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Probabilistic Model of Fatigue Strength

Abstract
A literature review relating to problems connected with the evaluation of the fatigue strength of materials was carried out concerning appropriate probabilistic models. It was found out that fatigue strength could be described by the following distributions: exponential, Weibull's, normal, Gumbel's, Ferecht's, Reyleight's, Gamma and log-normal. However, for modeling the problems of fatigue strength durability of textile materials, probabilistic models based on Weibull's theory and those based on the log-normal distribution seem to be most useful. The considerations presented also proved that many factors, mainly the kind of material used, the length of fibers in the assembly, the evenness of the thickness of the yarn and the system of spinning, influenced the fatigue strength of linear textile articles.

Key words: *probabilistic model, fatigue strength, stochastic process, material destruction.*

Introduction

Resistance examinations are a large group of examinations used for the assessment of linear textile products. This group of examinations includes: static breaking assessment, rub resistance assessment as well as consideration of rub forces at rub barriers. Evaluation of linear textile products' quality in the aspect of their static breaking is insufficient, and in some cases a material qualified as technologically useless in static breaking can fulfill technological criteria in changeable load conditions. Exclusive use of indicators defining only static resistance reflects the correctness of the yarn production process but do not predict its future usefulness in modification processes. The issue of resistance and fatigue strength as well as the use of statistic and probabilistic models is a supplement to examinations dealing with static resistance.

In the light of the literature review analysis it can be stated that evaluation of the fatigue strength issue is not completely recognised. The issue of fatigue strength evaluation and preparing adequate probabilistic models is worth a detailed analysis.

A highly statistical character is a characteristic feature of phenomena arising during textile product destruction in cyclic conditions.

In the classic approach fatigue strength is mainly assessed with statistical methods, as probabilistic models are rarely used due to their extensive mathematical apparatus. Despite this, in subject literature devoted to the estimation of statistical and fatigue strength outside the field of textiles, probabilistic models based on the following types of distribution can be found: exponential, Weibull's, normal, Gumbel's, Ferecht's, Reyleight's, Gamma, and log-normal. The distributions presented are often used for static and fatigue strength evaluation of material outside the field of textiles. The majority of the distributions mentioned require the use of advanced numeral techniques and the conduction of examinations on a sample of large quantity. Therefore when evaluating the fatigue strength of a material, in the beginning it must be assumed that a used probabilistic model is capable of giving information enabling the characterisation of changes occurring during the conduction of durability tests.

The aim of the article is to present probabilistic models for assessment of the fatigue strength of textile materials.

Stochastic approximation of textile objects

Events which can be classified as stochastic processes $X(\mathrm{T}, \mathrm{e})$, [4, 21] sets of random variables $X_{\mathrm{T}}(\mathrm{e})$ dependent on parameter T, belonging to a set of real numbers $\mathrm{T} \in (\mathrm{T_a}, \mathrm{T_b})$ take place during textile product manufacturing. Stochastic processes also include phenomena happening in the course of linear textile product destruction in cycle conditions. A stochastic process $X(\mathrm{T}, \mathrm{e})$ can be described as a set of random time functions assigned to elementary events e, where parameter T usually has a sense of time and in the case of fatigue strength evaluation is strictly related to the tension cycle (period) T as well as the frequency f. The probability density $f_1(x_1, \mathrm{T}_1)$ for process value x_1 at the moment T_1 is one of the stochastic process properties. A stochastic process is a random variable $X(\mathrm{T}_0, \mathrm{e})$ for a fixed parameter $\mathrm{T} = \mathrm{T}_0$. In the case of a fixed elementary event $\mathrm{e} = \mathrm{e}_0$ a stochastic process $X(\mathrm{T}, \mathrm{e}_0)$ is a time function $x(\mathrm{T})$ determined for $\mathrm{T} \in (\mathrm{T_a}, \mathrm{T_b})$ (called stochastic process realisation). The realisation $x(\mathrm{T})$ of a stochastic process $X(\mathrm{T}, \mathrm{e}_0)$ is an analogue of the x value, which is taken by the $X(\mathrm{e})$ random value for a fixed $\mathrm{e} = \mathrm{e}_0$. The probability density is more complex for any time sequence and multidimensional random variable $X(\mathrm{T}_1)$, $X(\mathrm{T}_2)$, ..., $X(\mathrm{T_n})$ characterised by n-dimensional density $f_n(x_1, \mathrm{T}_1; x_2, \mathrm{T}_2; ...; x_\mathrm{n}, \mathrm{T_n})$ consideration is necessary for any time sequence T_1, T_2, ..., $\mathrm{T_n}$. An orderly set of one-dimensional stochastic processes is called a multidimensional stochastic process. The value of a stochastic process expected $X(\mathrm{T})$ can be defined by time function:

$$\mathrm{E}[X(\mathrm{T})] = \int_{-\infty}^{+\infty} x_1 f_1(x_1, \mathrm{T})\mathrm{d}x_1 = m_x(\mathrm{T}) \quad (1)$$

The variance of a stochastic process is expressed in ***Equation 2***:

$$\begin{aligned}\mathrm{var}[X(\mathrm{T})] &= \mathrm{E}\left\{\{\mathrm{X(T)} - \mathrm{E[X(T)]}\}^2\right\} = \\ &= \int_{-\infty}^{+\infty} \{x - \mathrm{E[X(T)]}\}^2 f_1(x_1, \mathrm{T})\mathrm{d}x_1 = \delta_x^2(t)\end{aligned} \quad (2)$$

The expression (3) is called the self-correlation function of a stochastic process $X(\mathrm{T})$ (***Equation 3*** see page 62):

The self-correlation normalised function of a stochastic process $X(\mathrm{T})$ can be expressed as follows:

$$\begin{aligned}R_X(\mathrm{T}_1, \mathrm{T}_2) &= \frac{K_X(\mathrm{T}_1, \mathrm{T}_2)}{\sqrt{K_X(\mathrm{T}_1, \mathrm{T}_1) K_X(\mathrm{T}_2, \mathrm{T}_2)}} = \\ &= \frac{K_X(\mathrm{T}_1, \mathrm{T}_2)}{\delta_X(\mathrm{T}_1)\delta_X(\mathrm{T}_2)}\end{aligned} \quad (4)$$

because according to formulas (3) & (4):

$$K_X(\mathrm{T}, \mathrm{T}) = \mathrm{var}\, X(\mathrm{T}) = \delta_\mathrm{X}^2(\mathrm{T}) \quad (5)$$

A self-correlation normalised function value is contained in compartment [0,1]. The relationship between the two stochastic processes $X(\mathrm{T})$ and $Y(\mathrm{T})$ can be described by a mutual correlation function presented in ***Equation 6*** (see page 62):

The following expression is called a normalised mutual correlation function (***Equation 7*** see page 62).

$$K_X(T_1,T_2) = E\{\{X(T_1) - E[X(T)]\}\{X(T_2) - E[X(T_2)]\}\} =$$
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[x_1 - m_x(T_1)][x_2 - m_x(T_2)] f(x_1,T_1;x_2,T_2)\,dx_1 dx_2 \qquad (3)$$

$$K_{XY}(T_1,T_2) = E\{\{X(T) - E[X(T)]\}\{Y(T) - E[Y(T)]\}\} =$$
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[x - m_x(T_1)][y - m_y(T_2)] f(x,T_1;y,T_2)\,dx\,dy \qquad (6)$$

$$R_{XY}(T_1,T_2) = \frac{K_{XY}(T_1,T_2)}{\sqrt{K_X(T_1,T_1)K_Y(T_2,T_2)}} = \frac{K_{XY}(T_1,T_2)}{\delta_X(T_1)\delta_Y(T_2)} \qquad (7)$$

Equations 3, 6 and ***7.***

As in the case of dependence (4), a normalised mutual correlation function value is contained in a compartment [0,1].

The dependence or independence of statistical properties of a stochastic process on time is its characteristic feature. Consequently stationary and non-stationary stochastic processes can be distinguished. In the case of the issue presented, related to textile material durability, a stationary stochastic process occurs during technological processes. Thus it can be assumed that all multidimensional probability densities depend exclusively on the distance of moments $T_1, T_2, ..., T_n$ from each other, but do not depend on the moments' values. It can also be assumed that the statistical properties of a stochastic process do not change while moving all points $T_1, T_2, ..., T_n$ along the time axis by the same value of T_0, meaning that:

$$f_n(x_1, T_1; x_2, T_2; ...; x_n, T_n) = f_n(x_1, T_1+T_0; x_2, T_2+T_0; ...; x_n, T_n+T_0) \qquad (8)$$

Analogous to formula (8), a multidimensional stationary process is defined, assuming that components of the multidimensional stationary process are related to each other in a stationary way. The formula above also implies that one-dimensional stationary process density can be expressed by the following formula:

$$f_1(x_1, T_1) = f_1(x_1, T_1 + T_0) \qquad (9)$$

In this case the stationary process density does not depend on time at all:

$$f_1(x_1, T_1) = f_1(x_1) \qquad (10)$$

Two-dimensional stationary process density is obtained from the following formula:

$$f_2(x_1, T_1; x_2, T_2) = f_n(x_1, T_1+T_0; x_2, T_2+T_0) \qquad (11)$$

in this case the stationary process density f_2 depends only on the difference $T_2 - T_1 = \tau$, hence

$$f_2(x_1, T_1; x_2, T_2) = f_n(x_1, x_2, \tau) \qquad (12)$$

Examination of the multidimensional densities of stochastic processes is difficult and arduous to conduct. As a result, expected values, variances and correlation functions are often used in practice. During the proceedings of putting formulas (10) and (11) into formulas (1), (2) and (3), the expected value, variance and correlation function (13), (14) & (15) are obtained:

$$E[X(T)] = \int_{-\infty}^{+\infty} x_1 f_1(x_1)\,dx_1 = m_x = const \qquad (13)$$

$$var[X(T)] = \int_{-\infty}^{+\infty}(x_1 - m_X)^2 f_1(x_1)\,dx_1 = $$
$$= \delta_x^2 = const \qquad (14)$$

$$K_X(T_1,T_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x_1 -$$
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x_1 - m_x)(x_2 - m_x) f(x_1,x_2,\tau)\,dx_1 dx_2 =$$
$$= K_X(\tau) \qquad (15)$$

The expected value and variance of a stochastic process are constant, whereas a self-correlation function depends on the difference $T_2 - T_1 = \tau$. The relations presented: (13), (14) & (15) are necessary conditions of a stochastic process, but are not sufficient. They can be fulfilled when beginning with a certain number n equation (8) does not occur. Then it can be recognised that a process is not a non-stationary stochastic one.

With stable machinery operation and proper resource choice, spinning processes can be classified as random stationary processes. These assumptions, as well as adhering to technological requirements, have direct reflection in the quality of yarn manufactured. On examination of spinning processes, the simplified assumption that a class of processes called ergodic processes can be found among stationary stochastic processes is often made, which is mainly sensible in physical parameter estimation of yarns. A stochastic process $X(T)$ is called an ergodic process if the value expected $EX(T)$ of the process calculated as the average of the set is equal to the average obtained after the time:

$$\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2} x(T,e)\,dT = 0, \qquad (16)$$

for nearly every implementation, thus with the probability equal to one, as presented in ***Equation 17***

$$P\left(\left\{e : \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2} x(T, e)\,dT = E[X(T)]\right\}\right) = 1 \qquad (17)$$

The necessary and sufficient principle of ergodicity of a stochastic process is the equation:

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T\left(1-\frac{\tau}{T}\right)K_X(\tau)\,d\tau = 0 \qquad (18)$$

and

$$\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2} K_X(\tau)\,d\tau = 0 \qquad (19)$$

Ergodicity occurs if condition $K_X(\tau) \to 0$, where $|\tau| \to \infty$, is fulfilled. The self-correlation function of a stationary process $X(T)$ can be calculated on the basis of one implementation of the process $X(T)$. In this case the self-correlation function of a stationary process is an expected value of a stochastic process:

$$Z(T) = [X(T) - m_x(T)][X(T + \tau_0) - m_x(T)] = X^0(T)X^0(T + \tau_0) \qquad (20)$$

where: $X^0(T)$ indicates process deviation from the standard value (central process), and τ_0 is a constant parameter, out of which:

$$X^0(T) = X(T) - m_x(T) \qquad (21)$$

Process $Z(T)$ is stationary because $X(T)$ is stationary. To make the process $Z(T)$ ergodic, its self-correlation function $K_Z(\tau)$ should fulfill condition (18).

In the case of X(T) with normal deviation, the necessary and sufficient condition of the process Z(T) ergodicity fulfils the equation:

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T(1-\frac{\tau}{T})[K_X^2(\tau)+ \\ +K_X(\tau+\tau_0)K_X(\tau-\tau_0)]d\tau=0 \quad (22)$$

The sufficient condition of the stationary process ergodicity is the equation:

$$\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}K_X(\tau)^2 d\tau=0 \quad (23)$$

as well as the condition $K_X(\tau)\to 0$, where $|\tau|\to\infty$.

Fulfilling the principle of ergodicity Z(T) enables the calculation of the self-correlation function $K_X(\tau)$ of process X(T) according to one realisation x(T, e) of process X(T). The self-correlation function of process X(T) equals:

$$K_N(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}x^0(T,e)x^0(T+\tau,e)dT \quad (24)$$

for two processes it equals:

$$K_{XY}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}x^0(T,e)y^0(T+\tau,e)dT \quad (25)$$

x^0(T, e) and y^0(T + τ, e) indicates the realisation of central processes x^0(T) and y^0(T).

Review of probabilistic models selected for potential use in the textile industry

In the case of fatigue strength, the choice of appropriate statistical and probabilistic models is not unequivocal . The issues presented above can only be reflected in technological processes; however, they do not fully show phenomena taking place during material destruction. A great deal of textile products realise their function in changeable tension conditions, which result in disturbances of technological processes [3, 13, 24, 27], causing economic losses. To avoid such events, a lot of research on textile material durability has also been conducted relating to fibres and yarns [4, 8, 9, 11, 15], as well as to physical phenomena occurring in the technological process [18, 33]. The existence of non-homogenous tension fields in textile products complicate a lot the calculation process of fatigue strength determination. The complex structure of textile products, their production technology and the way of loading them have an essential impact on the origin of non-homologous tension value areas and hence on the origin of non-homologous fatigue strength degree areas.

Analysing subject literature on fatigue strength [14, 15, 17, 32], two main groups of methods, taking into account the influence of tension non-homogeneity on fatigue strength, applicable to textile products, with special consideration of yarns, can be distinguished. The first group, more common in technical sciences, include deterministic methods in which fatigue strength is defined by values, without determination of deviations. This kind of simplification, which means the assumption of the existence of deterministic conditions, can be very helpful and correct in some conditions. Excluding random factors and related formalism, it enables the explanation of experiment performance planning in the simplest way [25].

Experiments that can be carried out in random conditions can also be included in the plan of experiment in deterministic conditions. They give the possibility of model parameter estimation mistake assessment for determined random variable variance – measurement conditions, if the number of measurements is higher than the number of parameters. Since the variance is nearly unknown, in the course of the experiment a minimum amount of measurements is desired. Model parameter estimation mistake assessment is difficult and usually impossible to conduct. The second group is probabilistic methods assuming that the material has defects of different kinds. The material destruction has its origin in the most vulnerable to destruction "defect", whose morphology and tension level around the "defect" are most likely to develop a rupture (burst). Considering the assumptions made, it can be stated that the size of the area vulnerable to changeable tensions influences the probability of rupture occurrence.

Conception of the method of the weakest link

Probabilistic methods of fatigue strength evaluation are based on the idea of "the weakest link". They are useful for description of phenomena taking place in non-homologous materials and materials with changeable tension distribution over time. These materials include textile products. Fatigue mechanisms which occur e.g. in yarns depend largely on the resource type they have been made of, the manufacturing technology and final operations. The construction and structure of yarns have a non-homogenous character. Yarns which have similar properties and are subjected to the same changeable loads exhibit fatigue strength distribution around the average value. This phenomenon can be explained by the weakest link theory that was originally used to explain the so-called scale effect and material statistical durability distribution [1, 4, 7].

Assumption of the probability distribution of the feature examined based on experimental data e.g. the amount of fatigue cycles preceding destruction at a fixed load is a starting point for determining parameters describing fatigue strength N. Statistic inference methods of fatigue strength estimation can be based on knowledge of the distribution form of the feature distribution examined and are called parametric methods. Graphic methods realised by function nets and analytic methods are distinguished [2]. Parametric methods are more efficient than nonparametric ones because they need a random sample of a smaller amount in order to obtain information on the fatigue strength of a material. Choosing the most adequate mathematical model for determining fatigue strength experimental data, physical aspects of the random events examined e.g. damage to the product examined (in this case the textile product) should be considered. Then, choosing a distribution function of rupture (damage) probability during the fatigue test, information about the kind of damage to which the product is subjected, is necessary. The practical usefulness of the mathematical models chosen for certain durability evaluation is different and dependant on the recognition degree of different aspects [8, 19]. However, [31] did not take into account the so-called scale effect [1, 7] in their considerations. In subject literature devoted to fatigue strength estimation, use of the following distributions is proposed [5, 17]:

- Exponential, used to model sudden damage, load problems and fatigue strength,
- Weibull's, used to model gradual damage, durability problems, especially fatigue strength, as well as serial and parallel reliability structures,
- Normal, used to model static loads, solve problems of durability model-

ling and model serial reliability structures,
- Gumbel's, used to model catastrophic damage, fatigue strength, load and durability problems, serial and parallel reliability structures,
- Ferecht's, useful for modelling of catastrophic damage and corrosive wear, Reyleight's, used to model load problems,
- Gamma, useful for modelling durability problems,
- Log-normal, used to model fatigue strength.

In conclusion, it can be noticed that mechanisms taking place in yarn largely depend on its structure, which is not homogenous at a microscopic scale. The non-homogeneity of the material has a random character and, to some extent, implicates the random character of fatigue strength. Samples having the same parameters in the quality scale and subjected to the same changeable loads exhibit fatigue strength distribution around the average value [13]. The phenomenon of random character can be explained by "the theory of the weakest link", which was originally used to explain the so called effect of scale and statistical durability distribution of the material durability border [1, 7].

The weakest link theory was created in the 1920 s by Tippet [26] and Peirce [23]. Weibull [28, 29] developed it, presenting a distribution of an exponential type, and in 1951, the same author [30] showed research results of the tear durability of Bofors' steel and Indian cotton fibres. This publication contributed to popularization of the distribution so much, that the name Weibull's distribution was introduced. Basic assumptions of the weakest link theory in a general approach are shown below [1]:
- An element, e.g. a constructive element, contains different types of statistically distributed defects,
- Fatigue rupture initiation occurrences in different links of the element are independent from each other,
- A rupture initiation takes place in a certain elementary area (link) of the element, which contains the most dangerous defect.

Delahay and Palin-Lic [5] claim that the process of material degradation is accelerated by the non-homogenous structure of the object. Different types of inclusions, contaminations, thin places, thick places, dislocations and preferentially oriented bands of material structure can be classified as defects. Each of the objects mentioned can constitute the weakest place of the material, from which the process of destruction begins. Considering the assumptions made it can be stated that a material subjected to a static load or an fatigue one is regarded as a serial connection of links, in which damage to one link can cause the destruction of the whole material. During the cyclic loading of a material with the same material and load, the most dangerous defect has different features resulting in rupture (splits) initiation. The probability of a rupture arising in the whole element in the compartment [0, N] means that rupture initiation will occur in an elementary subarea. In fatigue strength calculations a material is divided into subareas (i), and individual survival probability $P_{tr}^{(i)}$ is calculated for each subarea. The destruction probability of the whole element is determined according to the event independence rule:

$$P_z = 1 - \prod_{i=1}^{k_p} P_{tr}^{(i)} \qquad (26)$$

where: k_p - amount of all subareas.

Thus the subarea containing the most dangerous defect is the weakest link of the material. If the element area increases the probability of occurrence of a defect with rising destruction potential increases. This effect is shown in equation (26), where an increase in the subarea amount k decreases the product $\prod_{i=1}^{k_p} P_{tr}^{(i)}$ value, increasing the probability P_z of destruction of the material subjected to a load. Regarding the material as a continuous medium, the subarea size $\rightarrow 0$ and their amount $k \rightarrow \infty$ Delahay, Palin-Lic [5]. The assumption of the exponential distribution form leads to the replacing of the product in the equation by the summation or integration operations (in the case of continuous medium) of exponent e,

$$P_{tr} = P_{tr}^{(i)} \cdot P_{tr}^{(i+1)} \ldots = \\ = e^{-f(\sigma^{(i)})} \cdot e^{-f(\sigma^{(i+1)})} \ldots = e^{-f(\sigma^{(i)}) - f(\sigma^{(i+1)}) \ldots}$$

Such a form of probability distribution of a random variable was proposed by Weibull [28], making the distribution $P_{tr}^{(i)} = e^{-f(\sigma^{(i)})}$ dependent on the tension level $\sigma^{(i)}$. The probability distribution of a material (element) destruction takes the form [28]:

$$P_z = 1 - e^{-\frac{1}{\Omega_0}\int_\Omega g(\sigma)d\Omega}, \; g(\sigma) = \left(\frac{\sigma}{\sigma_0}\right)^m \text{ or } \\ g(\sigma) = \left(\frac{(\sigma - \sigma_u)}{\sigma_0}\right)^m \qquad (27)$$

where:

V_0 - volume or reference surface of the element characterised by the distribution described by dependence (27), [28],

$g(\sigma)$ - destruction risk function, whose form depends on the material properties $(x) = 0$ for $x \leq 0$ and $(x) = x$ for $x > 0$.

Weibull proposed a two- and three-parameter function form $g(\sigma)$, in which σ_u, σ_0, m are, respectively, parameters of the shift (often called threshold) as well as scale and shape of the distribution (27). The author (as above [28]) analysed his considerations separately for the destruction probability distribution of a material on its surface ($\Omega = A$) and for its volume ($\Omega = V$), in relation to different properties of the material. Consideration of material properties both on its surface and in its volume result from an additional mechanical preparation of its surface and the fact that the surface is influenced by different external factors, such as the climate of the environment, including temperature, pressure, moisture, etc. (which do not fully affect the material interior). If degradation, rupture and, in consequence, the destruction of the material arise on its surface ($\Omega = A$) as well as in its volume ($\Omega = V$), the probability P_z of destruction can be described by the function of two probabilities $P_{tr}(A)$ and $P_{tr}(V)$ product [22]:

$$P_z = (A + V) = 1 - P_{tr}(A) \cdot P_{tr}(V) = \\ = 1 - e^{-\int_A g_A(\sigma)dA - \int_V g_V(\sigma)dV} \qquad (28)$$

where $g_A(\sigma)$, $g_V(\sigma)$ are, respectively, functions describing probability distribution for the material volume $P_{tr}(V)$ and its surface $P_{tr}(A)$ separately.

In the case of the fatigue strength probability distribution of material destruction is the binomial function $P_z = f(\sigma_a, N)$ of tension amplitude σ_a, which indicates the number of cycles N preceding the element destruction. This form was presented by Weibull [30] and the idea developed by Delahay and Palin-Lic [5], but their research considered $P_z = f(\sigma_a, N)$ function type, approximately equal to the fatigue border level, determination, which means they were based on the issue of whether the element subjected to a load would be destroyed or not without a number of load cycle analyses. The distributions of rupture presented and the destruction initiation probability are based on a two- and three-parameter Weibull distribution, which was generally used for fatigue strength assessment in the aspect of the material durability of a temporary border. The weakest link theory presented by Weibull involved brittle materials characterised by a lack of plastic deformation occurrence. Weibull [30] analysed the quasi-statistical durability of materials, in which the destruction of an element was defined as the total split of the material, and the arising of one rupture was followed by the destruction of the whole material. In this case the hypothesis of the independence of ruptures causing the simultaneous destruction of the material in consecutive links (elementary areas of the material) is fulfilled. However, in the case of changeable loads in the range of any number of cycles, the hypothesis of the independence of ruptures in elementary links presented, any of which can lead to material destruction, has some restrictions. Material destruction occurs when a rupture of defined length appears. The destruction mechanism of yarn made by binding randomly located fibres together was described in the publication [8], where the destruction process is dependent on friction forces and cohesion. During yarn stretching in any section, fibre segments draw out and are held by the smallest friction force.

Under stretching elongation increases. Fibres located in the smallest transverse section break. Following these assumptions, it can be stated that total material destruction should be understood as the total split of its elements. This phenomenon was dealt with by Liu, Choi and Li [20], who made an analysis based on the fibres discrete modelling rule, in which yarn is considered as a stream of a great deal of discrete bounds of chains. Fibre movement during deformation and their final locations are dependent on the length of chains of the fibre stream. The calculation method used by Liu, Choi and Li [20] considered non-linear fibre behavior at a large tension. The theoretical models prepared enabled accurate prediction of yarn behavior during the deformation. In order to determine if the material examined was destroyed, it is sufficient to observe the sample surface, e.g. with a microscope, or detect a decrease in the material rigidity (e.g. for bending), as shown by Karolczuk and Macha [14-15]. Analysing the assumptions of the weakest link theory mentioned, it can be stated that the theory can be used in the range of any amount of cycles if the rupture length defining material destruction is achieved in the period dominated by the initiation mechanism, in which there is no clear interpretation between the micro-ruptures. Plastic deformations cause a slowdown in micro-rupture and fatigue tear development. In this case a too long rupture would be achieved in the period of tear propagation. A propagating fissure extends on consecutive links, destroying the possibility of rupture origin at these points and hence mitigating the links independence theory of Karolczuk and Macha [14, 15]. In conclusion, the weakest link theory can be used in any range of cycle amount if the rupture length defining the destruction of a material is achieved in the dominating rupture initiation mechanism. In the weakest link theory, use of another distribution, e.g. normal distribution, causes big problems in the aspect of calculation, namely the destruction probability P_z of an element having homogenous tension distribution σ_{eq} based on normal distribution:

$$P_z = \int_0^{\sigma} \frac{1}{\sigma_u \sqrt{2\pi}} e^{\frac{-(\sigma_{eq}-\sigma_g)^2}{(2\sigma_u^2)}} d\sigma_{eq} \qquad (29)$$

where σ_u and σ_g are parameters of normal distribution.

For elements of non-homogenous tension distribution σ_{eq}, they should be divided into links. For every i-th link, the survival probability $P_{tr}{}^{(i)}$, from which the total probability $\prod_{i=1}^{k_p} P_r^{(i)}$ can be obtained, should be determined. As mentioned above, each link is characterised by a certain feature - $\Omega^{(i)}$ ($A^{(i)} = \Omega^{(i)}$ or $A^{(i)} = \Omega^{(i)}$), which impacts the probability – $P_{tr}{}^{(i)}$ - scale effect [1, 7], and the total probability must consider the sum of these properties. Because of this, the probability must take an exponential form, where the exponent is replaced by the sum after multiplication of individual probabilities. In order to maintain this form, equation (29) should be changed in the following way:

$$P_{tr} = 1 - P_z = \qquad (30)$$

$$= 1 - \int_0^{\sigma} \frac{1}{\sigma_u \sqrt{2\pi}} e^{\frac{-(\sigma_{eq}-\sigma_g)^2}{(2\sigma_u^2)}} d\sigma_{eq} = e^y$$

subsequently, y is determined:

$$y = \ln\left[1 - \int_0^{\sigma} \frac{1}{\sigma_u \sqrt{2\pi}} e^{\frac{-(\sigma_{eq}-\sigma_g)^2}{(2\sigma_u^2)}} d\sigma_{eq}\right], \qquad (31)$$

finally, the equation below is obtained:

$$P_{tr} = e^{\ln\left[1 - \int_0^{\sigma} \frac{1}{\sigma_u \sqrt{2\pi}} e^{\frac{-(\sigma_{eq}-\sigma_g)^2}{(2\sigma_u^2)}} d\sigma_{eq}\right]} \qquad (32)$$

Such a form can be used to determine the individual probability $P_{tr}{}^{(i)}$, considering the scale effect:

$$P_{tr}^{(i)} = e^{\frac{\Omega^{(i)}}{\Omega_0}\ln\left[1 - \int_0^{\sigma} \frac{1}{\sigma_u \sqrt{2\pi}} e^{-(\sigma_{(eq)}^{(i)} - \sigma g)^2} d\sigma_{eq}^{(i)}\right]} \qquad (33)$$

If a continuous tension distribution σ_{eq} is assumed, the total probability of element destruction can be determined:

$$P_z = 1 - e^{\frac{1}{\Omega_0}\int_{\Omega} \ln\left[1 - \int_0^{\sigma} \frac{1}{\sigma_u \sqrt{2\pi}} e^{-\frac{(\sigma_{eq}-\sigma_g)^2}{2\sigma_u^2}} d\sigma_{eq}\right] d\Omega} \qquad (34)$$

The equation presented regards the destruction probability of the element of non-homogenous equivalent tension field σ_{eq}, with the use of normal distribution. The use of dependence (34) in fatigue strength calculation would require the conduction of the numeral integration process for each link (i) after equivalent tensions and subsequent repeated integration on the surface ($\Omega = A$) or in the volume ($\Omega = V$) of the element. For these reasons, normal distribution is not often used in fatigue strength calculation with probabilistic methods. Normal distribution is particularly important in the theories of communication, estimation and control. Thus it is a reasonable approximation of probabilistic properties observed for many physical systems.

As has been already mentioned, examination of a sum of many small, random factors – consideration of a phenomenon being the effect of the multiplicative mechanism impact on many such factors – was the genesis of normal distribution [7]. Assuming that quantity Y_n after the n-th event is equal the product of the Y_{n-1} - quantity before the event and random factor - W_n, and generalising, the following result is obtained, as given by Benjamin and Cornell [2]:

$$Y_n = Y_{n-1} W_n = Y_{n-2} W_{n-1} W_n = \ldots = \\ = Y_0 W_1 W_{2\ldots} W_n \qquad (35)$$

Many physical systems can be characterised with use of similar mechanisms, causing that an increase in the impulse response of the Y_n - Y_{n-1} system subjected to a random impulse at the Z_n entrance is proportional to the current value of the response Y_{n-1}. Finally a description of the phenomenon considered can be expressed in the following way:

$$Y_n - Y_{n-1} - Z_n Y_{n-1}$$

$$Y_n = Y_{n-1}(1 + Z_n) = = Y_{n-2}(1 + Z_{n-1})(1 + Z_n) = = Y_0(1 + Z_1)(1 + Z_2) \dots (1 + Z_n) \quad (36)$$

Substituting the expression $W_i = 1 + Z_i$ into formula (36), it can be stated that the random variable Y_n has the same multiplicative form as the variable discussed above. This model can be used to characterise the mechanisms of material existence origin [6]. The internal tension after n-cycles is equal to:

$$Y_n = g(Y_{n-1})W_n \quad (37)$$

In this expression, W_n indicates the internal tension of the n-th load, undergoing changes due to internal differences in the material at the macroscopic level. If $c_{n-1}(Y_n)$ is considered as the first approximation of $g(Y_{n-1})$, the following equation is obtained:

$$Y_n = W_1 W_2 \dots W_n \quad (38)$$

In the cases presented above, random variable Y is a product of a large amount of other variables, of which each one is difficult to exam and describe if considered separately. However, the distribution of variable Y can be recognised. To achieve it, both sides of equation (38) are logarithmed, giving:

$$\ln Y_n = \ln Y_0 + \ln W_1 + \ln W_2 \dots + + \ln W_n \quad (39)$$

Since quantities W_i are random variables, their functions $\ln W_i$ are also random variables. Considering the central border theorem, a hypothesis can be made that the sum of a certain number of these variables has approximately normal distribution. Thus it is assumed that $\ln Y$ undergoes normal distribution:

$$X = \ln Y \quad (40)$$

Assuming that X has normal distribution, the Y distribution variable is determined in the following way.

Random variable Y, whose logarithm undergoes normal distribution, can be regarded as a variable of logarithmic-normal or logarithmic distribution. In the case of mutually unequivocal transformation, the dependent variable density function has the following form:

$$f_y(y) = \left| \frac{d_g^{-1}(y)}{dy} \right| f_x(g^{-1}(y)) \quad (42)$$

in which

$$Y = g(X) = e^x, \; X = g^{-1}(Y) = \ln Y,$$

$$\left| \frac{d_g^{-1}(y)}{dy} \right| = \frac{1}{y} \quad ,$$

where X has normal distribution with the density function:

$$f_x(y) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - m_x}{\sigma_x} \right)^2 \right]$$

$$-\infty \le x \le \infty \quad (43)$$

Hence:

$$f_y(y) = \frac{1}{y\sigma_x \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln y - m_x}{\sigma_x} \right)^2 \right]$$

$$y \ge 0 \quad (44)$$

The random variable Y has log-normal distribution if its logarithm X undergoes normal distribution. Compartment $(0; +\infty)$ is a set of Y values, whereas $X \in (-\infty; +\infty)$. If $Y = 1$ and $X = 0$ but if $Y > 1$, quantity X acquires positive values.

In compartment $0 \le Y \le 1$, variable X acquire values $(-\infty; 0)$, because a logarithm contained between zero and one, whose basis exceeds one, is negative. Variable Y cannot gain negative values, as a logarithm of negative values is not defined [17]. Numeric calculations aimed at determining the form of this distribution are not complicated and can be used to show material fatigue strength statistics. Applying this type of distribution does not require many measurements, which is important in exhaustion examinations. The distributions described above are largely applied for static and fatigue strength assessment of materials outside of textiles. A part of these distributions describing the phenomena requires the application of advanced numeral techniques and conduction of examinations on samples of large quantity. Hence during material fatigue strength evaluation, in the beginning the assumption should be made that the probabilistic model applied is able to give information on characteristics of changes occurring during durability tests. Fatigue strength is the product durability determined in specific load and deformation conditions. Many factors impact this durability; however, it must be noted that knowledge of phenomena taking place during the action of forces at static stretch should be the basis of theoretical considerations.

Summary

The considerations presented prove that a lot of factors, mainly the kind of resource, fibre length in the stream, uniformity of yarn thickness and the spinning system, impact the fatigue strength of linear products. It is also influenced by the curve number [5]. When this number increases, the fatigue strength rises simultaneously (of course, only to a certain moment) [12]. These authors presented models describing the fatigue strength of polyester texture yarns and obtained fatigue curves consistent with theoretical ones. Probabilistic models can be also applied to fatigue strength estimation of cotton smooth yarns and flame-like fancy yarns [5].

Conclusions

On the basis of the literature review conducted and the author's own considerations it can be stated that:

1. Making use of indexes defining only strength proprieties can only apply to characterising the correctness of the technological process. However, it does not provide a full prognosis of usefulness in real conditions of use.
2. The lack of applying tools helping to design and predict the behavior of a given material in conditions of use can cause the generation of inevitable mistakes at the stage of their application.
3. Applying probabilistic methods based on the conception of "the weakest link" is a valuable supplement to designing textile materials.
4. The probabilistic models presented, with special regard to Gamma distributions and Weibull's distributions, are suitable for the description of phenomena occuring in materials with heterogeneous and changeable in time distributions of stresses, which include textile articles.

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