

Vibrations of a Rotating System Containing a Textile Element

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Abstract

The study examined the behaviour of a rotating system containing a textile element. A mathematical model is formulated and a numerical investigation is performed. It has been found that the layer of fibres limits the mass motion and stabilises the rotary motion, thereby preventing a decrease in the speed of the system.

Key words: nonlinear vibration, rotating body, fibre compression.

denotes the coordinate of the slider along the bar and w_0 the static eccentricity.

H) are the resilient element material constants, as defined in work [5].

The sum of projections of forces on the w -axis, that of the moments of forces with respect the axis of rotation and the equation of motor torque have form of the set of **Equation 1**.

With the non-linear elastic force and negative centrifugal force (2) we can associate energy function (3).

$$(B + m(v_0 + v)^2) \frac{d^2\alpha}{dt^2} + 2m \frac{d\alpha}{dt} \frac{dw}{dt} (w_0 + w) - M = 0,$$

$$F_w = \frac{\partial E}{\partial w} = \frac{k w}{\left(1 - \frac{w}{L}\right)^3} - \left(\frac{d\alpha}{dt}\right)^2 m (w_0 + w), \quad (2)$$

$$m \frac{d^2 w}{dt^2} - m \left(\frac{d\alpha}{dt}\right)^2 (w_0 + w) +$$

$$E = \frac{k}{2} \frac{w^2}{\left(1 - \frac{w}{L}\right)^2} - \frac{1}{2} m \left(\frac{d\alpha}{dt}\right)^2 (w_0 + w)^2, \quad (3)$$

$$+ \frac{c \operatorname{sgn}\left(\frac{dw}{dt}\right) \left(\frac{dw}{dt}\right)^2}{\left(1 - \frac{w}{H}\right)^3} + \frac{k w}{\left(1 - \frac{w}{L}\right)^3} = 0,$$

$$T \frac{dM}{dt} = C \left(\Omega - \frac{d\alpha}{dt} \right) - M.$$

Here, α is the angle of rotation of the bar, (T, C, Ω) the motor constants, and $(k, L, c,$

Numerical results and discussion

The set of **Equations 1** was numerically integrated using the Runge-Kutta method for mass $m = 5$ kg, inertia $B = 0.001$ kgm², eccentricity $w_0 = 0.001$ m, motor idle speed $\Omega = 100$ rad/s, motor

Introduction

The behaviour of fibre assemblies under compression has been studied in works [1 - 3]. In work [4] the author proposed a mathematical model of a layer of fibres submerged in a fluid. The properties of the layer were assumed to be determined by the bending elasticity of fibres and the resistance to fluid flow that is squeezed out of the layer. In paper [5] it was shown that the textile layer significantly restricts the amplitude of vibration of a supported mass. The application of textile items in supports of rotating bodies was studied in paper [6]. The stability of rotation of a disc carrying a mass supported with a cubic spring and moving along its diameter was investigated in works [7, 8]. In this paper, the behaviour of a mass moving along the groove of a rotating rod containing a textile element is studied. The paper is a continuation of works [6, 7].

Equations of motion

The system, shown in **Figure 1**, consists a rotating bar with a moment of inertia of B driven by motor torque M , a slider of mass m that can move along the bar in a groove and a textile resilient support. w

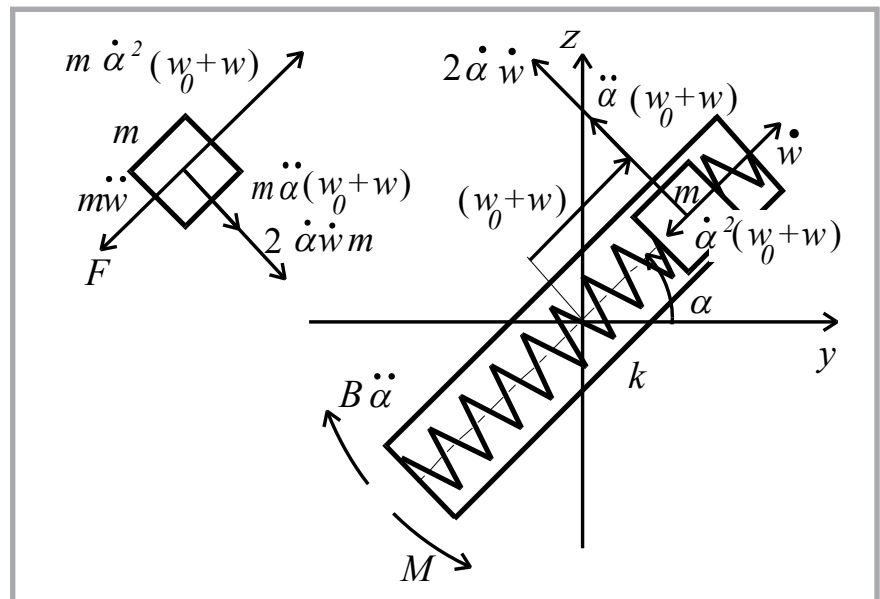


Figure 1. Mass moving along the rotating bar in a groove.

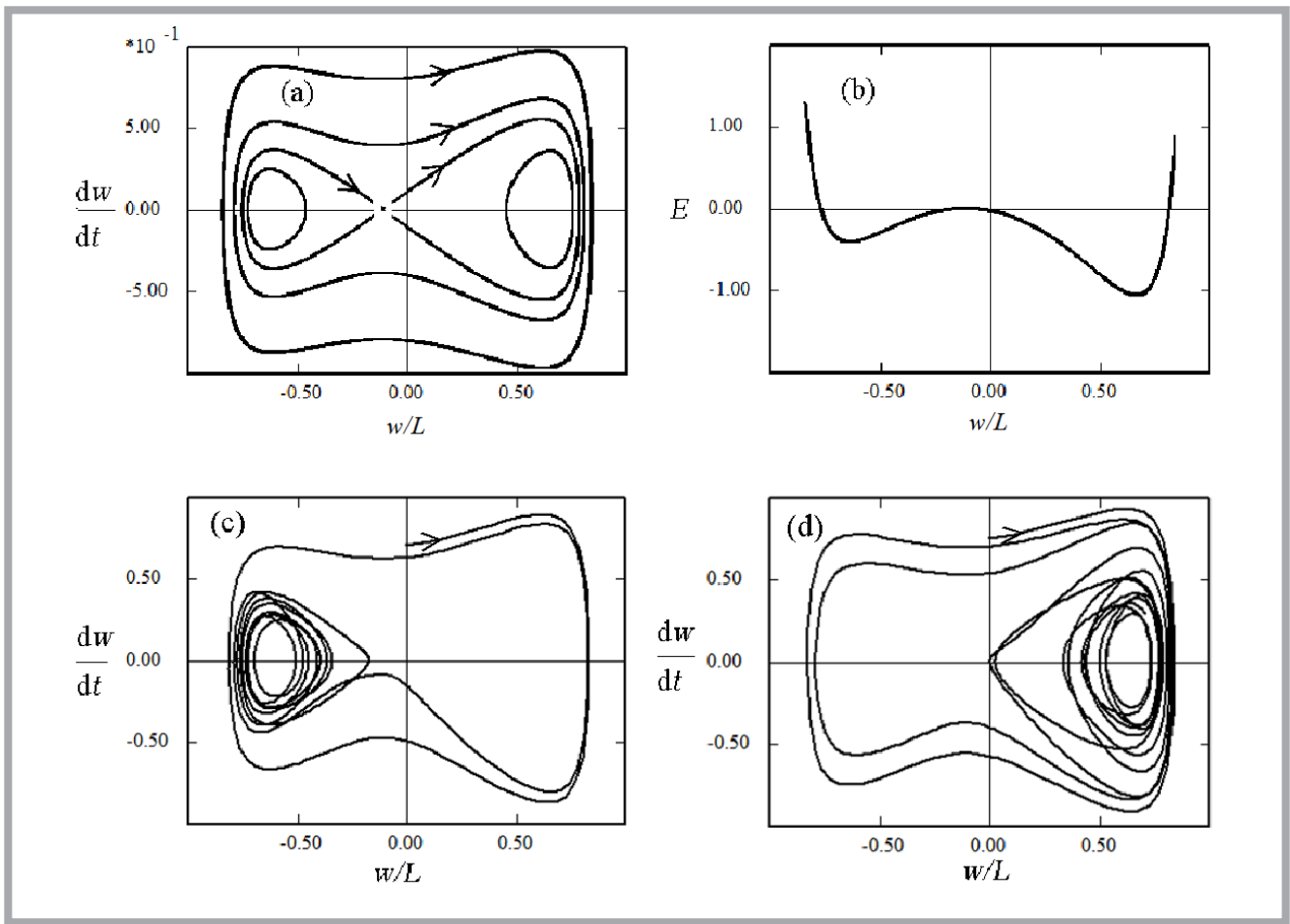


Figure 2. Radial velocity of mass dw/dt in m/s versus its displacement w : (a) for $(c = 0, C = 0)$ and various initial conditions; (c) for $(C = 1 \text{ kgm}^2/\text{s}, c = 0.05 \text{ kg/m}, w(0) = 0, dw/dt = 0.7)$; (d) for $(C = 1 \text{ kgm}^2/\text{s}, c = 0.05 \text{ kg/m}, w(0) = 0, dw/dt = 0.75)$; (b) energy E in Nm for $(c = 0, C = 0)$.

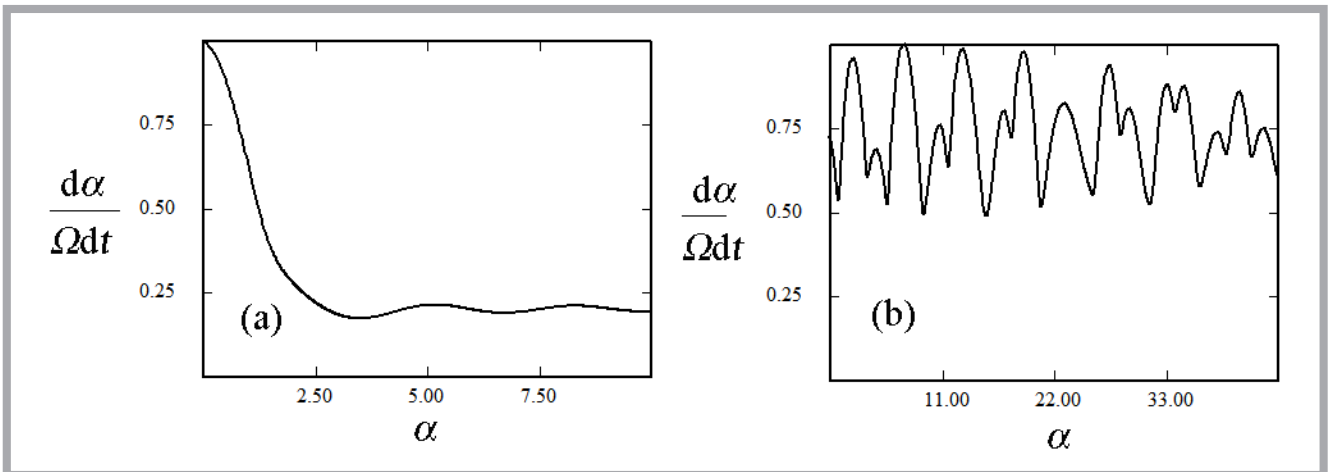


Figure 3. Angular velocity of rotating disc $d\alpha/dt$ versus the angle of revolution α in rad for (a) a linear spring of stiffness k and (b) for layer of fibres of stiffness $k/(1-w/L)^3$.

time constant $T = 2\pi/\Omega$, material parameters $H = 0.01 \text{ m}$, $L = 0.01 \text{ m}$, and $k = \Omega^2 m/25 = 2000$. The initial conditions were taken as follows: $\alpha = 0$, $d\alpha/dt = \Omega$, $M = C(\Omega - d\alpha/dt)$ and various values of w and dw/dt . In **Figure 2** the radial velocity dw/dt of mass and energy E versus the mass displacement w are shown. **Figures**

2.a and **2.b** show the results for a freely rotating system ($C = 0$) without damping ($c = 0$), and **Figures 2.c** and **2.d** show the results for the motor characteristic slope $C = 1 \text{ kgm}^2/\text{s}$ and damping coefficient of the material $c = 0.05 \text{ kg/m}$. The energy plot shown in **Figure 2.b** is bounded by vertical asymptotes at $w/L = \pm 1$. The an-

gular velocity $d\alpha/dt$ of the rotating disc versus the angle of revolution α is shown in **Figure 3**, (a) for a linear spring of stiffness k and (b) for a layer of fibres of stiffness $k/(1-w/L)^3$. It can be seen that the angular velocity could be maintained for a nonlinear restoring force, while it was not the case for a linear spring.

The behaviour of the nonlinear system can be understood by looking at the energy curve in **Figure 2.b**. The energy E has three extreme points that represent the state of equilibrium. There is one maximum and two minimums, one deeper than the other. The maximum represents an unstable state of equilibrium. The deeper minimum represents a stable equilibrium state. The other minimum is metastable, that is stable for sufficiently small disturbances. If, however, the disturbance goes over the maximum, then the point will move to the deeper minimum. The corresponding trajectories in the phase space are shown in **Figure 2.a**, where we see three types of closed curves representing periodic orbits, and two not closed homoclinic orbits. The behaviour of the system subjected to a damping force and driven by an electric motor is shown in **Figures 2.a** and **2.b**. It can be seen that the system when starting from a slightly different initial condition moves toward one of two attracting points associated with energy minimums. The results shown in **Figure 2** resemble the behaviour of the solutions of Duffing's equation.

Conclusions

1. As opposed to a linearly elastic spring, the fibrous element limits the

mass motion, which stabilizes the rotation of the system, not allowing for a decrease in the rotational speed of the driving motor.

2. The rotating system containing a textile element that can move along its diameter exhibits three states of equilibrium: metastable, unstable and stable.
3. Restricting the motion on the rotating plane [6] to that along the line on the rotating body changed the unstable saddle to a meta-stable attractor.
4. The model of textiles under compression analysed can be applied for studying the behaviour of fabric handling by manipulators [9, 10].



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