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Inverse Problem of Textile Material Design at Low Temperature Solved by a Hybrid Stochastic Algorithm

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Abstract
The inverse problem of textile material design (IPTMD) aims to determine textile materials with optimum thermal conductivities for the thickness designed in terms of the thermal comfort requirements of the wearer. In this paper, an IPTMD is presented on the basis of the physical nature of steady heat and moisture transfer in a human body-clothing-environment system. A globally convergent algorithm, the modified particle collision algorithm (MPCA), is proposed and its validity is verified. The MPCA is applied to solve the IPTMD for single-layer textile materials at low temperature. Numerical simulation results of the IPTMD proved the suitability of the IPTMD and effectiveness of the MPCA in solving complex global optimisation problems. The encouraging results indicate that the modelling method above and optimisation algorithm can be used for further applications.

Key words: *inverse problem, textile design, hybrid stochastic algorithm, optimisation method.*

Introduction

Under low temperature, clothing acts as a resistance to heat and moisture transfer between the human body and the environment, by which means it retards heat dissipation and protects the human body against cold. However, it also obstructs the transport of redundant heat or perspiration generated, for instance, by physical activity. The accumulation may result in a remarkable change in temperature and/or relative humidity (RH) in the microclimate and affect the sensation of comfort accordingly. Generally with respect to cold-weather clothing, the protective performance and physiological comfort are two main concerns of both the wearer and designer [1 - 3].

Coupled heat and moisture transfer through clothing is a comprehensive process involving a variety of mechanisms, which has been widely investigated experimentally and theoretically since 1940’s [4 - 8]. The physics of heat flow by conduction, radiation and convection, water vapour flow by diffusion, and phase change regarding evaporation and condensation has been studied under diverse circumstances. Mathematical models have been proposed to determine the field of temperature and moisture as well as the flux of heat and mass in the objective system. The problems considered in these researches are direct problems of heat and mass transfer.

By contrast, inverse problems in the field of heat and mass transfer derive unknown parameters, such as physical/structural parameters of an object, and/or boundary condition parameters, from measurable data. For instance, Huang et al [9] and Dantas et al [10] proposed inverse problems to estimate thermophysical and boundary condition parameters in Luikov’s theory by means of temperature and moisture measurements. Xu et al [11 - 14] proposed inverse problems of textile design to determine, in terms of thermal comfort requirements, the thickness or thermal conductivity of textile materials from knowledge of the boundary conditions of the temperature, RH and moisture flux. The inverse problems were solved by the regularisation method together with numerical optimisation methods. However, the optimisation methods adopted so far in Xu’s studies have been local optimisation methods, i.e. the Hooke-Jeeves’s algorithm (HJA), the golden-section search algorithm and Cai’s direct search algorithm, which were found to be sensitive to initial values and might diverge with improperly selected ones [11]. Furthermore due to the complexity inherent in the physics of heat and moisture transfer in fibrous assemblies, the uniqueness of numerical solutions of inverse problems has not yet been manifested theoretically. Therefore it is essential to introduce globally convergent optimisation methods when dealing with such inverse problems.

Among global optimisation methods, stochastic methods have been successfully applied to solve complex optimisation problems [15, 16]. Simulated annealing and modified versions are conventional methods of this kind, yielding global optima, yet are sensitive to the initial temperature and annealing schedule. To overcome such drawbacks, hybrid stochastic algorithms inspired by the physics of nuclear particle collision reactions (PCA) were put forward and proved to be effective in solving global optimisation problems [17, 18].

In this paper, an inverse problem of textile material design (IPTMD) is presented and a modified PCA (MPCA) is proposed to solve the IPTMD. The validity of the MPCA is verified and numerical results of the IPTMD are provided and discussed.

Theory

Consider a human body-textile-environment system where the human body is the source of heat and moisture flux. It is assumed that the system is in a steady state during the process of heat and mass transfer, the textile material is uniform in its geometrical structure and material properties and the structure keeps stable despite variations in environmental conditions. The pore structure in the textile material is composed of parallel cylindrical pipes aligned in the direction of heat and mass transfer. Based upon these assumptions, the heat and moisture transfer in the objective system can be modelled by the coupled ordinary differential equations described below, associated with initial and boundary conditions [14, 19]. According to mass conservation, the mass flux of water vapour m_v in $kg \cdot m^{-2} \cdot s^{-1}$ in parallel cylindrical pore textile materials can be expressed by ***Equation 1a***:

$$k_1 \frac{\varepsilon(x) \cdot r(x)}{\tau(x)} \cdot \frac{p_v(x)}{T(x)^{3/2}} \cdot \frac{dp_v(x)}{dx} + m_v(x) = 0 \quad (1a)$$

where, x in m is the distance from the inner surface of the textile material, k_1 a parameter related to the molecular weight of water and the gas constant, T in K the temperature, p_v in Pa the water vapour pressure, and ε in %, r in m and T are the porosity, average pore radius, and effective tortuosity of the textile material, respectively.

Concerning the condensation of water vapour on the surface of fibres, we have:

$$\frac{\mathrm{d}m_{\mathrm{v}}(x)}{\mathrm{d}x} + \Gamma(x) = 0 \qquad (1b)$$

where, Γ in kg·m^{-2}·s^{-1} is the condensation rate of water vapour and is given by the modified Hertz-Knudsen equation [20]:

$$\Gamma(x) = -k_2\left[p_{\mathrm{sat}}(T(x)) - p_{\mathrm{v}}(x)\right]\frac{1}{\sqrt{T(x)}} \qquad (1c)$$

where, k_2 is an experiential parameter, and p_{sat} in Pa is the saturation water vapour pressure at T, expressed by the Antoine equation [21]

$$P_{\mathrm{sat}}(T) = \qquad (1d)$$
$$= 1000 \times \exp\left[16.6536 - \frac{4030.183}{(T - 273.15) + 235}\right]$$

According to heat energy conservation, we have:

$$\kappa\frac{\mathrm{d}^2T(x)}{\mathrm{d}x^2} + \lambda\Gamma(x) = 0 \qquad (1e)$$

where, κ in W·m^{-1}·K^{-1} is the thermal conductivity of the textile material, λ in kJ·kg^{-1} is the latent heat of condensation of water vapour.

The boundary conditions for ***Equation 1*** are

$$T(0) = T_0,\ T(L) = T_L, \quad m_{\mathrm{v}}(0) = m_{\mathrm{v},0},\ p_{\mathrm{v}}(0) = p_{\mathrm{v}\,0} \qquad (2)$$

where, L in m is the thickness of the textile material, T_0 and T_L are the temperatures on the inner and outer surfaces of the textile material, respectively, $m_{\mathrm{v},0}$ the water vapour flux generated by the human body, and $p_{\mathrm{v},0}$ the water vapour pressure near the surface of the human body.

The combination of ***Equations 1*** and ***2*** comprises a direct problem of heat and moisture transfer in textiles, from which the fields of temperature and water vapour pressure, as well as the heat flux and mass flux of water vapour can be derived when all the related parameters are predefined.

By contrast, the IPTMD aims to determine a textile material with optimum thermal conductivity for a thickness designed to meet the thermal comfort requirements of the wearer. As is generally recognised, the ideal perception of thermal comfort is achieved when the thermal comfort indexes for in between the human skin's surface and the inner surface of the clothing meet the conditions of $T = 32 \pm 1$ °C and $RH = 50 \pm 10\%$.

Given the environmental condition $(T, RH) \in [T_{\min}, T_{\max}] \times [RH_{\min}, R_{\max}]$ (where $T_{\min}$ and $T_{\max}$, $RH_{\min}$ and $RH_{\max}$ are, respectively, the minimum and maximum temperatures and RHs in a certain region during a defined period of time), the IPTMD can be described by ***Equation 1*** together with boundary conditions [14]:

$$T(0) = T_0,\ T(L) = T_L,\ m_{\mathrm{v}}(0) = m_{\mathrm{v},0}, \quad p_{\mathrm{v}}(L) = p_{\mathrm{v}\,L} \qquad (3)$$

where, $p_{\mathrm{v},L}$ is the environmental water vapour pressure.

Due to the ill-posed character of the problem, a regularised function is constructed by formulae:

$$J(\kappa) = \sum_{i=1}^{N_T}\sum_{j=1}^{N_{RH}}\left[RH^0_{\mathrm{cal}}(x) - RH^{0*}\right]^2 + \delta\cdot\kappa^2 \qquad (4)$$

where, N_T and N_{RH} are, respectively, the numbers of discretised subintervals of $[T_{\min},T_{\max}]$ and $[RH_{\min},RH_{\max}]$ used in the numerical calculation, RH^{0*} the empirical value of RH in a comfortable state, RH^0_{cal} the RH on the inner surface of the textile material numerically obtained by solving ***Equations 1*** and ***3*** with the finite difference method described in [14], and $\delta\cdot\kappa^2$ the penalty term introduced to suppress the instability.

By solving the IPTMD, we find κ^*, which satisfies:

$$J(\kappa^*) = \min J(\kappa),\ 0 < \kappa < 1 \qquad (5)$$

Numerical algorithm

Herein an MPCA is proposed to solve the IPTMD given by ***Equation 5***. The MPCA describes a stochastic process where solution states evolve out of random events and along a so-called Markov chain. Taking a case of constrained minimisation as an example:

$$\min f(\boldsymbol{x})$$

where,

$$\boldsymbol{x} = \{(x_i) | x_i \in R \text{ and } x_{i,\mathrm{InfLim}} \le x_i, \quad x_i \le x_{i,\mathrm{SupLim}}, i = 1,2,\cdots,n\}$$

is a configuration of variables, f a function, and f: $R^n \to R$.

Let $S(\boldsymbol{x_i})$ (i=1, 2, …) represent the solution states decided by configurations $\boldsymbol{x}_i = (x_{1i}, x_{2i}, \cdots, x_{ni})$ and Ω a solution space made up of all the possible solution states of the problem. By assuming that function $f(\boldsymbol{x})$ satisfies the transitive relation: $f(\boldsymbol{x}_a) < f(\boldsymbol{x}_b)$, $f(\boldsymbol{x}_b) < f(\boldsymbol{x}_c) \to f(\boldsymbol{x}_a) < f(\boldsymbol{x}_c)$ and each state $S(\boldsymbol{x_{i+1}})$ is solely dependent on the previous one $S(\boldsymbol{x_i})$, the optimum solution of the problem $\boldsymbol{x}^*$ is an extreme state of Ω with the property of $f(\boldsymbol{x}^*) < f(\boldsymbol{x})$ for any other $\boldsymbol{x_i}$ in Ω. With the MPCA, solution state S_i evolves from the initial state $S_0 = S(\boldsymbol{x}_0)$ to the optimum one $S^* = S(\boldsymbol{x}^*)$ along a specific path, namely, a Markov chain $S(\boldsymbol{x}_1)$, $S(\boldsymbol{x}_2)$, …. , $S(\boldsymbol{x}^*)$ in Ω where $f(\boldsymbol{x}_0) > f(\boldsymbol{x}_1) > f(\boldsymbol{x}_2) > \ldots > f(\boldsymbol{x}^*)$.

The flow chart of the MPCA is given in ***Figure 1*** (see page 42).

In comparison to the original PCA [17, 18], the MPCA proposed here has a simpler yet effective schedule.

To verify the validity of the MPCA, it is applied to global optimisation test functions mentioned in reference [17-19]. Correct results were obtained for all the cases. Here the results for the classical Rosenbrok's function are presented for illustration. The Rosenbrok's function is formulated as:

$$z(x, y) = (1 - x)^2 + 100(y - x^2)^2$$

The global optimum of the function, $(x^*, y^*) = \min z(x,y)$ is located at (1, 1) with a function value of 0 and lies in a long, narrow, parabolic shaped valley.

The MPCA is used to solve the problem of $\min z(x, y)$. One hundred independent runs are implemented with random initial values in the range of [-2, 2]. The total number of function evaluations in global and local searches, the optimum solution (x^*, y^*) and optimum function value f^* are recorded. The statistical results are shown in ***Table 1***.

Table 1. *Optimisation results of Rosenbrok's function yielded by the MPCA.*

Search step	No. of function evaluation			(x*, y*)	f*
	Average	Max	Min		
(1e-3, 1e-3)	11044	23574	841	(1.000 ± 5e-4, 1.000 ± 1e-3)	5e-7 ± 3e-7
(1e-1, 1e-1)	222142	1081390	831	(1.000 ± 5e-4, 1.000 ± 1e-3)	5e-7 ± 3e-7

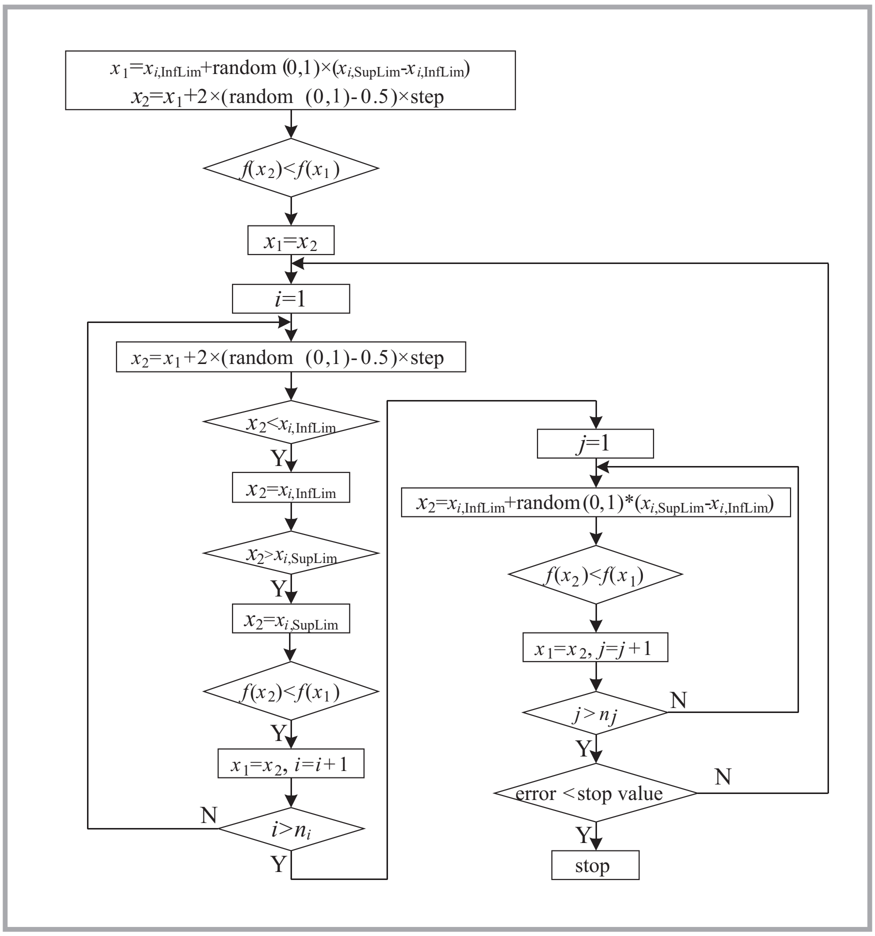

***Figure 1.** Flow chart of the MPCA.*

***Table 2.** Optimisation results of Rosenbrok's function yielded by the HJA.*

(x_0, y_0)		(2, 2)	(-2, -2)	(2, -2)	(-2, 2)
(x^*, y^*)	step=1e-3	(6.680, 43.876)	(1.000, 1.000)	(6.800, 41.405)	(1.044, 1.091)
	step=1e-1	(1.841, 3.392)	(1.000, 1.000)	(2.887, 8.338)	(1.000, 1.000)

By means of the MPCA, all the simulations converge to the global optimum irrespective of the initial values and search steps.

For comparison, the HJA is used to solve the same problem. Simulations are carried out with the following parameters: truncation error 10^{-6}, acceleration rate 1.2 and deceleration rate 0.8. The initial values are chosen in the domain of [-2, 2]. The simulation stops when the search step is lower than the truncation error. The numerical results are listed in ***Table 2***.

The results show that the convergence of the HJA is sensitive to the initial values as well as search steps. It is difficult for the HJA to yield a correct result for the global optimum. More examples which illustrate difficulties of the convergence to the global optimum of the Rosenbrok function are given in reference [22].

The above shows that the MPCA is applicable as an effective tool to solve complex optimum problems such as the IPTMD for single-layer textile materials at low temperature.

Simulations and results

In this section numerical simulations are carried out for two cases of environmental conditions under low temperature:

Case 1:

$(T_1, RH_1) \in [-15\ °C, 0°C]\times[40\%, 70\%]$

Case 2:

$(T_2, RH_2) \in [0\ °C, 15\ °C]\times[40\%, 70\%]$

In simulations, the above intervals of T_i and RH_j (i,j = 1, 2) and the thickness of the textile material L are discretised into 10, 10 and 20 isotropic subintervals, respectively. Simulations are carried out using the following parameters: $k_1 = 2\times10^{-3}$, $k_2 = 1\times10^{-5}$, $r = 10^{-5}$ m, ε = 90%, τ = 1.2, $\lambda = 2260\times10^3$ J·kg^{-1}, $\delta=10^{-4}$, search step 10^{-4}, and truncation error 10^{-6}, and under the following boundary conditions:

$m_v(0) = 3.3084\times10^{-5}$ kg·m^{-2}·s^{-1},

$T(0)$ = 305.15 K &

RH^{0*} = 50%.

Thermal conductivity data used for determination of textile materials are listed in ***Table 3*** [24].

Textile material determination is performed in two steps:

Step 1: For a defined thickness, the optimum thermal conductivity is yielded by the IPTMD and MPCA.

Step 2: The textile material with the thermal conductivity closest to the optimum value is evaluated from ***Table 1*** and is suggested as the optimum to be used in the case under consideration.

Repeat steps 1 and 2 until the optimum thermal conductivities and textile materials are determined for a series of thicknesses.

Results for Case 1: Under the environmental condition of $T_1 \in$ [–15 °C, 0 °C], $RH_1 \in$ [40%, 70%], the optimum thermal conductivity for a textile material with a thickness of 8.10 mm is found to be 0.02395. In ***Table 1***, down has a thermal conductivity of 0.024, which is the value closest to 0.02395, hence down is suggested as the optimum textile material in this case. Similarly for a textile material with a thickness of 9.90 mm, PET is the material suggested. For case 1, the optimum thermal conductivities and textile materials are determined for a thickness range of [8.10, 9.90], the results of which are listed in ***Table 4***.

Results for Case 2: Similar to case 1, under environmental condition $T_2 \in$ [0 °C, 15°C] & $RH_2 \in$ [40%, 70%], the optimum thermal conductivities and textile materials are determined for a thickness range of [5.85, 7.00]. In this case, down and PET are suggested to be used for textile materials with a thickness of 5.85 mm and 7.00 mm, the results of which are listed in ***Table 5***.

In both cases, the MPCA suggested convergence to the global optimum in all simulations and the IPTMD yields reasonable results for textile materials with a series of thicknesses under two envi-

Table 3. Thermal conductivity for some textile materials.

Textile material	κ, W·m-1·K-1
Down	0.024
PVC	0.042
PAN	0.051
Wool	0.052 ~ 0.055
Viscose	0.055 ~ 0.071
Cotton	0.071 ~ 0.073
PET	0.084

Table 4. Simulation results of the IPTMD by using the MPCA for Case 1, where $T_1 \in [-15\ °C, 0\ °C]$, $RH_1 \in [40\%, 70\%]$.

Thickness, mm	Optimum κ, W·m-1·K-1	Textile material suggested
8.10	0.02395	Down
9.00	0.04232	PVC
9.25	0.05039	PAN
9.35	0.05413	Wool
9.50	0.06065	Viscose
9.70	0.07101	Cotton
9.90	0.08395	PET

Table 5. Simulation results of the IPTMD by using the MPCA for Case 2, where $T_2 \in [0\ °C, 15\ °C]$, $RH_2 \in [40\%, 70\%]$.

Thickness, mm	Optimum κ, W·m-1·K-1	Textile material suggested
5.85	0.02442	Down
6.40	0.04225	PVC
6.55	0.04986	PAN
6.60	0.05279	wool
6.75	0.06292	viscose
6.85	0.07098	cotton
7.00	0.085	PET

ronmental conditions. This suggests that the IPTMD is reasonably defined and the MPCA is an effective tool in solving complex optimisation problems such as the IPTMD.

Conclusions

An IPTMD for single-layer textile materials at low temperature is presented on the basis of the physical nature of steady heat and moisture transfer in a human body-clothing-environment system.

A hybrid stochastic algorithm MPCA is proposed to solve the IPTMD. The validity and global convergence of the algorithm are verified by application to various global optimisation test functions.

Numerical simulation results of the IPTMD proved the suitability of the IPTMD and effectiveness of the MPCA in solving complex global optimisation problems.

The modelling method and optimisation algorithm used in this paper can be further extended to the design of multi-layer textile materials. They offer a theoretical basis and opportunity to design clothing which meets comfort requirements prior to making actual samples.

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