Robert Drobina,
Andrzej Włochowicz,
Mieczysław S. Machnio,
Elżbieta Drobina,
Stanisław Lewandowski

Fatigue Curves Elaborated for Selected Worsted Wool Yarns

Institute of Textile Engineering and Polymer Materials, University of Bielsko-Biala,
ul. Willowa 2, 43-309 Bielsko-Biała, Poland
e-mail: rdrobina@ath.bielsko.pl

Abstract
There is not any elaborated methodology for making diagrams of fatigue strength curves for yarns presented in world wide literature. Thus, we proposed our own methodology for measuring the properties of fatigue strength, using worsted wool yarns as an example. The object of the research presented was worsted wool single yarn with linear density of 16 tex and 25 tex produced with the use of a FIOMAX 2000 ring spinning machine from MESDAN. In order to prepare a graphical presentation of fatigue strength we carried out tests on seven groups measuring series, which corresponded with the specific amplitude of load through a given cycle. We increased the number of series with respect to the stochastic character of the yarn strength properties. As a result of this we obtained a very good fitting of Wöhler characteristics with experimental observations.

Key words: *fatigue curves, Wöhler fatigue curves, yarn, worsted wool yarns.*

Introduction

Fatigue research allows to measure the strength properties of prepared specimens or of final goods related to the impact of loads originating in repeated changeable forces. Investigations and routine fatigue strength tests of various construction elements and different materials have been carried out for yarns, but according to a literature review of the problem discussed here, a methodology for designing a Wöhler fatigue curve for yarns has so far not been formulated. Below we present the procedure of elaborating the Wöhlercurve used in mechanical engineering as a basis for our considerations concerned with yarns [1 - 6].

The number of changes, i.e. cycles of tensions recorded to yarn break, indicate how long the material is able to withstand up to the moment of its destruction, which depends not only on the force values of forms, but also on the character of the variability of tension. Fatigue strength, measured by the number of cycles, increases together with a decrease in tension amplitude. The variability of tensions with time can be periodical, programmed or completely random. The characteristic quantities measured in order to obtain fatigue strength are [1]:

- the nominal tension, determined as the quotient of the largest axial load and the initial cross section of the specimen tested in the case of axial loading, or the quotient of the largest bending moment and the index of the cross-section at bending,
- the cycle of tension, i.e. variability of tension with time which is characterised by frequency *f*,
- the value and the kind of tension and the index of asymmetry *R* of the cycle given by

$$R = \sigma_{min}/\sigma_{max} \qquad (1)$$

where: σ_{min}, σ_{max} are the minimum and maximum tensions of the cycle.

- the cyclic tension average, which is characterized by the average value of the cycle – σ_m, given by

$$\sigma_m = (\sigma_{max} + \sigma_{min})/2 \qquad (2)$$

- the amplitude σ_a of the tension cycle, given by

$$\sigma_a = (\sigma_{max} - \sigma_{min})/2 \qquad (3)$$

The calculation of the fatigue strength may be performed by using various kinds of changeable loads, for example, compressing, bending or twisting (torque), and loads of different kinds during the cycle. Therefore it is possible to distinguish symmetrical one-sided, and two-sided cycles as well as the so-called zero suppression cycles. One-side tension is understood as tension in which both σ_{max} and σ_{min} have the same sign. In the case of two-sided tensions not only does the value change, but also the sign of the tension. Pulsing tensions (or zero suppressed tensions) are a special kind of one-sided tension, where the absolute values σ_{max} and σ_{min} are equal.

In practice, the fatigue strength of materials, for example, constructional material, is measured with the use of a strength testing machine designed for fatigue tests, and equipped with appropriate software. The specimen tested is subjected to periodical, variable tensions. The first sample is loaded relatively high values of σ_m and σ_a. Then N_{01}, the number up to break of the first sample is recorded. The second sample is loaded with smaller values of σ_m and σ_a so that we obtain $N_{02} > N_{01}$. Further specimens are loaded by decreasing the values of σ_m and σ_a, up to the moment that the sample material will withstand a very large number of cycles without break, (considered in practice as infinite). For example, the boundary number of fatigue cycles for steel and other iron alloys equals $N_G = 10^7$ [1]. This number equals $2 \cdot 10^7 \div 10^7$ for non-ferrous metals.

The fatigue strength can be determined as the computed value of the tension σ_{max},which appears in the fatigue cycle, determined by σ_m and σ_a values, which cause the destruction of the sample material after a conventionally accepted number of fatigue cycles – N_G. The results are usually presented in graphical form in the coordinate system "tension – number of cycles", e.g. by the so-called Wöhler fatigue curve shown in Figure 1 (see page ...) [1].

Determining a Wöhler fatigue curve is expensive as well as time and labour-consuming. A few attempts last from a dozen or so, to several hundred hours. The Wöhler plot enables the assigning of the fatigue strength to a given series of cycles of fatigue loads determined by the ratio σ_m/σ_a. It is possible to obtain a number of series of Wöhler plots for the same material and the same kind of ten-

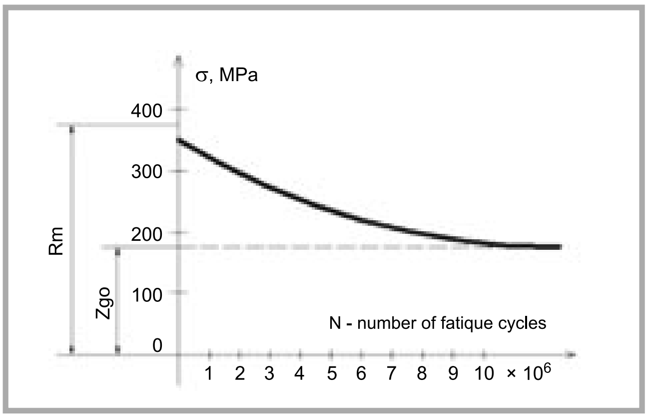

***Figure 1.** Plot of the Wöhler fatigue curve (for symmetrical cycle series) performed for steel [1].*

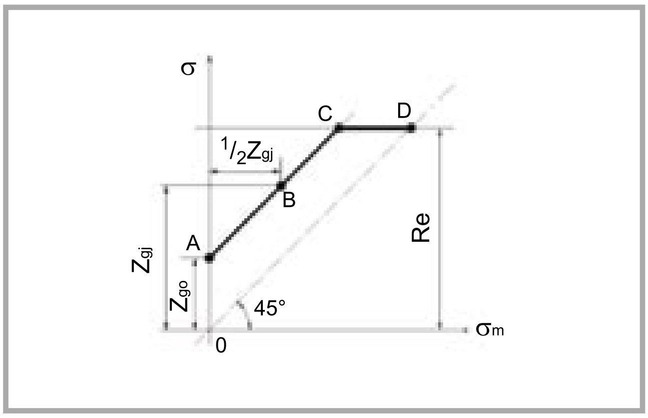

***Figure 2.** Simplified Smith graph plotted for plastic materials (steel, alloys of non-iron metals) [2].*

sion (e.g. tensile-compressive) by changing the values of this ratio.

During the computing of the fatigue strength of machine elements determined for various cycles for a given material, Smith and Haigh fatigue plots are made [2], [3]. The Smith plot is the most often applied way of presenting all results concerning constructional materials. The values of average tension σ_m are drawn on the abscissas axis, whereas the values of σ_{max} and σ_{min} corresponding with the values of fatigue strength are drawn on the ordinates axis. (Figure 2). Across the points $A(0, Z_{go})$ and $B(1/2Z_{gj}, Z_{gj})$ a straight line is drawn and a horizontal straight line determined by the R_e value. From the zero point of the coordinate system a straight line is drawn at an angle of 45° to the $0 - \sigma_m$ axis. This latter line assess point B by crossing the horizontal line. In this way we obtain the upper branch of the Smith graph formed by the straight line segments *AC* and *CD*.

Investigations concerning the elaboration of a Wöhler fatigue curve for yarns we began in the year 2000s, selecting worsted woal yarns as the first research object. Despite the conference information [17], our investigation results has not yet been published. The aim of the research presented in this article was to elaborate and propose our own methodology concerning the design of a Wöhler fatigue curve for linear textile products, especially for yarns. Knowledge of fatigue properties allows to predict breakages of yarns in subsequent textile processes. It is very interesting to determine to what degree a degree yarn fulfills the criteria of technological usefulness for the subsequent production stages of spinning, weaving and knitting, with temporary tensions which may be similar to the maximum values of its strength. On the other hand, while using ready-made textile products, we have to deal with alternating loads, far from the maximum level and of low frequency. Therefore, ranges of tension which are much lower than the material strength were also taken into consideration during examinations.

Experimental

Material of research

Worsted single wool yarn with linear density of 16 tex and 25 tex, produced with the use of a FIOMAX 2000 ring spinning machine from SUESSEN, was the object of our tests. The Physical prop-

***Table 1.** Physical properties of tested yarns, and basic properties of top and fibres used for their production, before the winding operation.*

Analyzed parameter of top and yarn	Unit	Linear mass of yarn	
		16 tex	25 tex
Mean value of fibre diameter	μm	19.0	19.2
Mean length of fibres	mm	65.0	66
Maximum length of fibres	mm	139.0	137
Fibre contents shorter than 40 mm	%	10.0	18.7
Direction and number of twisted yarn	rev./m	Z790	Z630
Breaking force of yarn R_{mn}	cN	82.9	152
Breaking tenacity of yarn W_t	cN/tex	5.18	6.19
Coefficient of variation of breaking force $V(R_{mn})$	%	10.64	13.95
Breaking elongation of yarn E_{rn}	%	8.81	9.78
Coefficient of yarn mass variation CV Uster	%	20.27	16.47

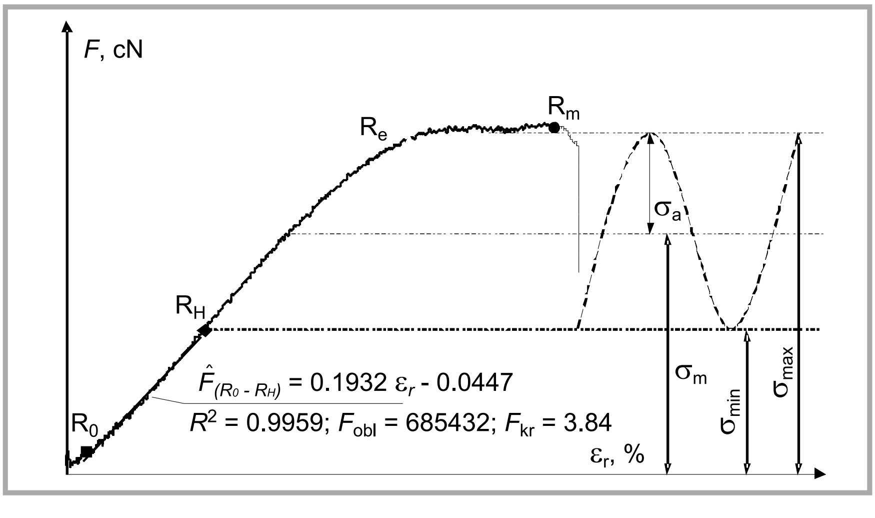

***Figure 3.** The tension curve of the yarn with characteristic points; R_0 – the end of straightening the yarn, R_H – the value of force assumed as the limit of elasticity (limit at the linear course of the curve, R_e – the value of force assumed as the beginning of plasticity, R_m – tensile strength of yarn, σ_{min} – minimal value of load cycle, σ_{max} – maximal value of load cycle, σ_m – average value of load cycle, σ_a – amplitude value of load cycle.*

erties of tested yarns and basic properties of fibres and top used for producing the yarn are listed in Table 1 (see page 55).

Methods

The yarns were subjected to variable loads of fatigue cycles executed up to the final moment of breaking. In order to design a graphic presentation of fatigue strength, tests were executed for seven groups of measuring series, corresponding with the specific loud amplitude of a given cycle. The number of series was increased with respect to the stochastic character of the yarn strength properties.

Our research was based on the tension curve, which is shown with its characteristic points in Figure 3.

Changing the load amplitude value σ_a of the cycle, we checked the relation between the cycle load amplitude and the number of load cycles N to the final destruction in the subsequent measuring series.

Fatigue tests were performed while ensuring a constant value of amplitude load σ_a during each cycle (loading – unloading) and stable frequency of cycle of 1.5 Hz.

In each series of tests the quantities: σ_{min}, σ_{max} and σ_m were also assumed as constant, where:

$$\sigma_m = (\sigma_{max} + \sigma_{min})/2 \text{ const.} \qquad (4)$$

It was necessary to assign a range of amplitude of load cycles – σ_a, before starting the fatigue strength test (see Figure 3):

$$\sigma_a = (\sigma_{max} - \sigma_{min})/2 \qquad (5)$$

The values of σ_{max} and σ_{min} were assessed using a 5544 ISTRON tensile tester with Merlin and Test Profiler software. The minimal value of the load cycle was assumed as the average value of the modulus of initial elasticity in cN/tex

$$\sigma_{min} = M_{F_p} = \text{const.} \qquad (6)$$

was determined on the basis of 50 measurements executed for every kind of yarn $T_{tp} = [16, 25]^T$ tex [18].

The modulus of the initial elasticity was determined on the basis of a segment of the straight line R_0 - R_H of the tension curve (see Figure 3) consistent with Hook’s law, where the point R_H was established on the conventional elasticity limit. Hook’s law is theoretically fulfilled within the straight line segment R_0 - R_H, because elastic strains appear instantaneously within this range, and delayed elastic strain appears only in a limited range.

Taking into account traditional denotation used in textile science, we decided that in this article the linear elastic limit will be called ‘the initial modulus of elasticity’, in cN/tex.

The maximum value of the load cycle – σ_{max} was determined on the basis of the average value of the yarn breaking tension – W_t in cN/tex, and taking into consideration the ratio α_c of the load over the cycle

$$\sigma_{max} = \alpha_c \cdot W_t = const. \qquad (7)$$

The following ratios of load – α_c were arbitrarily accepted on the basis of experience:

$$\alpha_c = [0.98, 0.95, 0.85, 0.75, 0.65, 0.55, 0.50]^T$$

Mean breaking yarn tension, in cN/tex, was determined from 50 tests for each yarn. $T_{tp} = [16, 25]^T$ tex [18].

The acceptance of the maximum top load cycle as value $\sigma_{max_g} = 0.98 \cdot W_t$ was caused by the fact that in the case of fatigue tests – for $\sigma_{max} > 0.98 \cdot W_t$, over 90% of samples were destroyed in a number of fatigue cycles of $N_p \leq 1$ and $N_n \leq 1$.

However, accepting the minimum of the higher value of the load as $\sigma_{max_d} = 0.50 \cdot W_t$ was caused by the fact that the fatigue strength was similar to the range of the fatigue strength limit, which is assumed as the fatigue strength of the material.

On the other hand, a higher minimum load cycle ratio smaller than $\alpha_c \leq 0.5$ causes an unlimited work of the material at cyclic alternating loads.

The distance between the jaws (500 mm) of the tensile tester and the method of mounting the yarn was the same as for measurements of static strength. The number of fatigue cycles N_n to the moment of yarn breaking was assessed using a 5544 ISTRON tensile tester and 20 measurements were carried out in each series.

Results

The values of the particular loads accepted for the tests on16 tex and 25 tex

***Table 2.** Values of the particular loads accepted for tests with use of an Instrem tensile tester for 16 tex yarns; $\sigma_{n\,m} = (\sigma_{n\,min} + \sigma_{n\,max})/2$, $\sigma_{n\,a} = (\sigma_{n\,max} - \sigma_{n\,min})/2$, n - subsequent series of cycles (n = 1, ..., 7) .*

Ratio of load of cycle α_c	Minimal load at every cycle $\sigma_{n\,min}$, cN/tex	Maximal load at every cycle $\sigma_{n\,max} = \alpha_c W_t$, cN/tex	Average load at every cycle $\sigma_{n\,m}$, cN/tex	Range of amplitudes of a load at every cycle $\sigma_{n\,a}$, cN/tex
0.98	1.33	5.08	3.20	1.87
0.95		4.92	3.13	1.80
0.85		4.40	2.87	1.54
0.75		3.89	2.61	1.28
0.65		3.37	2.35	1.02
0.55		2.85	2.09	0.76
0.50		2.59	1.96	0.63

***Table 3.** Values of the particular loads accepted for test with the use of an Instrom tensile tester for 25 tex yarns.*

Ratio of load of cycle α_c	Minimal load at every cycle $\sigma_{n\,min}$, cN/tex	Maximal load at every cycle $\sigma_{n\,max} = \alpha_c W_t$, cN/tex	Average load at every cycle $\sigma_{n\,m}$, cN/tex	Range of amplitudes of a load at every cycle $\sigma_{n\,a}$, cN/tex
0.98	1.76	5.96	3.86	2.10
0.95		5.78	3.77	2.01
0.85		5.17	3.46	1.70
0.75		4.56	3.16	1.40
0.65		3.95	2.86	1.10
0.55		3.34	2.55	0.79
0.50		3.04	2.40	0.64

yarns, with the use of a INSTRON tensile tester, are presented in Tables 2 and 3, respectively.

The results of tests of the numbers of fatigue cycles at subsequent loads of the cycle for yarns 16 tex and 25 tex are shown in Tables 4 and 5.

Analysing the course of variabilities for 16 tex for yarns of the number of fatigue cycles at the subsequent loads of each cycle as a function of the value of load in each fatigue cycle up to point of destruction of the yarns, it is possible to observe an increase in fatigue strength for the ratio of load of α_c = 55%. A sudden drop in fatigue strength appears within the range of $\alpha_c = 98 \div 95\%$. This is caused by a small fatigue resistance to the maximum temporary loads. The mean limited strength $P_{g,\alpha}$ = 50% equals 120577 cycles for the ratio of load of a cycle of α_c = 50%, i.e. in the area of loads, after which the yarn reaches the linear limit of elasticity. Similar indications are valid for 25 tex yarn with the difference that $P_{g,\alpha}$ = 50% equals 162234 cycles.

On the basis of measurement results of the number of fatigue cycles in the subsequent series of load cycles presented in Tables 4 and 5, and in order to clearly illustrate the dependences Wöhler curves were drawn and are presented in Figures 4 and 5 (see page 60).

Drawing Wöhler curves for 16 tex and 25 tex yarns was possible thanks to the statistics determined with the use of variance test analysis, and according to the double classification for the number of fatigue cycles in the subsequent load cycle series amounted to:

- $F_{calc} = 163.91 > F_{critic} = 6.61$, for yarns 16 tex,
- $F_{calc} = 67.461 > F_{critic} = 6.61$, for yarns 25 tex.

Analysing the course of the Wöhler fatigue curve for yarns with linear density 16 tex (see Figure 4), we can state that the fatigue strength begins to grow at the moment when the ratio of the load of the cycle exceeds the value α_c = 65%. A sudden increase in fatigue strength occurs when the ratio of the charge of the series equals α_c = 55%. The mean limited strength $P_{g,\alpha=50\%}$ = 120577 cycles occurs when the load cycle ratio equals $\alpha_c \leq 0.50 \cdot W_t$. This causes unlimited work of the material at periodicalternating loads.

Table 4. *Number of fatigue cycles at subsequent loads of cycle for 16 tex yarns; S – standard deviation, V – coefficient of variation.*

Item number	Number of fatigue cycles at subsequent loads of cycle N_n, cycles for α_c						
	98%	95%	85%	75%	65%	55%	50%
1	2	12	117	101	16102	62375	136288
2	2	18	565	359	12916	49875	96823
3	2	17	406	1286	10295	46442	59613
4	2	21	186	301	946	62717	136315
5	2	17	90	1099	6785	77699	126320
6	2	23	166	933	14844	49518	92697
7	2	28	626	1925	9300	77235	107461
8	2	30	425	1585	11068	45671	132837
9	1	16	207	1190	8774	76930	110317
10	2	18	148	1508	9088	62282	127244
11	2	11	208	265	8827	22835	90431
12	2	12	358	589	7913	72228	75892
13	2	30	189	458	9746	25578	123558
14	1	18	280	1330	9699	51076	132656
15	2	30	242	2085	8528	52310	131039
16	2	23	256	1626	11144	26149	108022
17	2	19	432	893	8590	56216	183637
18	2	11	443	1582	7528	65499	189768
19	1	24	205	732	8187	73873	105085
20	2	30	331	703	7890	66001	145548
Nn	2,0	20,0	294,0	1027,5	9408,5	56125,5	120577,5
S	0,3	6,6	148,7	583,6	3109,9	16998,0	31633,3
V	18,3	32,4	50,6	56,8	33,0	30,2	26,2

Table 5. *Number of fatigue cycles at subsequent loads of cycle for 25 tex yarns; S – standard deviation, V – coefficient of variation.*

Item number	Number of fatigue cycles at subsequent loads of cycle N_n, cycles for α_c						
	98%	95%	85%	75%	65%	55%	50%
1	2	46	657	1661	16348	62375	192581
2	2	396	69	2887	1914	49875	138547
3	2	32	695	4377	2893	66168	171170
4	2	51	238	253	4398	69223	208879
5	2	88	156	5345	6877	62757	226024
6	2	62	86	4435	7564	74425	225565
7	2	42	83	1202	4756	69389	82447
8	2	110	140	3333	5687	76733	153678
9	2	126	128	1934	8765	69786	229248
10	2	21	313	1544	12345	63601	172527
11	2	64	270	3722	5674	74311	147014
12	1	43	138	2345	8943	56746	130375
13	2	123	199	2691	7654	68984	194747
14	2	137	340	1972	4339	61701	151374
15	1	128	154	4378	9743	59913	132570
16	2	187	580	2987	3476	64346	84844
17	2	143	46	5876	8345	65623	140492
18	2	38	310	2667	2457	67843	146471
19	2	89	294	1234	4055	51314	167163
20	2	22	233	723	3495	52671	148980
Nn	2,0	97,0	256,0	2778,0	6486,0	64389,0	162234,0
S	0,3	84,6	189,1	1540,5	3603,9	7534,8	41716,9
V	15,3	86,8	73,7	55,4	55,5	11,7	25,7

Analysing the course of the Wöhler fatigue curve for yarns with linear density 25 tex (see Figure 5), we indicated the same behavior. The mean limited strength of $P_{g,\alpha=50\%}$ = 162235 cycles occurs when the cycle load ratio is

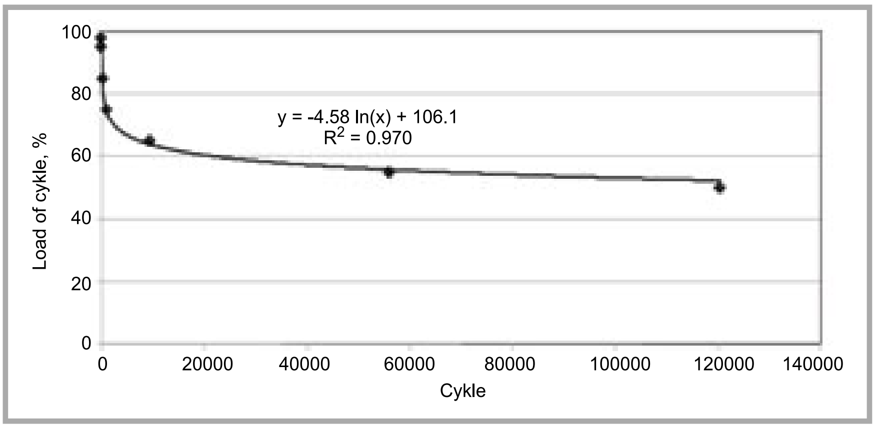

Figure 4. *Fatigue Wöhler curve for yarns with linear density 16 tex.*

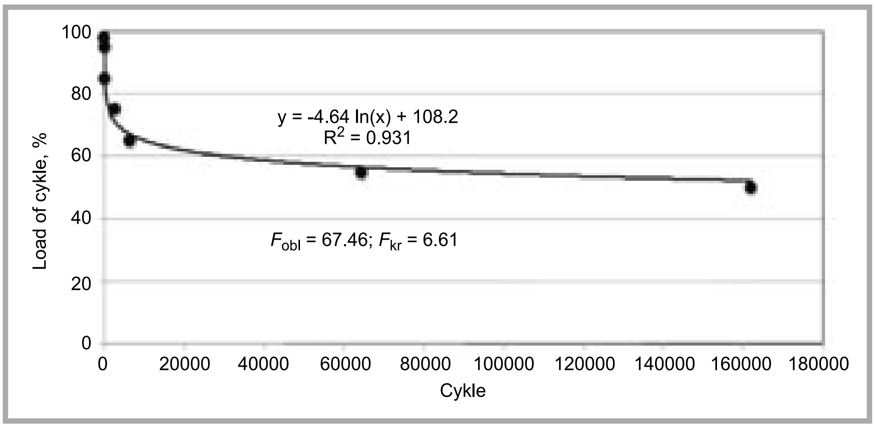

Figure 5. *Fatigue Wöhler curve for yarns with linear density 25 tex.*

$\alpha_c \le 0.50 \cdot W_t$ which causes the same effect as for 16 tex yarn.

Conclusions

Analysing the presented investigations, it is possible to indicate that:

1. The linear density of worsted wool yarns twisted with the same twist ratio - α_m = 100 is not significantly influenced by the course of the Wöhler curve up to the ratio of load α_c = 55%. Differences in variability of the Wöhler curves of the tested yarns are visible only at limited fatigue strength of yarns, i.e., when α_c = 50%.

2. The fatigue strength limit of the tested yarns grows simultaneously with the increase in their linear density.

3. Plotting Wöhler fatigue curves is very laborious, but it helps to interpret the phenomena occurring during fatigue investigations.

4. Fatigue Wöhler curves allow to illustrate the functional relationship between the ratio of the periodically acting load, expressed in the percentage of static strength of the material and the corresponding number of load cycles, causing the destruction of the tested yarns.

5. Knowledge of the problems related to the dynamic fatigue strength of yarns is very significant when analysing the production and use of ready-made textile goods. Knowledge of the fatigue strength of textile products not only has theoretical and cognitive importance but also utility, particularly important in the case of products de voted to technical articles, (such as tyre cords, driving and conveyor belts, transporters, etc), as well as work clothes.

References

1. Ciszewski A., Radomski T.: „Materiały konstrukcyjne w budowie maszyn", PWN, Warsaw 1989.
2. Osiński Z., Bajon W., Szucki T. „Podstawy konstrukcji maszyn", PWN, Warsaw 1980.
3. Niezgodziński M. E., Niezgodziński T. „Wytrzymałość materiałów", PWN, Warsaw 1998.
4. Kocańda S., Szela J. „Podstawy obliczeń zmęczeniowych", PWN, Warsaw 1985.
5. Nowak M., Włochowicz A. „Analysis of fatigue resistance changes and microstructure of polyamide as a function of the thermal state of the mould and thermal processing" (in Polish);Mechanika Teoretyczna i Stosowana, Polskie Towarzystwo Mechaniki Teoretycznej i stosowanej, kwartalnik, vol. 17, issue 4, Warsaw 1979, PWN.
6. Włochowicz A., Nowak M., „Investigation of the submicroscopic structure of polyamide after fatigue tests" (in Polish) Scientific lectures of TUŁ, Textile Science 1971.
7. Włochowicz A., Nowak M., „Changes of the polyamide 6 structure in the process of fatigue destruction" (in Polish) Mechanika Teoretyczna i Stosowana, Polskie Towarzystwo Mechaniki Teoretycznej i stosowanej, kwartalnik, vol. 19, issue 1, Warsaw 1981, PWN.
8. Żurek W., „Struktura liniowych wyrobów włókienniczych", Wydawnictwa Naukowo-Techniczne, Warszawa 1989, p. 356.
9. Włochowicz A., Linek M., „Application of the Zurkow – Abasow theory for description fatigue resistance features polyethylenotereftalate fibres", Acta Polymerica 1981/32, No. 8, pp. 445-542.
10. Urbańczyk G., „Wybrane zagadnienia z nauki o włóknie." Stowarzyszenie Włókienników Polskich, Łódź 1969 r., pp. 69-79.
11. Urbańczyk G., „Fizyka Włókna", Politechnika Łódzka, Łódź 2002 r., pp. 144-235.
12. Miklaszewski J., „Pomiary wytrzymałości własności materiałów", Biblioteczka Metrologii, Education Materials, Bydgoszcz 1981r., s.39-42.
13. Gizmajer A., „Pomiar siły" , Poradnik legalizatora nr 3, Wydawnictwa Normalizacyjne „Alfa", Warszawa 1987r., pp. 77-82.
14. Jakubowicz A., Orłoś Z., „Wytrzymałość materiałów"", Wydawnictwa Naukowo-Techniczne, Wydanie szóste zmienione, Warszawa 1984, pp. 580-613.
15. Nowak M., Zawadzki J., „Fatigue resistance of polymer materials" (in Polish), Research work of the Institute of Material Science and Technical Mechanics, Wroclaw 1976.
16. Przygocki W., Włochowicz A., „Fizyka Polimerów" Wydawnictwa Naukowe PWN, Warsaw 2001.
17. Józkowicz I., Włochowicz A., Machnio M.S., Drobina R., „Fatigue Properties of Spliced Connections of Yarn-Ends" 9th International Cotton Conference 'Future of Cellulose Fibres Considering Development Trends of the Textile-Clothing Industry, Gdańsk, 6-7.09.2007.
18. Mańczak K.; „Metody identyfikacji wielowymiarowych obiektów sterowania", Ed. WNT, Warsaw 1979 pp. 134-172.

Received 15.11.2007 Reviewed 15.01.2008