross-section. The global directional description is place-dependent and can change along the particular component. The equations have the form of ***Equation (11)***.

The structures are homogenised using the efficient and simple *rule of mixture*. The diffusion coefficient of water vapour in air is equal to $D_a = 2{,}5e^{-5}$. The material porosity and sorption coefficient of water vapour in the material are introduced as a constant in the form of ***Equation (12)***.

Heat conduction coefficients for orthotropic material are defined as [20] ***Equation (13)***.

The cross-transport coefficient λ_w and heat capacity c are assumed according to [20] ***Equation (14)***.

The primary problem is defined using the transport equations – ***Equation (1)*** and the set of conditions – ***Equation (2)***. Let us optimise the material thickness in respect of the minimal moisture flux density on the surface of the sealing tape $\Gamma_{ext} = \Gamma_C \cup \Gamma_r \cup \Gamma_3$, cf. ***Equation (9)***. The problem is solved using the direct and adjoint approaches to sensitivity analysis. The direct approach is defined by ***Equation (4)***. The time varies from the initial value $t_0 = 0$ to the final – $t_k = 300\ s$, and the discrete increase is assumed as $\Delta t = 10\ s$. The surface film conductance is equal to $h = 8\ W/(m^2K)$, and the surrounding conditions are $T_\infty = 20°C$, $w_{f\infty} = 0{,}06\ kg/m^3$. The external boundary is defined by line segments of mean curvature $H = 0$. The optimisation problem is defined by ***Equation (8)*** and the final form of the sensitivity expression can be described in respect of ***Equation (5)***, as ***Equation (15)***. The alternative is the adjoint approach defined by ***Equation (6)*** and sensitivity expression ***Equation (7)*** simplified in respect of ***Equation (9)*** to the form of ***Equation (16)***.

The optimization procedure is iterative and consists of both the analysis and synthesis stage. Threads are located within

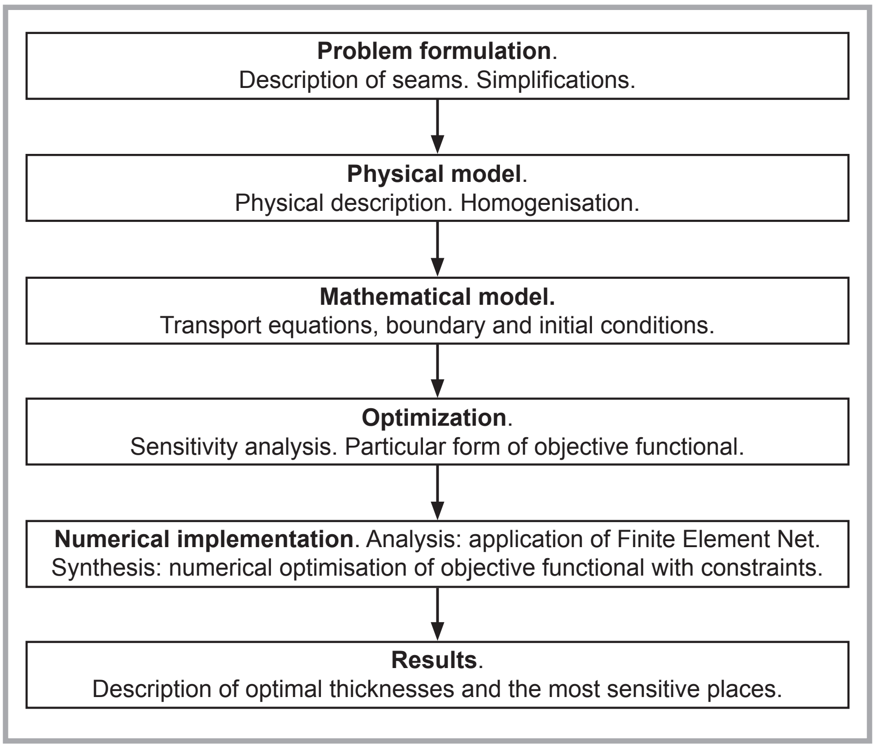


Figure 2. *Algorithm of thickness optimisation of textile layers in seam pack.*

$$\mathbf{D}^{(i)} = \begin{vmatrix} D_{xx}^{(i)} & 0 \\ 0 & D_{yy}^{(i)} \end{vmatrix};$$

$$\text{PES fabric: } D_{xx}^{(1)} = (1.12 - 410W_C - 8200W_C^2)\cdot 10^{-13};\ D_{yy}^{(1)} = (1.09 - 406W_C - 8250W_C^2)\cdot 10^{-13}.$$
$$\text{Membrane: } D_{xx}^{(2)} = 1.05\cdot 10^{-14};\quad D_{yy}^{(2)} = 1.35\cdot 10^{-13}.\ \text{Tape: } D_{xx}^{(3)} = 1.85\cdot 10^{-14};\quad D_{yy}^{(3)} = 1.85\cdot 10^{-13}. \qquad (11)$$
$$\text{PES thread: } D_{xx}^{(4)} = (1.20 - 380W_C - 8200W_C^2)\cdot 10^{-14};\ D_{yy}^{(4)} = (1.40 - 450W_C - 8250W_C^2)\cdot 10^{-13}.$$

$$\text{PES fabric: } \varepsilon^{(1)} = 0.150;\ \eta^{(1)} = 0.320;\quad \text{Membrane: } \varepsilon^{(2)} = 0.050;\ \eta^{(2)} = 0.050;$$
$$\text{Tape: } \varepsilon^{(3)} = 0.060;\ \eta^{(3)} = 0.060;\quad \text{PES thread: } \varepsilon^{(4)} = 0.110;\ \eta^{(4)} = 0.220 \qquad (12)$$

$$\mathbf{\Lambda}^{(i)} = \begin{vmatrix} \lambda_{xx}^{(i)} & 0 \\ 0 & \lambda_{yy}^{(i)} \end{vmatrix}; \quad \begin{matrix} \text{PES fabric: } \lambda_{xx}^{(1)} = 28.8\cdot 10^{-3}; & \lambda_{yy}^{(1)} = 33.0\cdot 10^{-3}; \\ \text{Membrane: } \lambda_{xx}^{(2)} = 66.0\cdot 10^{-3}; & \lambda_{yy}^{(2)} = 75.0\cdot 10^{-3}; \\ \text{Tape: } \lambda_{11}^{(3)} = 58.0\cdot 10^{-3}; & \lambda_{22}^{(3)} = 62.0\cdot 10^{-3}; \\ \text{PES thread: } \lambda_{xx}^{(4)} = 25.3\cdot 10^{-3}; & \lambda_{yy}^{(4)} = 30.0\cdot 10^{-3} \end{matrix} \qquad (13)$$

$$\text{PES fabric: } \lambda_w^{(1)} = 2522.0\quad c^{(1)} = 1610.9\qquad \text{Membrane: } \lambda_w^{(2)} = 2800.0\quad c^{(2)} = 1820.0$$
$$\text{Tape: } \lambda_w^{(3)} = 2100.0\quad c^{(3)} = 1510.0\qquad \text{PES thread: } \lambda_w^{(4)} = 2340.0\quad c^{(4)} = 1590.0 \qquad (14)$$

Equations (11), (12), (13) *and* ***(14).***

Table 2. *Initial and optimal material thicknesses, number of iterations and reduction in objective functional for minimisation of the moisture flux density:*

a) threads and needle channel of the same dimensions.

Design variable / optimal parameters	$g_1 \cdot 10^{-3}$m	membrane$\cdot 10^{-3}$m	$g_2 \cdot 10^{-3}$m	No of iterations	Reduc.obj. functional
Initial thickness	0.40	0.015	0.12	–	–
Optimal thickness seam 2.02.02, LSb-1	0.45	0.015	0.13	19	17.48%
Optimal thickness seam 4.03.03, SSz-3	0.44	0.015	0.13	24	12.22%
Optimal thickness seam 2.04.03, LSc-2	0.45	0.015	0.13	20	13.01%

b) threads and needle channel of different dimensions.

Design variable / optimal parameters	$g_1 \cdot 10^{-3}$m	membrane$\cdot 10^{-3}$m	$g_2 \cdot 10^{-3}$m	No of iterations	Reduc.obj. functional
Initial thickness	0.40	0.015	0.12	–	–
Optimal thickness seam 2.02.02, LSb-1	0.43	0.015	0.13	21	10.81%
Optimal thickness seam 4.03.03, SSz-3	0.42	0.015	0.14	35	9.27%
Optimal thickness seam 2.04.03, LSc-2	0.42	0.015	0.14	30	11.59%

Table 3. *Initial and optimal material thicknesses, number of iterations and reduction in objective functional for equalised moisture distribution:*

a) threads and needle channel of the same dimensions.

Design variable / optimal parameters	$g_1 \cdot 10^{-3}$m	membrane$\cdot 10^{-3}$m	$g_2 \cdot 10^{-3}$m	No of iterations	Reduc.obj. functional
Initial thickness	0.40	0.015	0.12	–	–
Optimal thickness seam 2.02.02, LSb-1	0.38	0.015	0.11	15	7.22%
Optimal thickness seam 4.03.03, SSz-3	0.36	0.015	0.11	17	9.16%
Optimal thickness seam 2.04.03, LSc-2	0.36	0.015	0.11	17	10.01%

b) threads and needle channel of different dimensions.

Design variable / optimal parameters	$g_1 \cdot 10^{-3}$m	membrane$\cdot 10^{-3}$m	$g_2 \cdot 10^{-3}$m	No of iterations	Reduc.obj. functional
Initial thickness	0.40	0.015	0.12	–	–
Optimal thickness seam 2.02.02, LSb-1	0.36	0.015	0.10	19	6.25%
Optimal thickness seam 4.03.03, SSz-3	0.37	0.015	0.10	24	9.16%
Optimal thickness seam 2.04.03, LSc-2	0.37	0.015	0.10	22	10.10%

the needle channel of the same dimensions. The complex structure at the analysis stage is successively approximated during heat and moisture transport by the same finite element net of the serendipity family according to Zienkiewicz [24]. Let us introduce plane rectangular 4-nodal elements of the nodes in corners.

$$F_P = \int_0^{t_f} \left\{ \int_{\Gamma_{ext}} \left[h_w \left(w_f^p - w_{f\infty}^p \right) + q_{w,n} v_n^p \right] d\Gamma_3 + \int_{\Sigma} \left] q_w \mathbf{v}^p \cdot \boldsymbol{\upsilon} \right[\right\} dt; \qquad p = 1.2 \tag{15}$$

$$F_p = \int_0^{t_f} \left\{ - \int_{\Gamma_{ext}} \left(\mathbf{q}_{w\Gamma} \cdot \nabla_{\Gamma} v_n^p \right) d\Gamma_{ext} + \int_{\Gamma_{ext}} q_{w,n} v_n^p \, d\Gamma_{ext} + \int_{\Sigma} \left] q_w \mathbf{v}^p \cdot \boldsymbol{\upsilon} \right[\right\} dt. \tag{16}$$

$$\left(F_w \right)_p = \int_0^{t_f} \left\{ \int_{\Gamma_{ext}} \left[\frac{n}{w_{f0}} \left(\frac{w_f}{w_{f0}} \right)^{n-1} w_f^p + \left(\frac{w_f}{w_{f0}} \right)^n ,_n v_n^p \right] d\Gamma_{ext} + \int_{\Sigma} \left] \left(\frac{w_f}{w_{f0}} \right)^n \mathbf{v}^p \cdot \boldsymbol{\upsilon} \right[d\Sigma \right\} dt; \quad p = 1.2 \tag{17}$$

Equations (15), (16) *and* ***(17).***

The total structure has 900 elements of 2600 nodes. The state variables are determined primarily, and the set of additional problems and adjoint problem by solution of the basic finite element equation. The directional minimum at the synthesis stage is determined using the second-order Newton method or alternatively the first-order method of the steepest descent. Thus it is necessary to introduce additionally a limitation concerning the difference in material thickness which cannot be greater than *±15%* in relation to the initial one. The thicknesses, the number of iterations necessary to obtain the optimum, and the reduction in the objective functional related to the initial one are listed in ***Table 2a***. The optimal thicknesses of PES fabric are close to the imposed limit, whereas the differences for the sealing tape are insignificant. The optimal objective functional is reduced between *13,01%* and *17,48%* in respect of the initial value.

The alternative thickness distribution is determined for a thread of a dimension equal to *90%* of that of the needle channel, ***Table 2b***. Thus irrespective of the transport within the material, heat and mass are transmitted through the void spaces between the thread and fabric. The optimal thicknesses of PES fabric are always greater than the original, but significantly smaller than the upper limit. The optimal dimensions of the sealing tape are convergent with those specified for the previous case. The number of iterations required to determine the minimum is now considerably greater than previously, while the objective functional decreases relatively slightly (*9.27%* to *11.59%*).

The difference in the optimal dimensions of material layers is the result of the different heat and mass transfer conditions. In the first case, heat and mass are transported only across the material, whereas in the second this occurs simultaneously through the material and voids around the thread. Therefore threads are the most sensitive element of the seam structure, considerably influencing the optimum.

The criterion can also be the equalised moisture distribution on the upper surface of the sealing tape. The objective functional describes the global measure of local moisture concentration in fibers w_f on the surface $\Gamma_{ext} = \Gamma_C \cup \Gamma_r \cup \Gamma_3$, cf. ***Equation (10)***, for the exponent equal to $n = 25$. Optimal shapes are determined using the direct method. Optimality conditions are defined again by ***Equation (8)***. The primary problem is determined using ***Equations (1)*** and ***(2)***, and the direct approach by ***Equation (4)***. The sensitivity expression can be simplified in respect of ***Equation (5)*** to the form of ***Equation (17)***.

The structure is approximated at the analysis stage using the same finite element net for the primary and the set of additional problems. The directional minimum at the synthesis stage is calculated using the Newton procedure and alternatively the method of steepest descent. The initial and optimal values are listed in ***Table 3*** for the same and different diameters of the thread and needle channel. The first case (***Table 3.a***, the same diameters) is characterised by the thicknesses of PES fabric and sealing tape close to the lower limit. The minimal objective functional is obtained, respectively, in *15*, *17*, *17* steps and reduced between *7,22%* and *10,01%* to the initial value. Similar results were obtained in the second case (***Table 3.b***, different diameters), but the number of iterations required to determine the minimum is greater.

Conclusions

Optimisation of the material thickness in a seam pack subjected to coupled heat and mass transport is a complex engineering problem. Coupled transport is described by means of the transport equations and a set of boundary and initial conditions. The problem is next solved using the material derivative concept and first-order sensitivity approach i.e. the direct and adjoint method of sensitivity analysis. The variational approach of the finite element method gives a clear physical interpretation of the problem solved. Thus the objective functionals are the moisture flux density and the measure of moisture distribution on the external boundary of the sealing tape. Minimisation of the flux density secures an adequate moisture level on the user's skin. Minimisation of moisture distribution causes equalized water vapor concentration within the seam, which is important in the wide seam.

The results obtained are logical and indicate that the optimal thicknesses satisfy the true physical problems. Thus the method proposed seems to be a promising tool for generating the optimal thicknesses of different materials within the seam pack. Additionally the numerical optimization is cheaper and more universal than the complex analysis of prototypes and provides some practical benefits.

Moreover the results obtained allow to determine the most sensitive component of the structure, which are the sewing threads inside the needle channels, where the heat and mass can be transported in the voids between the thread and material. These free spaces allow unrestricted heat and mass transfer, which significantly affects the distribution of optimal thicknesses. A possible future course of action can be searching for such objective functionals and structural responses that will identify other sensitive components within the seam.

The seam pack has a very small thickness and consists of a few different components. Therefore optimisation should include the constraints imposed on the dimension differences, which cannot be greater than *15% in* relation to the initial dimensions. These are, in fact, additional constraints irrespective of the inequality constraint of the structural cost imposed.

The paper presented is a theoretical optimisation of material thickness using computer analysis. The problem should be compared with corresponding tests, which is beyond the scope of the paper presented. Thus it is necessary to introduce real seams subjected to different temperatures and moisture concentrations. The problem is now being analyzed and the necessary test stand is in preparation.

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