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Interaction of the Driving Motor and Axially Moving Beam in Textile Machines

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Abstract
In this paper a mathematical model for a motor driven reciprocally moving beam is formulated. The partial differential equation of the beam is replaced by an ordinary differential equation, primarily describing the mode of vibration. Using the principle of virtual work, an equation for the driving system is derived. The set of equations is solved numerically. It is demonstrated that motion close to a frequency twice higher than the frequency of natural vibration is unstable.

Key words: *stability of motor drive, reciprocally moving beam, textile machines.*

Introduction

Axially moving beams find wide application in textile machines such as weaving, knitting, stitching and sewing machines. In order to perform a technological task, they are brought to periodical motion by a motor through a linkage or cam mechanism. It has usually been assumed that during the motion of elastic elements, the speed of the driving motor can be kept constant. This is not always true as has been shown in the case of the torsional vibration of shafts [1], flexural vibrations resulting from the rotation of a shaft [2] and rotary oscillatory motion [3]. In paper [1] it was shown that two periodic attractors coexists in the resonant region, which is bounded by two catastrophic bifurcations. In paper [2] it was demonstrated that motion over the critical speed cannot be maintained since the speed of the motor decreases to the critical value. In paper [3] the difference between critical and resonant frequencies was observed. The purpose of this paper is to study the interaction between the motor and the flexural vibration that results from the axial motion of the beam. The behaviour of sliding beams at an assumed constant speed of the driving motor was previously studied in papers [4 - 7].

Equation of motion

Scheme of the beam considered is shown in ***Figure 1***. Its support is moving axially in a z-direction. The beam is driven by a motor and reciprocating mechanism, not shown in the figure.

The motion of the beam element is presented by Equation (1).

Here, EI denotes beam stiffness, w transverse deflection of the beam, x coordinates along the beam, z displacement of the beam support, t time, μ mass of unit length of the beam, χ internal damping coefficient.

The following analysis is restricted to the first mode of vibration. In this case, the transverse deflection w can be assumed as a product of space Y and time function W. It is convenient to use the transformations shown in the set of Equation (2).

The admissible function Y was assumed to be of the form of the deflection function of a cantilever beam, subject to a concentrated force. This simplification gives good results [7] as compared to the results obtained for the exact modal function of the vibrating beam [5].

The length of the beam is denoted as l, the angle of revolution of the driving motor as α. Substituting relationships (2) into equation (1), multiplying by Y and integrating with respect to $\xi \in <0, 1>$, one obtains the ordinary differential Equation (3).

Summing up the virtual works in the system, one obtains an equation relating the angular rotation α of the driving motor with the resulting motion z of the beam

$$M_\alpha \mathrm{d}\alpha + F\mathrm{d}z = 0 \qquad (4)$$

were M_α and F are described by Equation (5).

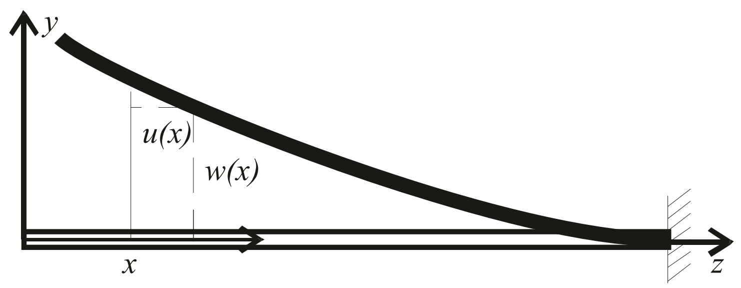

Figure 1. *Axially moving beam.*

$$EI\left(1+\chi\frac{\partial}{\partial t}\right)\frac{\partial^4 w}{\partial x^4}-\frac{\partial}{\partial x}\left(\mu x\frac{\partial^2 z}{\partial t^2}\frac{\partial w}{\partial x}\right)+\mu\frac{\partial^2 w}{\partial t^2}=0 \qquad (1)$$

$$w(x,t)=v(\xi,t),\ \xi=\frac{x}{l},\ v(\xi,t)=Y(\xi)W(t) \qquad Y=\xi^3-3\xi+2$$
$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2}=\frac{\mathrm{d}^2 z}{\mathrm{d}\alpha^2}\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2+\frac{\mathrm{d}z}{\mathrm{d}\alpha}\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} \qquad (2)$$

$$\frac{\mathrm{d}^2 W}{\mathrm{d}t^2}+\omega_1^2\chi\frac{\mathrm{d}W}{\mathrm{d}t}+\omega_1^2 W+\frac{105}{66l}\left(\frac{\mathrm{d}^2 z}{\mathrm{d}\alpha^2}\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2+\frac{\mathrm{d}z}{\mathrm{d}\alpha}\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}\right)W=0,\ \omega_1=\sqrt{\frac{12EI}{\mu l^4}\frac{35}{33}} \qquad (3)$$

Equations; 1, 2, 3.

$$M_\alpha = -A\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}+M \qquad F = -\mu l\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} - \int_0^l \mu\frac{\mathrm{d}^2 u}{\mathrm{d}t^2}\,dx$$

$$u(x,t)=\frac{1}{2}\int_x^l\left(\frac{dw}{dx}\right)^2 dx = \frac{1}{2}\frac{W^2}{l}\left(\frac{24}{5}-\frac{9}{5}\xi^5+6\xi^3-9\xi\right) \qquad (5)$$

$$\int_0^l \frac{\mathrm{d}^2 u}{\mathrm{d}t^2}dx = \frac{3}{4}\frac{\mathrm{d}^2\left(W^2\right)}{\mathrm{d}t^2}=\frac{3}{2}\left(W\frac{\mathrm{d}^2W}{\mathrm{d}t^2}+\left(\frac{\mathrm{dW}}{\mathrm{d}t}\right)^2\right)$$

$$\left(A+\mu l\left(\frac{\mathrm{d}z}{\mathrm{d}\alpha}\right)^2\right)\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}+\mu l\frac{\mathrm{d}z}{\mathrm{d}\alpha}\frac{\mathrm{d}^2z}{\mathrm{d}\alpha^2}\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2+\frac{3}{2}\mu\left(W\frac{\mathrm{d}^2W}{\mathrm{d}t^2}+\left(\frac{\mathrm{d}W}{\mathrm{d}t}\right)^2\right)\frac{\mathrm{d}z}{\mathrm{d}\alpha}=M \qquad (6)$$

$$\frac{\mathrm{d}\alpha_1}{\mathrm{d}t}=\alpha_2$$

$$\frac{\mathrm{d}\alpha_2}{\mathrm{d}t}=\frac{M_5-\mu l\frac{\mathrm{d}z}{\mathrm{d}\alpha}\frac{\mathrm{d}^2z}{\mathrm{d}\alpha^2}\alpha_2^2-\frac{3}{2}\mu\left(-\omega_1^2\chi W_3W_4-\omega_1^2W_3^2-\frac{105}{66l}\frac{\mathrm{d}^2z}{\mathrm{d}\alpha^2}\alpha_2^2W_3^2+W_4^2\right)\frac{\mathrm{d}z}{\mathrm{d}\alpha}}{\left(A+\mu l\left(\frac{\mathrm{d}z}{\mathrm{d}\alpha}\right)^2-\frac{3}{2}\frac{105}{66l}\mu W_3^2\left(\frac{\mathrm{d}z}{\mathrm{d}\alpha}\right)^2\right)}$$

$$\frac{\mathrm{d}W_3}{\mathrm{d}t}=W_4 \qquad (8)$$

$$\frac{\mathrm{d}W_4}{\mathrm{d}t}=-\omega_1^2\chi W_4-\omega_1^2W_3-\frac{105}{66l}\left(\frac{\mathrm{d}^2z}{\mathrm{d}\alpha^2}\alpha_2^2+\frac{\mathrm{d}z}{\mathrm{d}\alpha}\frac{\mathrm{d}\alpha_2}{\mathrm{dt}}\right)W_3,$$

$$\frac{\mathrm{d}M_5}{\mathrm{d}t}=\frac{C}{T}\left(\Omega-\alpha_2\right)-\frac{M_5}{T}$$

Equations; 5, 6, 8.

Here, M denotes the motor torque and A the mass moment of rotor's inertia.

Substituting equations (5) into equation (4), one obtains the Equation (6).

The driving torque M of the motor can be found from equation

$$T\frac{\mathrm{d}M}{\mathrm{d}t}=C\left(\Omega-\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)-M \qquad (7)$$

Here, T denotes the time constant, Ω is the idle angular velocity, and C is equal to the negative slope of the motor characteristic.

For the purpose of numerical integration, the set of Equation (3, 6, 7) is rewritten in the form of a set of first order differential equations (8) and the variables are numbered.

The purpose of replacing each second order equation by the equivalent set of two first order equations is to make it possible to use the available numerical algorithm of integration of the first order equations.

Resuls and discussion

Numerical calculations were performed for $z = S(1\text{-}\cos\alpha)/2$, $l = 1$, $S = l$,

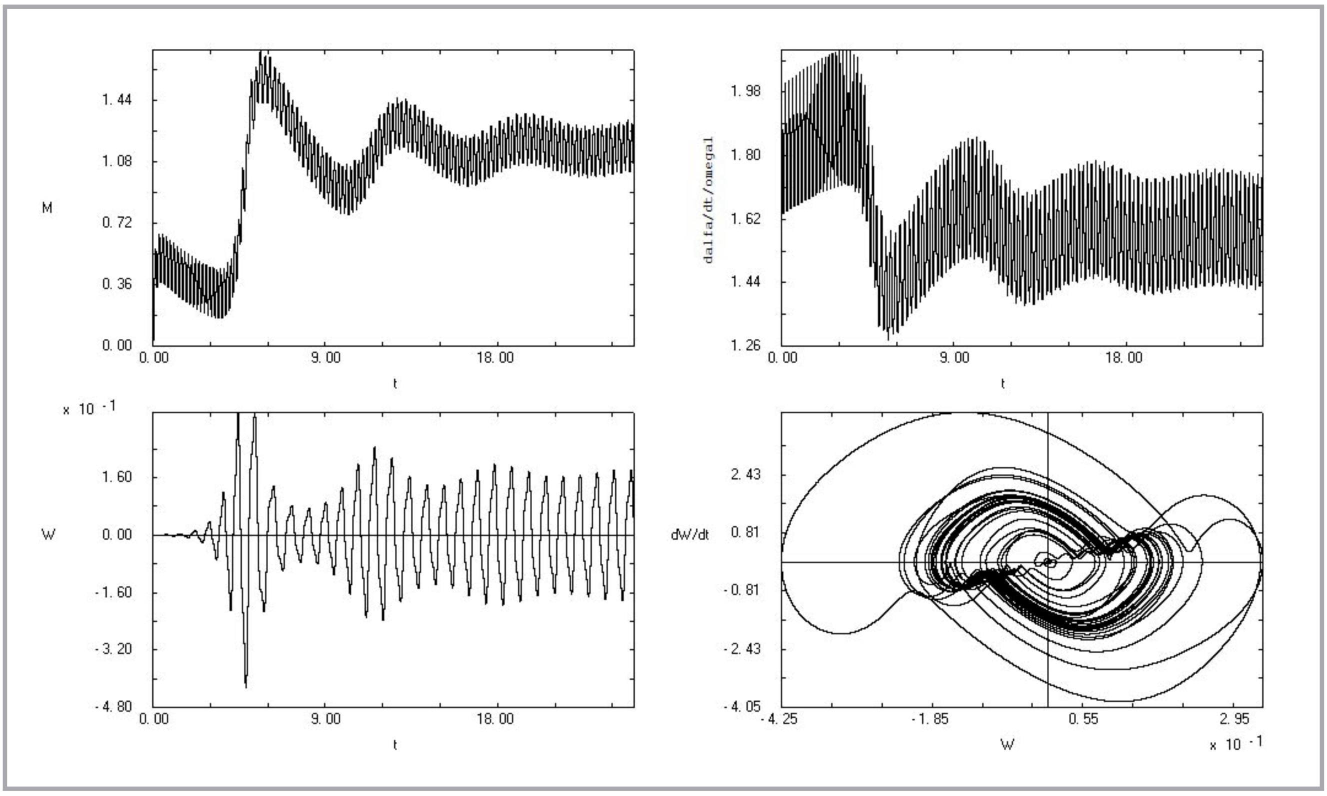

Figure 2. *Motor torque M, quotient of angular speed of the motor dα/dt and first circular frequency of natural vibration of the beam ω_1, transverse deflection of the beam w(0,t)=2W(t) versus time t and dW/dt versus W.*

$\mu = 2.5$, $\omega_1 = 9$, $\chi = 0.1$, $A = 1.25$, $C = 0.3$, $T = 0.1$, $\Omega = 2\omega_1$, and the initial conditions $\alpha(0) = 0$, $d\alpha(0)/dt = \Omega$, $W_3(0) = 0.001$, $dW_3(0)/dt = 0$, $M(0) = 0$.

The system parameters were chosen thus so as to observe the behaviour of the beam in the vicinity of the parametric resonance. In order to avoid long time calculations, which are necessary for starting the system, the value of the initial angular velocity of the motor was chosen to be equal to the idle angular velocity. Since we are interested only in the qualitative behaviour of a system disturbed from the dynamic equilibrium state, the initial transverse displacement of the beam could have any value.

From ***Figure 2***, one can observe that the motion is unstable when it starts from a frequency equal to twice the value of the circular frequency of the beam's natural vibration. The initial angular velocity $d\alpha/dt$ of the motor could not be maintained.

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