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# Probabilistic Model of the Fatigue Durability of Knitted Fabrics Produced from Standard Smooth and Fancy Flame Cotton Yarns

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**Abstract**
*The possibility of the introduction of probabilistic models based on Weibull's theory applied to modelling the problems of fatigue durability and reliable structures as well as models based on log-normal distribution is presented. It was found that the assessment of the quality of knitted fabrics on the basis of classic static durability is not sufficient and it can often lead to incorrect conclusions, having no grounds in real conditions of use. The question of the fatigue durability of knitted fabrics produced from standard smooth and fancy flame cotton yarns was considered. Quantitative description by means of probabilistic and statistical models was proposed. Attention was mainly given to working out probabilistic models based on graphic methods, enabling verification of experimental data.*

**Key words:** *knitted fabric, fatigue durability, probabilistic model, Weibull's expansion, log-normal expansion.*

## Introduction

Recently the production of clothing articles has not only been dependent on their functionality and destination, but it has also been created by designers of fashion. Assortment enriching is achieved by the production of clothing articles whose surfaces are colourful and of a complex structure [4, 5].

The introduction of modification to existing production technologies makes possible their adaptation to the production of a wide assortment of yarns with programmed changes of structure [4]. They give special paternal and textural effects to clothing articles and also expand possibilities in the range of their production. Interest in decorative yarns also results from the current tendencies of fashion promoting clothing articles of a well-developed spatial structure achieved by decorative effects of yarns (flames, flickers, flies, etc.). The production of thin fancy yarns poses a huge challenge for manufacturers. Probabilistically appearing faults in yarn make up a substantial source of breaks during the production of finished articles. Weak places appearing in the finished articles comprise a risk. The classic methods of the assessment of clothing articles can lead to incorrect conclusions, particularly in real use conditions [1, 2, 4, 7 - 9]. Fatigue destruction is one of the forms of the destruction of textile material most often observed. It is very dangerous because it is usually unexpected. In the course of the development of experimental methods for the strength of materials, it was noticed that textiles often undergo destruction at considerably lower stresses than the strength of a given material defined on the basis of static tests [9, 10]. Such destructions (cracks, for example) proceed without any perceptible plastic deformations. Among other factors, the imperfect elasticity of material is the cause of this damage [2, 3, 15].

Analysing the problems presented, it can be noticed that the issue of the fatigue strength of knitted fabrics produced from standard smooth and fancy flame cotton yarns is very significant from the point of view of further processing these yarns. Therefore we decided on closer examination of problems connected with the assessment of the fatigue strength of knitted fabrics by way of a probabilistic model.

## Materials

### Parameters of yarns used for investigations

Standard smooth and fancy flame cotton yarns of 25 tex linear density were used for the production of plain stitch knitted fabrics. The verification of probabilistic models was conducted for yarns of 50 tex linear density. The linear densities and numbers of twist of the yarns used to produce the knitted fabrics are presented in ***Table 1***. The characteristic of static strength of the yarns mentioned is presented in ***Table 2*** (see page 76).

### Parameters of knitted fabrics used for investigations

Knitted plain stitch fabrics designated for investigations were produced on an experimental work-stand of a circular sinker top machine with the number of needling equal to 16 and number of locks - 1. Investigations of the static strength of the yarns mentioned were conducted using an Instron 5544 tensile tester equipped with Merlin software and Test Profiler. The distance between the clamps of the tensile tester amounted to 100 mm. Measurements were made at a constant rate of elongation (CRE), meanwhile the width of the samples of knitted fabrics equaled 50 mm. Characteristics of parameters of the knitted fabrics produced are given in ***Tables 3*** and ***4*** (see page 76).

Analysing the results of the investigations presented in ***Table 3***, it can be noticed that among the physical proprieties of knitted plain stitch fabrics produced

***Table 1.*** *The linear masses and numbers of twist of yarns used to producing the knitted fabrics [16 - 20].*

| Parameter analysed | Symbol | Unit | Smooth yarn | | Fancy yarn | |
|---|---|---|---|---|---|---|
| | | | 25 tex | 50 tex | 25 tex | 50 tex |
| Linear mass | $Tt_n$ | tex | 24.77 | 51.47 | 26.06 | 57.65 |
| Coefficient of variation of mass of yarn determined from segments of 200 m | $V(Tt_n)$ | % | 11.31 | 9.10 | 28.65 | 51.25 |
| Number of twist | $T_{mn}$ | rev/m | 676.9 | 485.8 | 920.7 | 727.5 |
| Coefficient of variation of the number of twist | $V(T_{mn})$ | % | 3.42 | 4.99 | 5.83 | 4.57 |
| Number of twist in decorative effect | $T_{m\,ef}$ | rev/m | - | - | 87.5 | 82.5 |
| Coefficient of variation of the number of twist in decorative effect | $V(T_{m\,ef})$ | % | | | 20.51 | 31.22 |

*Table 2. Characteristic of static strength of yarns used to produce the knitted fabrics[16 - 20].*

| Parameter analysed | Symbol | Unit | Smooth yarn | | Fancy yarn | |
|---|---|---|---|---|---|---|
| | | | 25 tex | 50 tex | 25 tex | 50 tex |
| Average value of breaking force | $\overline{R}_m$ | cN | 363.65 | 716.21 | 340.78 | 518.79 |
| Coefficient of variation of breaking force | $V(R_m)$ | % | 8.31 | 5.86 | 9.93 | 8.04 |
| Breaking tenacity | $\overline{W}_t$ | cN/tex | 14.55 | 14.32 | 13.63 | 10.38 |
| Relative mean value of breaking elongation | $\overline{\varepsilon}_r$ | % | 7.00 | 6.60 | 6.86 | 5.40 |
| Coefficient of variation of breaking elongation | $V(\varepsilon_r)$ | % | 5.76 | 5.27 | 8.91 | 8.05 |
| Modulus of initial elasticity | $\overline{M}p$ | cN/tex | 2.51 | 2.85 | 2.64 | 2.15 |
| Coefficient of variation modulus of initial elasticity | $V(Mp)$ | % | 22.43 | 17.13 | 12.72 | 19.17 |

*Table 3. Characteristic of physical parameters of knitting fabrics.*

| Parameter analysed | Symbol | Unit | Knit fabric from smooth yarn | | Knit fabric from fancy yarn | |
|---|---|---|---|---|---|---|
| | | | 25 tex | 50 tex | 25 tex | 50 tex |
| Length of the thread in needle loop | L | mm | 4.9 | 4.5 | 3.9 | 4.1 |
| Number of courses | $n_r$ | 1/10 cm | 111 | 142 | 126 | 157 |
| Number of wale | $n_k$ | 1/10 cm | 99 | 93 | 105 | 91 |
| Thickness of knitted fabrics | d | mm | 0.60 | 0.86 | 0.83 | 1.20 |
| Contractibility after washing along courses | $\lambda_r$ | % | 6.6 | 10.0 | 6.3 | 13.8 |
| Contractibility after washing along wale | $\lambda p_r$ | % | 9.8 | 4.2 | 22.0 | 18.4 |
| Resistance to threads snagging along wales | $Z_k$ | - | 4 | 4-5 | 4 | 4-5 |
| Relative durable elongation of knitted fabrics under tension in the wale direction | $\varepsilon_t$ | % | 13.9 | 4.1 | 14.5 | 4.2 |
| Relative total elongation of knitted fabrics under tension in the wale direction | $\varepsilon_c$ | % | 25.5 | 7.8 | 27.6 | 9.4 |
| Relative elastic elongation of knitted fabrics under tension in the wale direction | $\varepsilon_s$ | % | 10.6 | 3.7 | 13.1 | 5.2 |
| Strength at the puncture of the ball | P | daN | 26 | 61 | 24 | 42 |

*Table 4. Characteristics of parameters of static strength of knitted fabrics about plain stitch [4].*

| Strength investigations of knitted fabrics were executed on samples with dimensions of about 100 × 50 mm | | | | | | |
|---|---|---|---|---|---|---|
| Parameter analysed | Symbol | Unit | Knit fabric from smooth yarn | | Knit fabric from fancy yarn | |
| | | | 25 tex | 50 tex | 25 tex | 50 tex |
| Mean breaking force along courses | $\overline{R}_{mr}$ | N | 110.05 | 268.97 | 80.94 | 397.20 |
| Mean breaking force along wale | $\overline{R}_{mk}$ | N | 196.60 | 460.10 | 188.70 | 421.29 |
| Relative mean breaking elongation along courses | $\overline{\varepsilon}_{rr}$ | % | 118.05 | 115.51 | 185.00 | 114.93 |
| Relative mean breaking elongation along wale | $\overline{\varepsilon}_{rk}$ | % | 141.71 | 134.60 | 86.30 | 110.12 |
| Modulus of initial elasticity along courses | $\overline{M}p_r$ | MPa | 0.083 | 0.075 | 0.196 | 0.089 |
| Modulus of initial elasticity along wale | $\overline{M}p_k$ | MPa | 0.140 | 0.121 | 0.167 | 0.930 |

from fancy flame cotton yarns in relation to knit fabrics produced from standard smooth cotton yarns, the vast majority of them underwent change. The use of fancy flame yarns caused a decrease in the resistance of knitted fabrics to ball bursting and growth of their contractibility after washing. Their usable proprieties worsen in effect.

Analysing the results of the investigations presented in ***Table 4***, it can be generally found that in the case of applying fancy flame cotton yarns to producing knitted fabrics, in relation to knitted fabrics produced from standard smooth cotton yarns, the static strength undergoes a decrease, whereas the modulus of initial elasticity grows .

## Methodology of investigations

Knitted fabrics produced from fancy flame cotton yarns as well as standard smooth cotton yarns were subjected to changeable cyclic fatigue loads till the moment of their total destruction. Investigations of the fatigue strength were conducted at ten levels of strain, carrying out 10 positive repetitions of tests for each of the levels defined. Tests in which a crack was detected were accepted as the criterion of destruction. The investigations of the fatigue strength of knitted fabrics were conducted using an Instron 5544 tensile tester equipped with Merlin software and Test Profiler. The distance between the clamps of the tensile tester amounted to 100 mm, whereas the width of the samples equaled 50 mm.

Wöhler's diagram is a classic diagram enabling the determination of the fatigue of a material. The procedure of obtaining the graph mentioned ran as follows: Samples of knitted fabrics produced from smooth yarns as well as fancy yarns were subjected to changeable cycles $\sigma_m$ until the sample underwent total destruction (break). The largest value of the maximum stress not generating the destruction (break) was defined as unrestricted fatigue durability $N_{unrest.}$ (cycles). The investigation was interrupted after achieving the quantity defined. The graphic way of drawing Wöhler's curve is presented in ***Figure 1***.

Changing the values of the amplitudes of the cycle load $\sigma_a$ in consecutive measuring series, where $\sigma_{max}$ was a fixed value, a search for a causal relationship between the amplitude of the cycle load and the number of cycles $N$ leading the knitted fabrics investigated to fatigue failure was conducted. The investigations of fatigue durability were carried out and assessed at a constant amplitude of loads $\sigma_a$ during each cycle (loading-unloading) and stable frequency of the cycle $fc$ = 4 Hz, which approximately reflects existing conditions in the majority of technological processes [9, 10]. In literature of the subject, it was found that investigations dedicated to the fatigue strength of textile articles had been conducted in the range

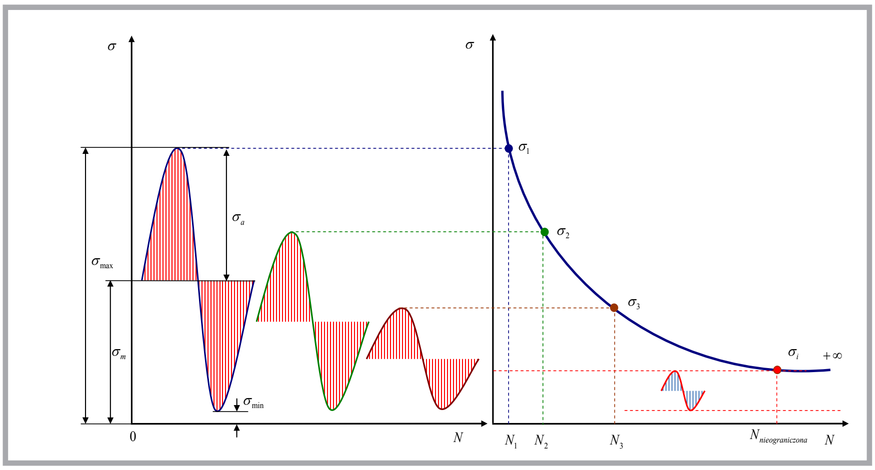


***Figure 1.** Scheme of preparing the Wöhler's graph of studied knitted fabrics.*

of frequency $fc$ = 1 - 5 Hz [1, 7, 8] and higher [14].

During the fatigue investigations, the following constant quantities were accepted [6]:

- $\sigma_{min}$ - minimal load in each cycle,
- $\sigma_{max}$ - maximal load in each cycle,
- $\sigma_m$ - average load of cycle - $\sigma_m =$

$$= \frac{\sigma_{max} + \sigma_{min}}{2} \qquad (1)$$

Before the beginning of the investigations, the range of amplitude of the load of the cycle - $\sigma_a$ at which investigations of fatigue durability were performed was determined (***Figure 1***)

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \qquad (2)$$

The minimal $\sigma_{min}$ and maximal $\sigma_{max}$ cycle load was programmed with an Instron 5544 tensile tester and Test Profiler software. The average value of the modulus of initial elasticity of knitted fabrics from standard smooth cotton yarns $\overline{Mp}_r$ and from fancy flame cotton yarns $\overline{Mp}_k$ was arbitrarily accepted as the minimum value of the load of the cycle $\sigma_{min}$:

$$\_\sigma_{min} = \overline{M}p_r = \overline{M}p_k = const \qquad (3)$$

determined from 10 measurements conducted for every knitted fabric.

The maximal stress of the cycle $\sigma_{max}$ was calculated on the basis of the average stress breaking knitted fabrics produced from standard smooth yarn $\sigma_{dn}$ and knitted fabric produced from fancy flame yarn $\sigma_{dp}$. The coefficient of the real strain of the cycle $\sigma_s$ dependent on the level of stresses in a given cycle was introduced to characterise the maximum stress of the cycle. It was arbitrarily assumed that the fatigue durability would be determined at the ten levels of maximal stress [4, 6]:

$$\sigma_s = [0.95 \cdot \bar{\sigma}\ 0.85 \cdot \bar{\sigma}\ 0.75 \cdot \bar{\sigma}\ 0.65 \cdot \bar{\sigma}\ 0.55 \cdot \sigma\ 0.50 \cdot \bar{\sigma}\ 0.40 \cdot \bar{\sigma}\ 0.30 \cdot \bar{\sigma}]^T, \text{MPa}$$

The coefficient of the real stress of the cycle $\sigma_s$ was accepted arbitrarily on the basis of earlier experiments [5]. In the case of fatigue investigations at the strain $\sigma_{max} > 0.98 \cdot \sigma$ in MPa, above 99% of samples of knitted fabrics underwent destruction at the number of fatigue cycles $N_p \leq 1$ and $N_n \leq 1$ (where: $N_p$ - number of fatigue cycles determined for knitted fabrics produced from fancy yarns, $N_n$ - number of fatigue cycles determined for knitted fabrics produced from smooth yarns). Hence the maximum upper value of the load of the cycle was accepted at the level $\sigma_{max} = 0.98 \cdot \sigma$ in MPa. Thus this range was close to that of stresses causing the failure of knitted fabrics. In turn, the acceptation of the bottom value of the cycle load $\sigma_{max} = 0.30 \cdot \sigma$ resulted from the fact that the fatigue durability was close to the range of fatigue limit strength considered as the fatigue strength of the material causing its unrestricted work at periodically changeable loads. Investigations were carried out for the run of changeable, one-sidedly positive tensing loads, making the assumption that $\sigma_a \neq 0$, $\sigma_m > 0$, $\sigma_{min} > 0$, as well as $f_c = 4$ Hz.

Phenomena taking place during the process of destroying standard smooth cotton yarns as well as fancy flame yarns have a remarkably statistical character in conditions of cyclic loads. We can generally define these phenomena as fatigue phenomena and the changes occurring we can define as fatigue changes, which successively develop until the total destruction of the material. Then we can talk about the fatigue of the given material. The course of changeable loads usually has a random character dictated by conditions of use of the material. Knowledge of criteria of the fatigue durability of yarns has a large practical significance because it makes possible the determination of the strength or fatigue durability of these yarns produced in various technological conditions. The classic methods of assessment of fatigue durability are mainly estimated by means of statistical methods. Probabilistic models are applied very seldom on account of very complex mathematical apparatus. However, we can find probabilistic models based on many types of distributions (exponential, Weibull's, normal, Gumbel's, Ferecht's, Reyleight's, Gamma and log-normal) [1, 3, 4, 6, 7, 14]. The distributions presented are also suitable for assessment of the static and fatigue durability of materials from outside the field of the textile industry. However, applying advanced numeric techniques and conducting investigations on large groups is required for the majority of these distributions. This considerably raises the price of applying these methods in practice. In the course of the assessment of the fatigue durability of a given material, we should a priori assume that the probabilistic model applied allows to characterise changes occurring during strength tests.

It was established as a result of conducting preliminary investigations [4] that in the majority of cases the dispersions of fatigue durability fulfilled the log-normal distribution or one similar to Weibull's in the range of the given fatigue durability. It was arbitrarily assumed that the fatigue durability of the knitted fabrics considered may be described by the log-normal distribution and also Weibull's distribution. In order to verifywhether the distributions determined have a log-normal or Weibull character, a so-called grid of probability was used. The log $N$ value was plotted on the axis of abscissa and the probability $P$ was plotted on the axis of ordinate. Thanks to this, a full fatigue graph was obtained from a probabilistic point of view, for co-ordinates $P$, log $N$, and for $\sigma_s = const$.

The results presented were arranged according to diminishing values and the probability of the destruction of sample $P$ was obtained in accordance with the following formula:

$$P = \frac{i - 0.5}{n} \cdot 100\ \% \qquad (4)$$

where:
$i$ – serial value of test, and $n$ – number of samples investigated at the accepted level of stresses.

The equation above takes into account the probability of fatigue destruction equal to 50% [6].

$$\sigma_S = [0.95\cdot\bar{\sigma}\ 0.85\cdot\bar{\sigma}\ 0.75\cdot\bar{\sigma}\ 0.65\cdot\bar{\sigma}\ 0.55\cdot\bar{\sigma}\ 0.50\cdot\bar{\sigma}\ 0.40\cdot\bar{\sigma}\ 0.30\cdot\bar{\sigma}]^T, \text{MPa}$$

Logarithms of the numbers of cycles to the failure of samples were settled in the sequence of growing values:

$$\log N_1 \le \log N_2 \le \ldots \le \log N_i \ldots \le \log N_n$$

for each stress $\sigma_S$. The results of investigations were presented in the coordinates system, $\sigma_S$, $N$ for $P = const$as diagrams $\sigma_S$, $N$, $P$ in the form of a family of fatigue curves relating to the probability of fatigue destruction assumed. The method of the largest likelihood was applied in the estimation of parameters of the probabilistic models. The method of the smallest squares is crucial among those commonly known and applied. The defect of the method of the smallest squares is the fact that it is too sensitive to observations "protruding" from remaining ones, which is why the results of estimation can be distorted. The basic premise of the use of the method of the largest likelihood is the fact that it is based on the function of likelihood (hence the name of the method). Moreover this method is very useful for models with an additive random component of an assumed type of distribution.

This method is based on the following idea: if the density of probability of the appearing random variable $E_n$ in observation $n$ at the parameters of distribution $\beta_0, \beta_1, ..., \beta_K$ equals $f(N_N|\beta_0, \beta_1, ..., \beta_K)$ accepting the independence of existence $E_1, E_2, ..., E_N$, we obtain the density of probability of the associative $N'$-dimension random variable in the following form:

$$f(N_1, N_2, \ldots, N_N | \beta_0, \beta_1, \ldots, \beta_K) = \prod_{n=1}^{N} f(N_n | \beta_0, \beta_1, \ldots, \beta_K) \qquad (5)$$

Then the function of density of probability in a given experiment is named as the likelihood function and it is denoted as $L$ [11, 12]:

$$L(N_1, N_2, \ldots, N_N; \beta_0, \beta_1, \ldots, \beta_K) = \prod_{n=1}^{N} f(N_n | \beta_0, \beta_1, \ldots, \beta_K) \qquad (6)$$

In the case of the estimation of unknown parameters $\beta_0, \beta_1, ..., \beta_K$ on the basis of the test $N_0, N_1, ..., N_K$ by the method of the largest likelihood, we can accept values $b_0, b_1, ..., b_K$ as estimators of these parameters. These values fulfill the following relationship:

$$f(N_0, N_1, ..., N_K| \beta_0, \beta_1, ..., \beta_K) = \max_{\beta_1,\ldots,\beta_K} L(N_1, \ldots, N_{N'}; \beta_0, \beta_1, \ldots, \beta_K) \qquad (7)$$

In order to determine optimal values $b_0, b_1, ..., b_K$, the partial derivatives of the function of likelihood $L$ in relation to parameters $\beta_K$ should be counted:

$$\frac{\partial L(\beta)}{\partial \beta_k} = \frac{\partial}{\partial \beta_k} \prod_{n=1}^{N} f(N_n | \beta_0, \beta_1, \ldots, \beta_K) \quad \text{for } k = 0, 1, ..., K \qquad (8)$$

and compared to the zero value. As a result we obtain the system of equations:

$$\frac{\partial}{\partial \beta_k} \prod_{n=1}^{N} f(N_n | \beta_0, \beta_1, \ldots, \beta_K) = 0 \quad \text{for } k = 0, 1, ..., K \qquad (9)$$

Reducing the system of equations obtained (8), we take a double-sided logarithm and we obtain:

$$\frac{\partial L(\beta)}{\partial \beta_k} = \frac{\partial}{\partial \beta_k} \text{h} \prod_{n=1}^{N} f(N_n | \beta_0, \beta_1, \ldots, \beta_K) \quad \text{for } k = 0, 1, ..., K \qquad (10)$$

We obtain the system of equations after their comparison with the zero value, equivalent to the system of equations (8), but in an additive form:

$$\frac{\partial(\beta)}{\partial \beta_k} \sum_{n-1}^{N} \ln f(N_n | \beta_0, \beta_1, \ldots, \beta_K) = 0 \quad \text{for } k = 0, 1, ..., K \qquad (11)$$

which makes possible easier determination of the value of optimal coefficients [11, 12].

We can describe fatigue durability using the log-normal distribution and Weibull's distribution. Numeric techniques were used to obtain distributions of the fatigue durability of standard smooth cotton yarns and fancy flame cotton yarns. Formal record of information gathered and processed, indispensable to draw the distributions assumed, was initially conducted using Microsoft Excel 2007 software. Detailed numeric analysis of fatigue durability was conducted by means of STATISTICA 9.0 software. In turn, verification of advanced hypotheses of the distributions of fatigue durability was conducted using STATISTICA 10.0 software. This programme made possible the selection of distributions of random variables. In order to verify hypotheses relating to the form of distribution, a level of confidence of 95% was accepted and Anderson's-Darling's test was applied, Being similar to Kołmogorow's-Smirnow's test. In order to demonstrate differences between the empirical cumulative distribution function and the model cumulative distribution function, Anderson's-Darling's test uses a different measure.

While the Kołmogorow's test relates to distance in the supreme sense, the Anderson's-Darling's test uses the mean-square distance. The statistic test has the following form [11 - 13]:

$$A^2 = n \int_{-\infty}^{+\infty} \frac{[F_n(x) - F(x)]^2}{F(x)[1 - F^*(x)]} f(x)\text{d}x \qquad (12)$$

This is the average weighed value of the squares of distances between the empirical and model cumulative distribution function. It is worth noting that if $x$ is very close to $d$ or $u$ then the weights of these quantities are very large (because of the size of the normative). For discreet data, the integral is reduced to the sum (***Equation 13***):

$$A^2 = -nF(x) + n \sum_{j=0}^{k} [1 - F_n(y_j)]^2 (\ln[1 - F(y_j)] - \ln[1 - F(y_j + 1)]) + n \sum_{j=0}^{k} F_n(y_j)^2 (\ln[F(y_j + 1)] - \ln[F(y_j)])$$

***Equation 13.***

Critical values are known and for 10%, 5% and 1% levels of confidence, they equal 1.933, 2.92 and 3.857, respectively. The following fact was found in consequence of the verification of hypotheses conducted of a form of the distributions assumed. Values of the approximate level of significance obtained indicated that there were no bases for the rejection of the hypothesis about the concordance of empirical distribution to each of the distributions checked. Verification of hypotheses about the distributions of the fatigue durability of knitted fabrics produced from standard smooth cotton yarns and fancy flame was performed in another step. The method of the largest likelihood was applied for the estimation of parameters of distribution.

Comparison of experimental data with the model assumed for selected distributions of probability was conducted by means of the graphic method. It was assumed that points relating to these data (within reasonable limits of error caused by the variation of testing) would create a straight line. In order to determine specific distributions, the functional grid relating to a given distribution (for example, the Weibull's distribution) was used. In practice, the functional grids of probability differ only in the scale indispensable for "straightening" the graph representing the model assumed. In the case of the Weibull's distribution, the functional grid was applied. If the model of this distribution is correct, then a line plotted on the functional grid of this distribution should create a straight line.

## Probabilistic model of the fatigue durability

**Probabilistic model based on the log-normal distribution for the fatigue durability of knitted fabrics produced from smooth and fancy flame cotton yarns**

The numeric procedure for determining the parameters of log-normal distribution by the method of the largest likelihood is presented below.

The density of log-normal distribution:

$$f(x;\upsilon,\zeta) = \frac{1}{\sqrt{2\Pi}\zeta X}\exp[-\frac{(\ln x-\upsilon)^2}{2\zeta^2}] \quad (14)$$

where: $\zeta$ - parameter of the shape, $\upsilon$ - parameter of the displacement.

$$\ln L(\upsilon,\zeta) = -\frac{-n}{2}\ln(2\Pi) - n\ln\zeta + \sum_{i=1}^{n}\ln x_i^{-1} - \sum_{i=1}^{n}\frac{(\ln x_i \upsilon)^2}{2\zeta^2} =$$
$$= \frac{-n}{2}\ln(2\Pi) - n\ln\zeta + \sum_{i=1}^{n}\ln x_i^{-1} - \frac{1}{2\zeta^2}\sum_{i=1}^{n}(\ln x_i - \upsilon)^2$$

***Equation 16.***

The likelihood function takes the following form:

$$(2\Pi)^{-\frac{n}{2}}\zeta^{-n}\prod_{i=1}^{n}x_i^{-1}\exp[-\sum_{i=1}^{n}\frac{(\ln x_i-\upsilon)^2}{2\zeta^2} =$$
$$=(2\Pi)^{-\frac{n}{2}}\zeta^{-n}\prod_{i=1}^{n}x_i^{-1}\exp[-\sum_{i=1}^{n}\frac{(\ln x_i-\upsilon)^2}{2\zeta^2} \quad (15)$$

Finding the logarithm of the likelihood function, we obtain, respectively ***Equation 16***.

We obtain two equations, where:

Equation I:

$$\frac{\partial \ln L}{\partial \upsilon} = -\sum_{i=1}^{n}\frac{2(\ln x_i - \upsilon)}{2\zeta^2}\cdot(-1) = 0 \quad (17)$$

Equation II:

$$\frac{\partial \ln L}{\partial \upsilon} = -\frac{n}{\zeta} - \frac{1}{2\zeta^3(-2)}\sum_{i=1}^{n}(\ln x_i \upsilon)^2 = 0 \quad (18)$$

The system of equations of the largest likelihood takes the following form:

$$\frac{1}{\zeta^2} = (\sum_{i=1}^{n}\ln x_i - n\upsilon) = 0 \quad (19)$$

$$\frac{1}{\zeta}(-n\frac{1}{\zeta^2}\sum_{i=1}^{n}(\ln x_i - \upsilon)^2) = 0 \quad (20)$$

and then:

$$\upsilon = \frac{1}{n}\sum_{i=1}^{n}\ln x_i \quad (21)$$

$$-\zeta^2 n + \sum_{i=1}^{n}\ln x_i - \upsilon)^2 =$$
$$= 0 \Rightarrow \zeta^2\frac{1}{n}\sum_{i=1}^{n}(\ln x_i - \upsilon)^2 \quad (22)$$

Then we solve the system of equations:

$$\hat{\upsilon} = \frac{1}{n}\sum_{i=1}^{n}\ln x_i \quad (23)$$

$$\hat{\zeta}^2 = \frac{1}{n}\sum_{i=1}^{n}(\ln x_i - \hat{\upsilon})^2 \quad (24)$$

We find the solution by means of numerical methods and we determine:

$$\hat{\zeta} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\ln x_i - \hat{\upsilon})^2} \quad (25)$$

The graphical test of the compatibility of distribution of the fatigue durability of the knitted fabrics analysed in the $P$% - log $N_N$ and P% - log $N_P$ systems with parameters of the level of stress:

$$\sigma_S = [0.95\cdot\bar{\sigma}\ 0.85\cdot\bar{\sigma}\ 0.75\cdot\bar{\sigma}\ 0.65\cdot\bar{\sigma}\ 0.55\cdot\bar{\sigma}\ 0.50\cdot\bar{\sigma}\ 0.40\cdot\bar{\sigma}\ 0.30\cdot\bar{\sigma}]^T, \text{MPa}$$

with log-normal distribution is presented in ***Figures 2 - 5*** (see page 80 - 81).

Analysing log-normal distributions of the fatigue durability of knitted fabrics produced from standard smooth and fancy flame cotton yarns presented in ***Figures 2 - 5***, it can be stated that:

- the numeric procedure of the method of the largest likelihood (formulas 5 -13) describing the type of log-normal distribution allows to obtain a linear diagram for all the knitted fabrics considered,
- the results of investigations in the form of points are grouped from both sides of the straight line of the diagram determined; however, this occurrence is caused by the inevitable dispersion of fatigue durability and the specifics of the material investigated,
- the distribution of fatigue durability for all the knitted fabrics analyzed can be described by log-normal distribution. This simultaneously confirms the fact that the fatigue failure of the knitted fabrics analysed proceeds in a random way across the spectrum of fatigue durability,
- the fatigue durability of knitted fabrics is subjected to considerable dispersions, but the larger dispersion of durability occurs in the course direction. This applies to knitted fabrics produced both from standard smooth and fancy flame yarns, larger dispersion occurs at loads below the average stress of the cycle $0.65\sigma_S$, dispersions of the border of fatigue bring about the necessity of applying different distributions for the description of the fatigue durability of knitted fabrics. The exponential distributions of probability are the most adequate in this case.

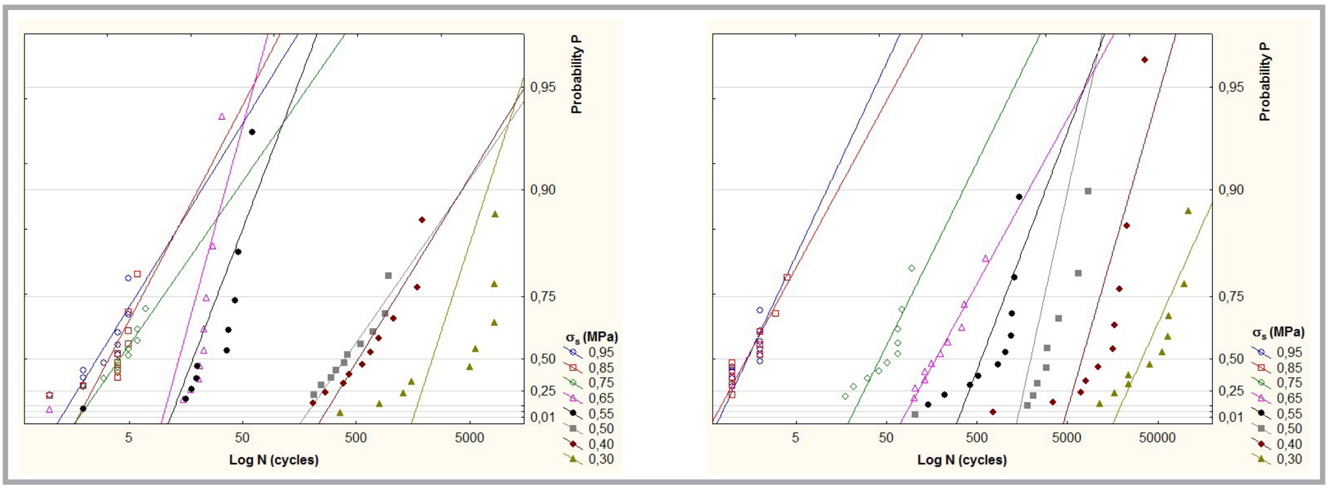


***Figure 2.*** *Log-normal distributions of fatigue durability determined along the wale (a) and courses (b) of knitted fabrics produced from standard smooth cotton yarn of 25 tex linear density.*

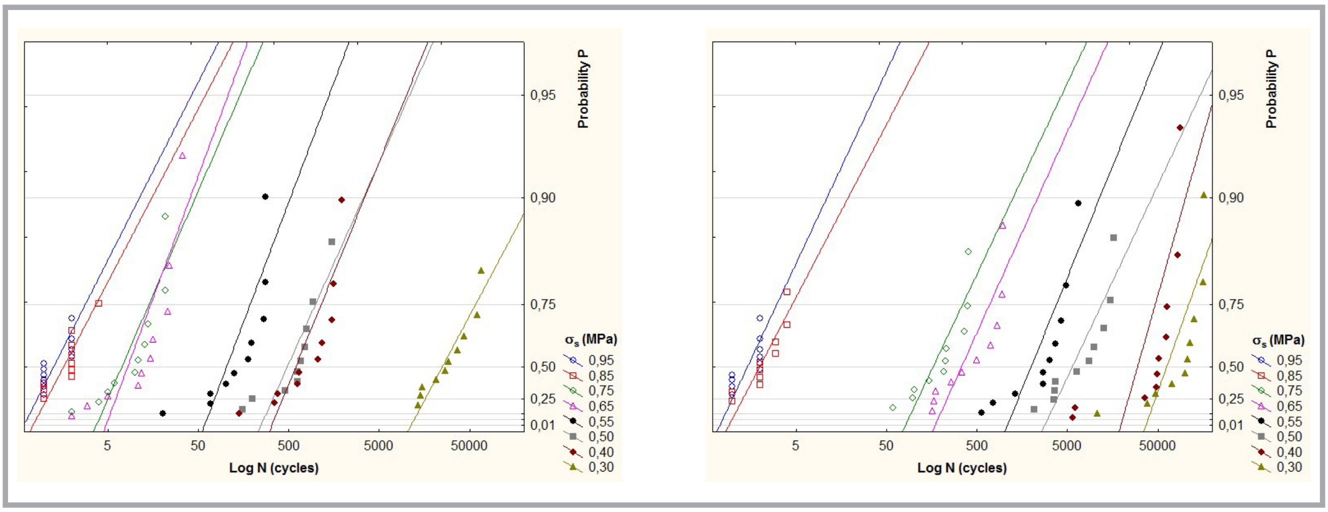


***Figure 3.*** *Log-normal distributions of fatigue durability determined along the wale (a) and courses (b) of knitted fabrics produced from cotton fancy flame yarn of 25 tex linear density.*

**Probabilistic model based on Weibull's distribution for the fatigue durability of knitted fabrics produced from smooth and fancy flame cotton yarns**

The characteristic feature of the function of probabilistic density of Weibull's distribution is the lack of a concrete shape. We obtain an exponential distribution for parameter $a = 1$, Rayleigh's distribution for parameter $a = 2$ and a normal distribution for parameter $a \approx 2.6$.

Using this property, we can obtain large agreement between theoretical values of distribution from experimental data. Weibull's distribution is often applied in the theory of reliability for mathematically modelling the distribution of the failure-free time of use of the object. It can also be applied for the assessment of the fatigue durability of knitted fabrics. In real conditions, in contrast to the data modelled, we can seldom afford to wait until the given constructional unit or material undergoes destruction. A model based on constant probability of failure is the initial one, taking into account random destruction of a given material; however, it does not take into account the factor of state of the material, which often depends on technological factors. Analysing the application of Weibull's distribution for the assessment of fatigue durability, the probability of the destruction of material possessing internal defects, which is recognised as "new" material, in relation to material having been exploited should be taken into account. Errors not detected in the assessment of material between technological processes are revealed in the first case. The second factor illustrates the normal wear of material and general aging of the whole system. Both of these factors are also random. The example presented can be successfully described by exponential distribution. Weibull's distribution takes into account two additional effects [13] and is sufficiently elastic to reproduce crucial structural phases of the course of the function of risk. The parameter of shape "*a*" is that which describes periods of the function of risk. The so-called "childhood" is an interval of time in which defects of the process of production are revealed. This period is concerned with the decreasing intensity of damage; the parameter of shape of Weibull's expansion $a < 1$ in this case. The so-called period of "normal exploitation" is an interval of time in which the occurrence of damage results from the random character of changes in durability.

This period is characterised by a uniform intensity of damage. The parameter of shape of Weibull's distribution is contained in the range $1 > a < 2$. The so-

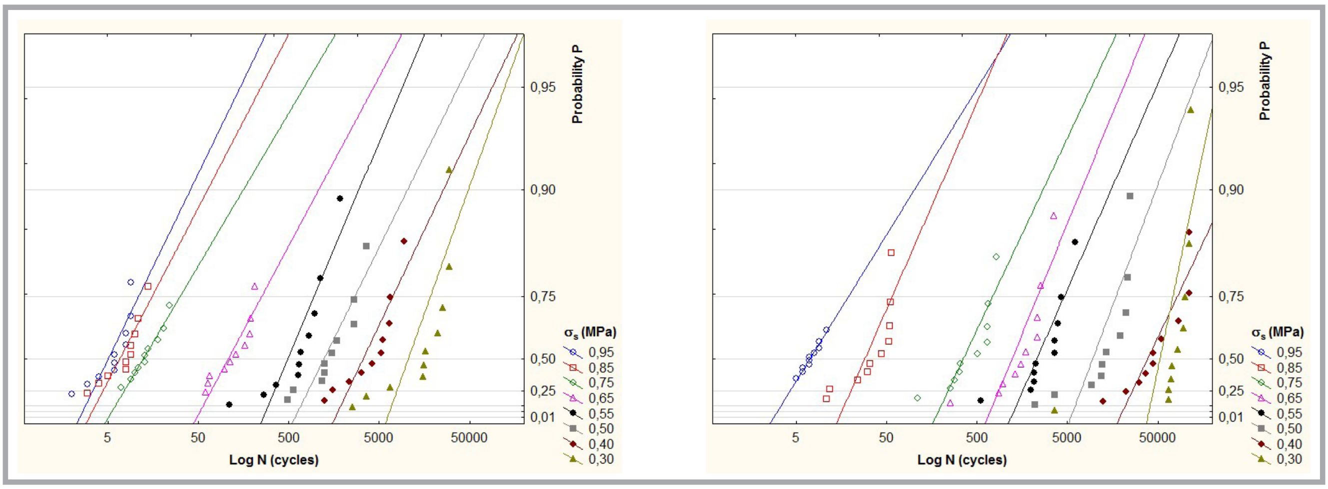


***Figure** 4. Log-normal distributions of fatigue durability determined along the wale (a) and courses (b) of knitted fabrics produced from standard smooth cotton yarn of 50 tex linear density.*

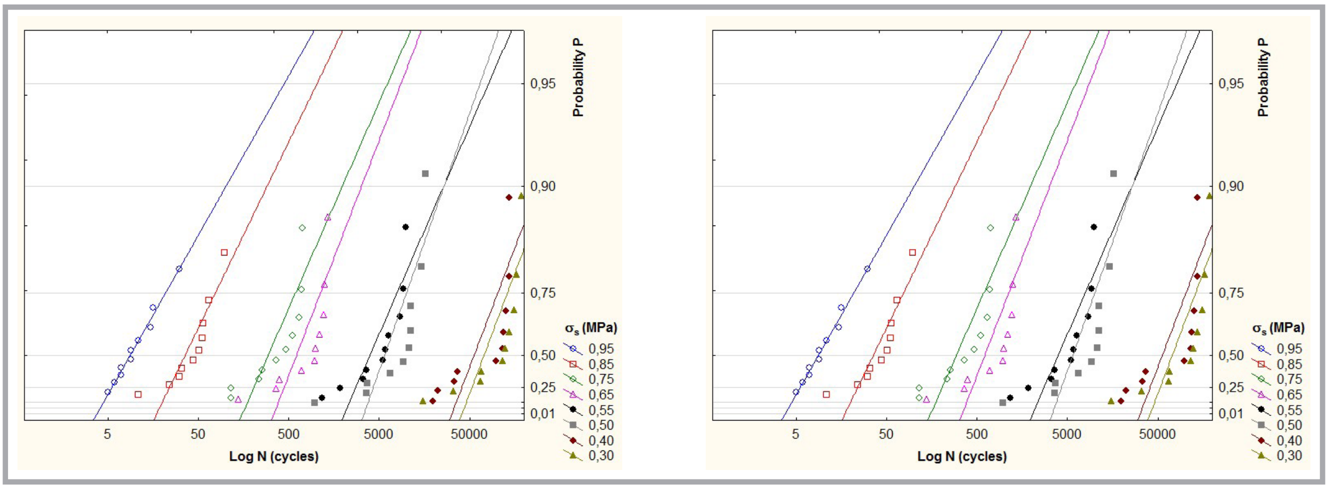


***Figure 5.** Log-normal distributions of fatigue durability determined along the wale (a) and courses (b) of knitted fabrics produced from cotton fancy flame yarn of 50 tex linear density.*

called period of "childhood" is an interval of time in which natural damage resulting from the real wear of the material are revealed. This period is concerned with the growing intensity of damage. The parameter of shape of Weibull's distribution is $a > 2$.

The numeric procedure of the method of the largest likelihood determining parameters of Weibull's distribution is presented below:

The density of Weibull's distribution

$$f(x_i) = \frac{a}{b} \cdot \left(\frac{x_i}{b}\right)^{a-1} \cdot \exp\left[-\left(\frac{x_i}{b}\right)^a\right] \quad (27)$$

where: $a$ - parameter of shape, $b$ - parameter of displacement.

The function of likelihood takes the following form:

$$L(x_1, x_2 \ldots, x_n) = \prod_{i=1}^{n} f(x_i; a; b) = \quad (28)$$

$$= \prod_{i=1}^{n} \frac{a}{b} \cdot \left(\frac{x_i}{b}\right)^{a-1} \exp\left[-\left(\frac{x_i}{b}\right)^a\right]$$

Finding the logarithm of the likelihood function, we obtain

$$\ln L(x_1, x_2 \ldots, x_n) =$$

$$= \ln \prod_{i=1}^{n} \frac{a}{b} \cdot \left(\frac{x_i}{b}\right)^{a-1} \exp\left[-\left(\frac{x_i}{b}\right)^a\right] =$$

$$= \ln \left(\frac{a}{b}\right)^n + \mathrm{h} \prod_{i=1}^{n} \left(\frac{x_i}{b}\right)^{a-1} +$$

$$+ \ln \prod_{i=1}^{n} \exp\left[-\left(\frac{x_i}{b}\right)^a\right] =$$

$$= n \ln\left(\frac{a}{b}\right) + \sum_{i=1}^{n} \mathrm{h} \left(\frac{x_i}{b}\right)^{a-1} +$$

$$+ \ln\left[\exp\left[-\sum_{i=1}^{n}\left(\frac{x_i}{b}\right)^a\right]\right] =$$

$$= n \ln a - n \ln b + \quad (29)$$

$$+ \sum_{i=1}^{n} \ln\left(\frac{x_i}{b}\right)^{a-1} - \sum_{i=1}^{n}\left(\frac{x_i}{b}\right)^a =$$

$$= n \ln a - n \ln b +$$

$$+ (a-1)\sum_{i=1}^{n}(\ln x_i - \ln b) - \sum_{i=1}^{n}\left(\frac{x_i}{b}\right)^a =$$

$$= \ln L = n\,(\ln a - \ln b + a \ln b - \ln b) +$$

$$+ (a-1)\sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n}\left(\frac{x_i}{b}\right)^a$$

$$\ln L = n\,(\ln a - 2 \ln b + a \ln b) +$$

$$+ (a-1)\sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n}\left(\frac{x_i}{b}\right)^a. \quad (30)$$

Then we solve the system of the largest likelihood:

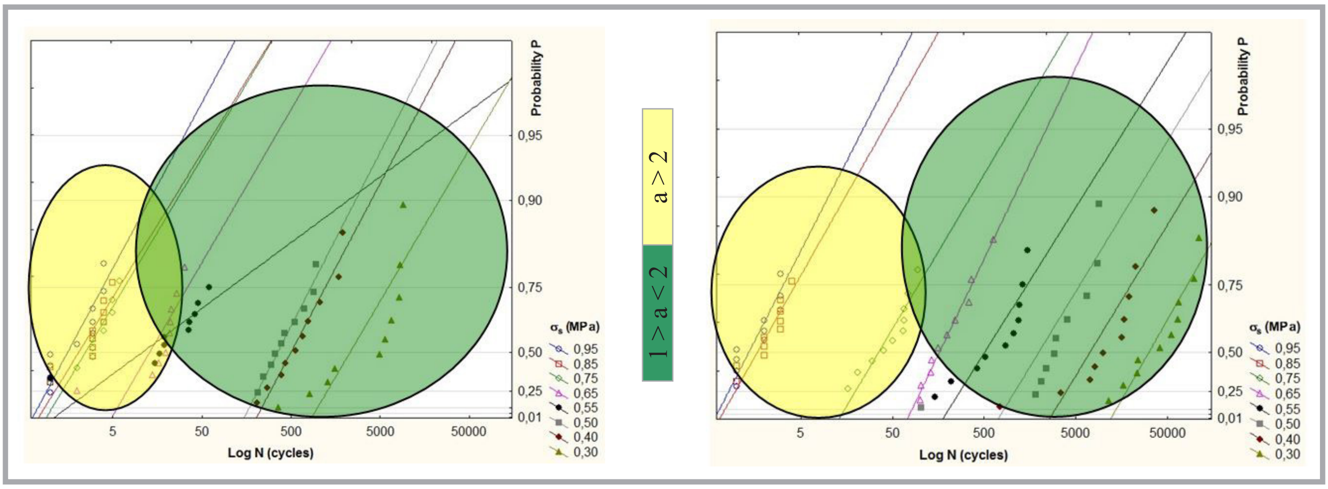


*Figure 6. Weibull's distributions of the fatigue durability of knitted fabrics - determined along the wale (a) and courses (b) - produced from standard smooth cotton yarn of 25 tex linear density, together with marked areas of material failure.*

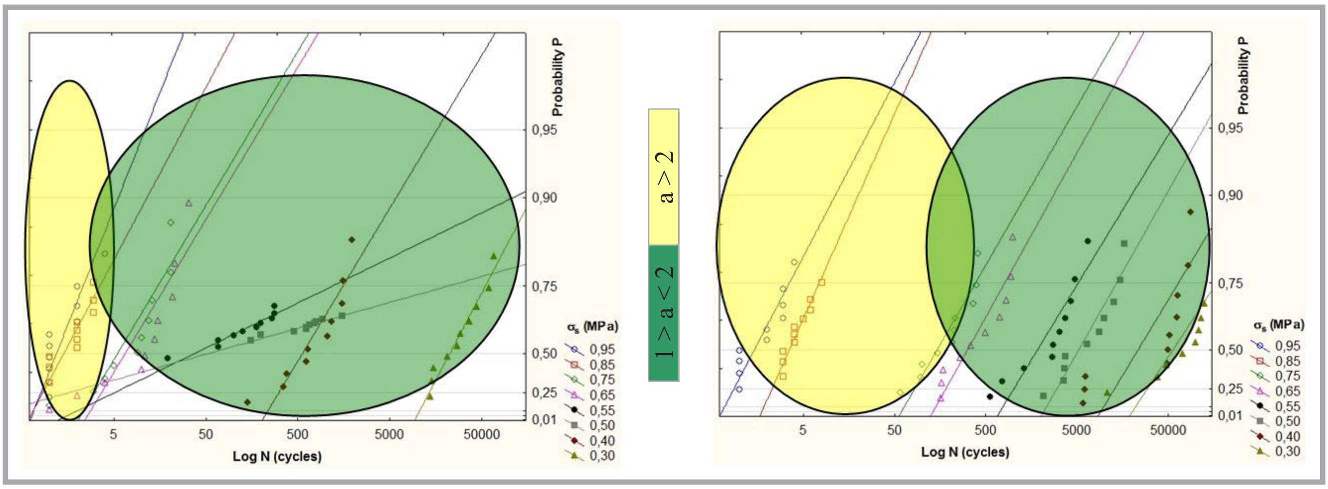


*Figure 7. Weibull's distribution of the fatigue durability of knitted fabrics - determined along the wale (a) and courses (b) - produced from fancy flame cotton of 25 tex linear density, together with marked areas of material failure.*

$$\frac{\sigma \ln L}{\sigma a} = 0 \text{ and } \frac{\sigma \ln L}{\sigma b} = 0 \quad (31)$$

The system of the largest likelihood takes the following form:

$$\frac{\sigma \ln L}{\sigma a} = \frac{n}{a} + n \ln b + \sum_{i=1}^{n} \ln x_i + - \sum_{i=1}^{n} \left(\frac{x_i}{b}\right)^a \ln\left(\frac{x_i}{b}\right) = 0 \quad (32)$$

$$\frac{\sigma \ln L}{\sigma b} = -\frac{2n}{b} + \frac{na}{b} + + \sum_{i=1}^{n} \frac{(x_i)^a}{(b^a)} - \frac{a}{b} = 0 / \cdot b \quad (33)$$

We obtain two equations, where:

I.

$$\frac{n}{a} + n \ln b + \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \left(\frac{x_i}{b}\right)^a \ln \frac{x_i}{b} = 0 \quad (34)$$

II.

$$-2n + na + \frac{a}{b^a} \sum_{i=1}^{n} x_i^a = 0 \quad (35)$$

$$\frac{a}{b^a} \sum_{i=1}^{n} x_i^a = 2n - na \quad (36)$$

$$b^a = \frac{a \sum_{i=1}^{n} x_i^a}{n(2-a)} \quad (37)$$

$$b = \left(\frac{a \sum_{i=1}^{n} x_i^a}{n(2-a)}\right)^{\frac{1}{a}} \quad (38)$$

After finding the logarithm we assume:

$$a \ln b = \ln \frac{a \sum_{i=1}^{n} x_i^a}{n(2-a)} \text{ or } \ln b = \frac{1}{a} \ln \frac{a \sum_{i=1}^{n} x_i^a}{n(2-a)} \quad (39)$$

we put the expressions obtained into equation I (***Equation 34***),

$$\frac{n}{a} + \frac{n}{a} \ln\left(\frac{a \sum_{i=1}^{n} x_i^a}{n(2-a)}\right) + \sum_{i=1}^{n} \ln x_i + - \sum_{i=1}^{n} \left(\frac{x_i^a}{\frac{a \sum_{i=1}^{n} x_i^a}{n(2-a)}}\right) \cdot (\ln x_i - \ln b) \quad (40)$$

Then we solve equation II (***Equation 35***),

$$\frac{n}{a} + \frac{n}{a} \ln\left(\frac{a \sum_{i=1}^{n} x_i^a}{n(2-a)}\right) + \sum_{i=1}^{n} \ln x_i +$$

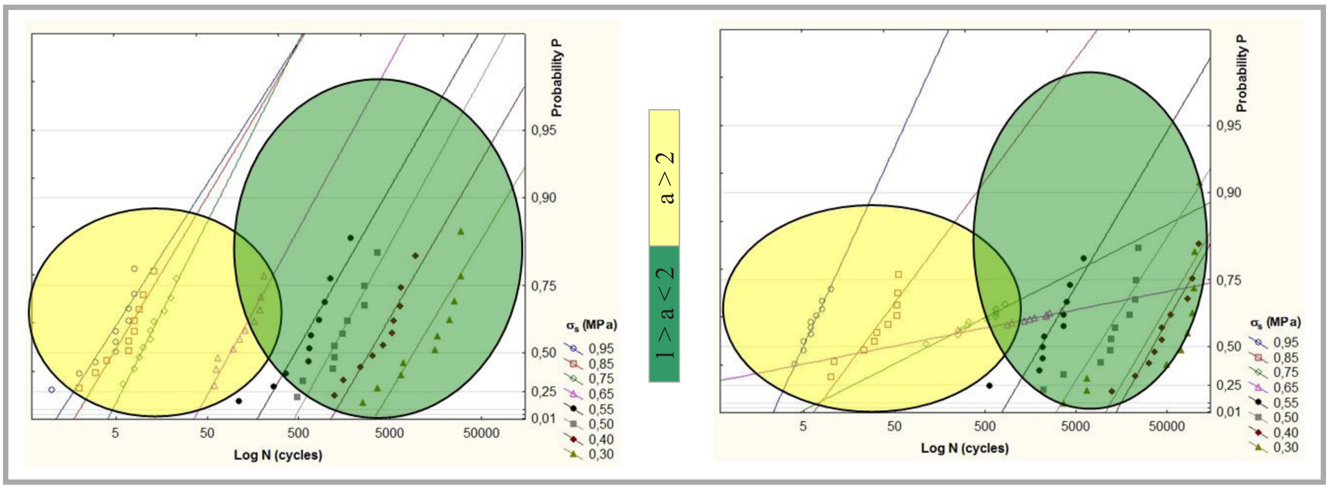


***Figure 8.** Weibull's distributions of the fatigue durability of knitted fabrics - determined along the wale (a) and courses (b) - produced from standard smooth cotton of 50 tex linear density, together with marked areas of material failure.*

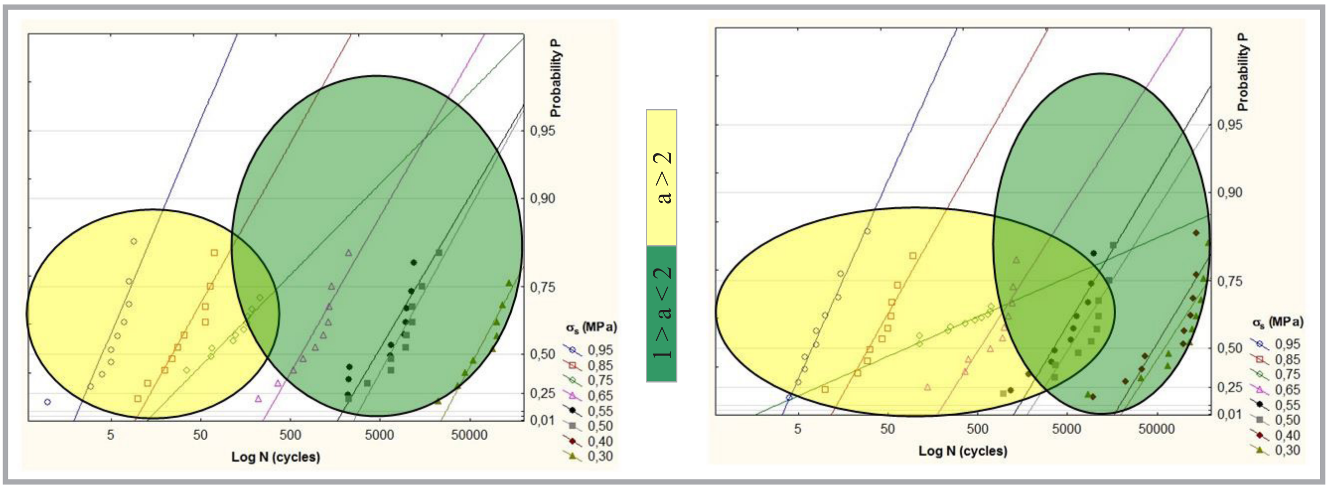


***Figure 9.** Weibull's distributions of the fatigue durability of knitted fabrics - determined along the wale (a) and courses (b) - produced from fancy flame cotton of 50 tex linear density, together with marked areas of material failure.*

$$-\sum_{i=1}^{n}\left(\frac{x_i^a}{\frac{a\sum_{i=1}^{n}x_i^a}{n(2-a)}}\right)\cdot\left(\ln x_i-\frac{1}{a}\ln\left(\frac{a\sum_{i=1}^{n}{}^{a}}{n(2-a)}\right)\right)=0 \quad (41)$$

We find the solution of this equation by means of numeric methods and then calculate the $b$ value from the following formula:

$$b=\left(\frac{a\sum_{i=1}^{n}x_i^a}{n(2-a)}\right)^{\frac{1}{a}} \quad (42)$$

As was mentioned earlier, numerical techniques were used to obtain distributions of the fatigue durability of knitted fabrics. Detailed numerical analysis of the fatigue durability was conducted with the help of STATISTICA 9.0 software. Verification of the hypotheses connected with the distributions of fatigue durability was conducted using STATISTICA 9.0 software, which made possible the selection of distributions of random variables, also Weibull's distribution. The 95% significance level was assumed for verification of hypotheses about the form of the distribution. The Anderson's-Darling's test was applied in this case.

Weibull's distributions of the fatigue durability of the knitted fabrics analysed are presented in ***Figures 6 - 9***. Analysing the diagrams, it can be stated that:

- in the case of knitted fabrics produced from standard smooth and fancy flame cotton yarns of 25 tex linear density, the area of failures relating to periodical ageing damage, in which natural damage resulting from the real wear of the material appear, is contained in the range of maximum stresses, i.e. (0.65 - 0.95) · $\sigma_S$. This is the result of too violent tension of the material, as a consequence of which the knitted fabric undergoes failure, but it does not become destroyed,
- the so-called period of normal exploitation can be observed for both kinds of knitted fabrics produced from yarns of 25 tex linear density in the range below 0.65 · $\sigma_S$,
- in the case of knitted fabrics produced from standard smooth and fancy flame cotton yarns of 50 tex linear density, the area of failures relating to growing intensity has a similar character for all knitted fabrics analysed. Only in the case of knitted fabrics produced from fancy flame yarn measured in the course direction does this area reache a tension equal to 0.50 · $\sigma_S$,
- fatigue durability can be described by means of intervals connected with the

character of material damage, resulting from the intensity thereof. This damage can be caused by factors connected with the production process, the random character or the failure of material resulting from its wear,
- it is not possible to unequivocally determine the magnitudes of tensions causing the definite event, or the character of failure of the material. It may turn out that at a given load of the cycle the features of failures of the material of an ageing character and also failures resulting from random damage appear,
- the character of Weibull's distribution also provides different valuable information which directly relates to the process of technological production of knitted fabrics from cotton yarns, both smooth and fancy flame,
- Weibull's distribution can also be useful in the theory of reliability.

## Conclusions

1. External influences, such as the starting effect of a machine engine wearing away its elements, the correctness of the performance of the technological process, as well as the behaviour of fibers while forming yarn, its processing and use are the main factors impacting the parameters of distribution of probabilistic models.
2. The dispersions of fatigue durability fulfill the log-normal distribution and Weibull's distribution, which is particularly useful for the description of damage at low levels of strains, close to the limit of fatigue. The probabilistic models obtained showed the necessity of working out a probabilistic method of calculation of the fatigue durability of yarns taking into account the influence of loads on the value of this durability. The models of fatigue durability proposed, formed using the graphic method and estimated by the method of the largest likelihood, were constructed with regard to the possibility of considering the influence of the load on the change in fatigue proprieties of yarns. In the models discussed, only the influence of the previous cycle of load was taken into account.
3. The intensity of damages caused by the ageing of knitted fabrics produced from cotton yarns of 50 tex linear density, both smooth and fancy flame, is larger than in the case of knitted fabrics produced from yarns of 25 tex linear density.
4. No damage in which defects of the process of production of the knitted fabrics or defects of the yarns applied appeared was observed.

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**Received 29.11.2012 Reviewed 09.05.2013**