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Vibrations of Elastically Supported Masses Separated by a Textile Layer

DOI: 10.5604/12303666.1227894

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Abstract
In this paper the problem of the transmission of vibration through a textile layer is presented. A mathematical model of a two degree of freedom system containing a textile layer and excited to vibrate by an electromagnet is formulated. The numerical simulation showsthat the textile layer increases the resonance frequency.

Key words: *transmission of vibration, textile layer.*

Introduction

In order to protect the health of machine operators from the effect of machine vibrations, various textile elements placed between the human body and rigid machine elements are used. In this problem we have to deal with such machines as electric or pneumatic hand tools and mobile machines. Vibration protecting elements such as gloves, armrests, seats and backrests are subject to oscillating compression. The compression characteristics of fibrous assemblies were explored in works [1,2]. The modelling and computer simulation of such systems can be found in work [3]. The compression behaviour of fabrics was studied in works [4,5]. The dependence between the force acting on the fibrous layer and the magnitude of its compression was proposed in work [6]. It was assumed that the gradual increase in the mutual contact areas of fibres which takes place during the compression of the fibre layer causes a reduction in the flexible bending portion of the fibres, which increases the layer stiffness. As the layer thickness decreases, its side surface through which the fluid is squeezed out decreases, causing an increase in the resistance to flow. The purpose of the present paper is to gain knowledge of the transmission of vibrations by textile elements.

Equations of motion

The system considered is shown in ***Figure 1***. It consists of two masses, m_d and m_u, separated by a textile layer k, two springs of stiffness k_u and k_d and a electromagnet of inductance L.

The dependence between the force F_{kc} acting on the fibrous layer and its deflection w was found in work [6] to be in the form of ***Equation (1).***

In ***Figure 1*** w_u and w_d denote coordinates of the upper and lower mass, respectively. Constants (k_u, k_d) denote elastic and (c_u, c_d) damping parameters of the upper (u) and lower (d) spring, respectively. Constants (k, L_1) denote elastic and (c, H_1) damping parameters of the nonlinear characteristic of the textile layer, defined in paper [1]. The parameters can be determined experimentally.

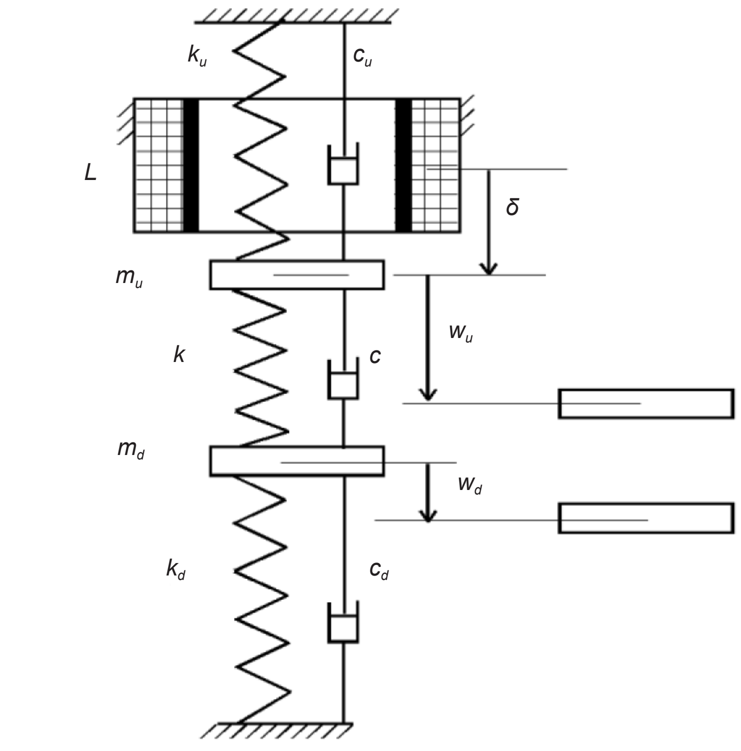

Figure 1. *Model of vibrating masses separated by a textile layer and excited to vibrate by an electromagnet.*

$$F_{kc} = \frac{kw}{\left(1-\dfrac{w}{L_1}\right)^3} + \frac{c\,\mathrm{sgn}\left(\dfrac{\mathrm{d}w}{\mathrm{d}t}\right)\left(\dfrac{\mathrm{d}w}{\mathrm{d}t}\right)^2}{\left(1-\dfrac{w}{H_1}\right)^3}, \quad w < H_1, \quad w < L_1, \tag{1}$$

Equation (1)

$$m_u \frac{d^2 w_u}{dt^2} + c_u \frac{dw_u}{dt} + k_u w_u + \frac{k(w_u - w_d)}{\left(1 - \frac{(w_u - w_d)}{L_1}\right)^3} + \frac{c\,\mathrm{sgn}\left(\frac{d(w_u - w_d)}{dt}\right)\left(\frac{d(w_u - w_d)}{dt}\right)^2}{\left(1 - \frac{(w_u - w_d)}{H_1}\right)^3} + F_E - m_u g = 0,$$

$$m_d \frac{d^2 w_d}{dt^2} + c_d \frac{dw_d}{dt} + k_d w_d - \frac{k(w_u - w_d)}{\left(1 - \frac{(w_u - w_d)}{L_1}\right)^3} - \frac{c\,\mathrm{sgn}\left(\frac{d(w_u - w_d)}{dt}\right)\left(\frac{d(w_u - w_d)}{dt}\right)^2}{\left(1 - \frac{(w_u - w_d)}{H_1}\right)^3} - m_d g = 0. \qquad (2)$$

$$L\frac{di}{dt} + \frac{dL}{dx} i + R i = u, \quad F_E = -\frac{1}{2} i^2 \frac{dL}{dx}, \quad x = \delta + w_u. \qquad (3)$$

$$L\frac{di}{dt} + Ri = U \sin \omega t, \quad i = I_s \sin \omega t + I_c \cos \omega t, \quad I^2 = I_s^2 + I_c^2$$

$$L(x) = \frac{1}{\omega}\sqrt{\left(\frac{U}{I}\right)^2 - R^2}\ . \qquad (4)$$

$$H = \frac{Iz}{4l}(\cos\beta_2 - \cos\beta_1) = \frac{Iz}{4l}\left(\frac{l + x}{\sqrt{r_0^2 + (l + x)^2}} - \frac{x - l}{\sqrt{r_0^2 + (x - l)^2}}\right) \qquad (5)$$

$$L(x) = \frac{L_{\max} - L_{\min}}{2}\sqrt{\left(\frac{r_0}{l}\right)^2 + 1}\left(\frac{1 + \frac{x}{l}}{\sqrt{\left(\frac{r_0}{l}\right)^2 + \left(1 + \frac{x}{l}\right)^2}} + \frac{1 - \frac{x}{l}}{\sqrt{\left(\frac{r_0}{l}\right)^2 + \left(1 - \frac{x}{l}\right)^2}}\right) + L_{\min},$$

$$\frac{dL}{dx} = \frac{L_{\max} - L_{\min}}{2}\sqrt{\left(\frac{r_0}{l}\right)^2 + 1}\left(\frac{\frac{1}{l}}{\sqrt{\left(\frac{r_0}{l}\right)^2 + \left(1 + \frac{x}{l}\right)^2}}\left(1 - \frac{\left(1 + \frac{x}{l}\right)^2}{\left(\frac{r_0}{l}\right)^2 + \left(1 + \frac{x}{l}\right)^2}\right) + \frac{\frac{1}{l}}{\sqrt{\left(\frac{r_0}{l}\right)^2 + \left(1 - \frac{x}{l}\right)^2}}\left(-1 + \frac{\left(1 - \frac{x}{l}\right)^2}{\left(\frac{r_0}{l}\right)^2 + \left(1 - \frac{x}{l}\right)^2}\right)\right). \qquad (6)$$

Equation (2), (3), (4), (5) and ***(6).***

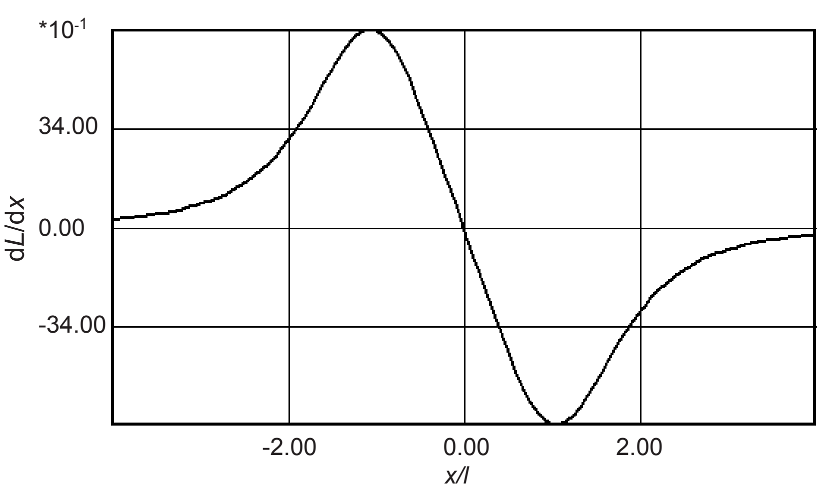

Figure 3. *First derivative of inductance dL/dx as a function of coordinate x, having its origin in the centre of coil, approximated by the magnetic field intensity function for R = 7 Ω, 2l = 0.056 m, r_0 = 0.032 m, L_{max} =0.364319 H, L_{min} =0.04 H.*

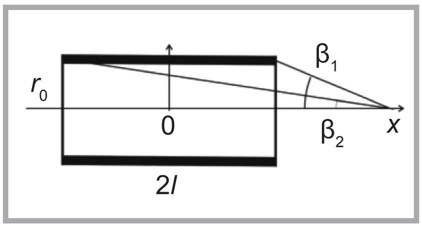

Figure 2. *Scheme for calculating the intensity of the magnetic field of the stationary coil, having z turns, at distance x from its centre.*

The equations of motion of masses m_u and m_d (***Figure 1,*** see page 131) are found to be in the form of ***Equations (2)***.

Assume the excitation force of the electromagnet to be described by ***Equations (3)***.

In ***Equations (3)***, *i* is the current intensity, *R* the resistance of the circuit, *u* feed voltage, *x* the position of the core centre, having its origin in the centre of the coil, and *δ* is the distance from the centre of the core to the centre of the coil at rest.

The methods of determination of the inductance *L* are described in papers [7-10]. Here the experimental method [7, 11] is explained in detail. For the stationary electromagnet, having a coil of resistance *R,* supplied with voltage $u = U\sin\omega t$, we measure the current *I* and calculate the inductance *L* as a function of the mutual position of the armature and electromagnet see ***Equations (4)***.

Using the results of measurements, we approximate the inductance function *L* by the function describing the intensity of the magnetic field *H* (***Equations (5)***) of the stationary coil (***Figure 2***) and by choosing proper parameters.

In ***Equations (6)***, defining the inductance *L* and its derivative d*L*/d*x* (***Figure 3***), the *x* coordinate specifies the position of the movable core centre and has its origin in the centre of the coil; *l* denotes half of the computational length of the coil; r_0 the computational radius of the coil; L_{max} is the maximum inductance of the coil, that is when the centre of the core coincides with the centre of the coil, and L_{min} is the minimum inductance of the coil when the core is in the end position.

Results

The set of differential ***Equations (2)***, ***(3)*** was solved numerically using the

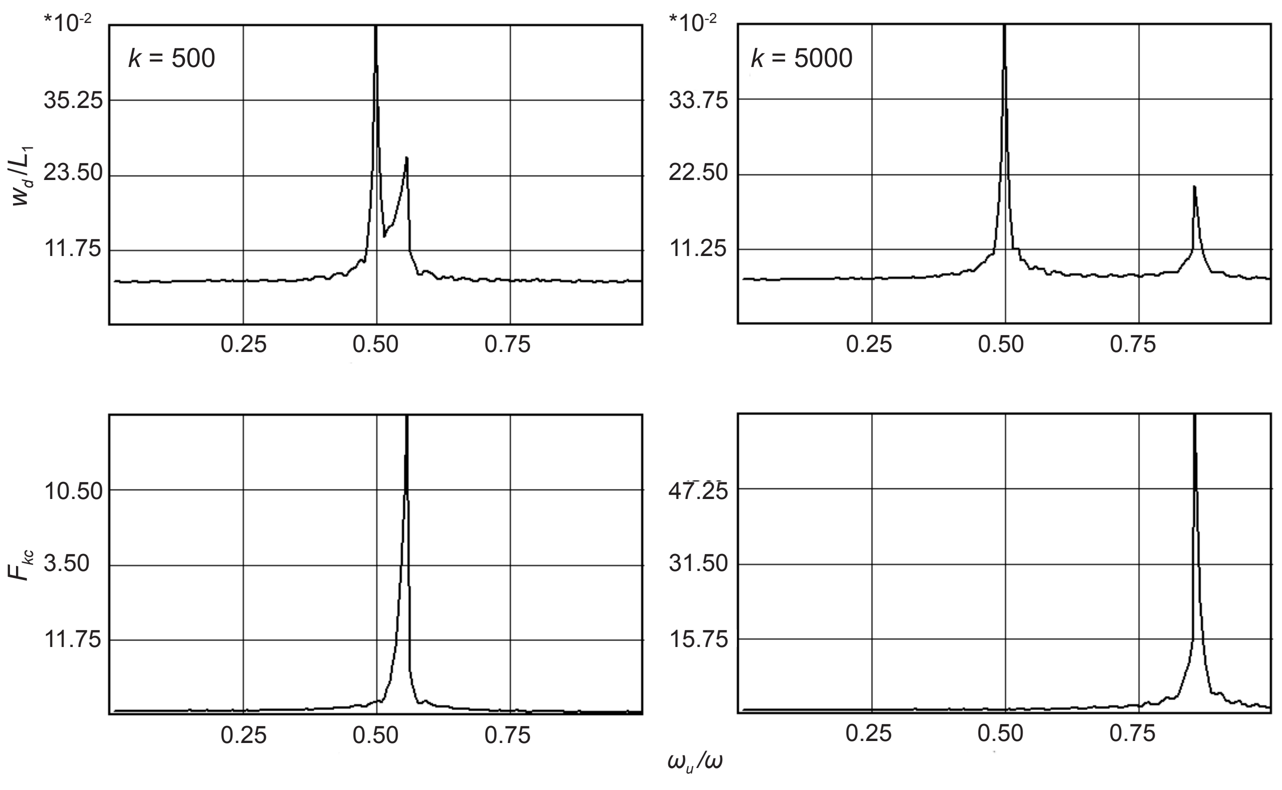

Figure 4. *Steady-state maximum displacements w_d of the lower mass and textile layer reaction force F_{kc} for increasing frequency of excitation ω_u.*

Runge-Kutta method. The integration was carried on until the difference between each period became negligible and the solution achieved steady-state. Calculations were performed for $u = U_m \sin(\omega_u t)$, $U_m = 24$ V, $L_1 = 0.03$ m, $H_1 = 0.03$ m, $k = 500$ and 5000N/m, $c = 0.1$ Ns^2/m^2, $k_u = k_d = 5000$ N/m, $c_u = c_d = 0.1$ Ns/m, $m_u = m_d = 1$ kg, $g = 9.81$ m/s^2, $L_{max} = 0.364319$ H, $L_{min} = 0.04$ H, $l = 0.028$ m, $r = 0.032$ m, $R = 40\ \Omega$, $\delta = l$, $Um = 24$ V. Initial conditions were $w_d(0) = w_u(0) = 0$, $\mathrm{d}w_d\mathrm{d}t(0) = \mathrm{d}w_u\mathrm{d}t(0) = 0$, $i(0) = 0$, $\omega = (k_d/m_d)^{0.5}$. The results are shown in ***Figures 4***.

The peak of the reaction force of the textile layer F_{kc}, shown in ***Figure 4***, shows the frequency of vibration of the masses when they move in the opposite direction, which results in textile layer compression.

Conclusion

In the absence of the textile layer, since both oscillators are the same, the resonance frequencies associated with mass motion in the same and opposite direction are equal. If the textile layer is present, those resonant frequencies are different, and that variation in resonant frequencies increases with an increase in the textile layer stiffness. In order to get practical results, further studies of the transmission of vibrations through various textile elements are needed.

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Received 29.02.2016 Reviewed 14.07.2016