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Variational Approach to Nonlinear Coupled Oscillators Arising in Sirospun Yarn Spinning

Abstract
A description of stable working conditions for spinning two-strand yarn is given. A theoretical model underlying Sirospun yarn spinning is also given. Based on the variational formulation established for the coupled oscillators arising in two-strand yarn spinning, a simple analytical model for the forces that determine the nonlinear oscillation during the yarn spinning, as a function of inlet velocities, strand-spacing, and forces acting on the strands is proposed, which reveals that resonance occurs when the convergence angle is near 127 degrees.

Key words: *two-strand yarn, Sirospun, Sirofil, dynamical model, variational principle, coupled nonlinear oscillator, resonance, three-strand yarn.*

Introduction

Recently, many concepts in applied mathematics and applied mechanics have been successfully applied to modelling the process of yarn spinning; for example, the conservation laws in fluid mechanics have been applied to establish a quasi-static model for two-strand yarn spinning [1], the homotopy perturbation method [2-4] has been applied to nonlinear problems arising in yarn spinning [5, 6], the technology of neural networks is applied to describing yarn spinning [7-9], intelligent systems have been applied as a basis for improving the position and competitiveness of the textile industry [10], the kinematic approach to the analysis of the sewing mechanisms of an overedge machine [11], and ancient Chinese mathematics to accurate identification of the shape of the yarn balloon [12]. This paper will apply the variational method to study the nonlinear phenomenon in the process of two-strand yarn spinning.

Stable working conditions

Traditionally, two-strand yarns have been used for weaving because they are stronger, and the twisting operation binds the surface fibres into the yarn structure so that it is smoother and more resistant to abrasion during weaving. Two-strand spun yarns are now widely used in the worsted industry. The strands are texturised to improve the bulk of the resultant yarns, which have been proved to possess more desirable properties. For example, the weaveability of the fabric formed by the Sirospun yarns is significantly improved over its counterpart yarns.

The dynamical character of the system strongly depends upon the convergent angles, and we should guarantee a stable working condition during the spinning procedure. The observation shows that a slight change in the densities and the velocities of the two strands may induce chaotic motion of the strands. Przybyl studied the stable working conditions of the twisting-and-winding system of a ring spinning frame [13]. Herein we suggest a very simple but effective approach to determining stable working conditions for two-strand yarn spinning.

We first assume the system is in a stable condition, and then a control volume is chosen as illustrated in Figure 1. Based on the characters of the system's dynamics (mass conservation) [1], the total mass in the control volume remains unchanged. This requires fulfilling the following formula:

$$\pi R^2 \rho u = \pi R_1^2 \rho_1 u_1 + \pi R_2^2 \rho_2 u_2 , \qquad (1)$$

or

$$R^2 \rho u = R_1^2 \rho_1 u_1 + R_2^2 \rho_2 u_2 . \qquad (2)$$

where:
ρ_1, ρ_2 - are the densities of the above two strands,
ρ - is the density of the spun yarn,
u_1, u_2 - are the velocities of the two strands,
u - is the velocity of the spun yarn,
R_1, R_2 - are the radii of the two strands,
R - is the radius of the resultant yarn.

If:

$$R^2 \rho u \neq R_1^2 \rho_1 u_1 + R_2^2 \rho_2 u_2 ,$$

the stable working condition is broken, and the spinning system must operate in an unsteady condition.

For multiple-strand yarn spinning, the stable working condition can be readily obtained as follows:

$$R^2 \rho u = \sum_{k=1}^{n} R_k^2 \rho_k u_k \qquad (3)$$

where ρ_k, u_k, R_k are respectively the density, velocity and radius of the k-th strand.

Three-strand yarn can be designed for smart fabric, and can have many advantages over two-strand yarn. Our group in Donghua University, Shanghai, China, is

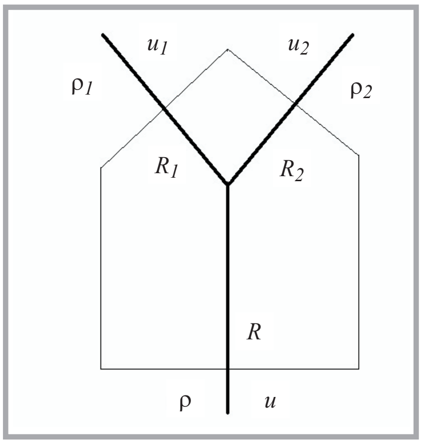

Figure 1. *Control volume for stable spinning process; ρ_1, ρ_2 - densities of the strands, ρ - density of the spun yarn, u_1, u_2 - velocities of the strands, u - velocity of the spun yarn, R_1, R_2 - radii of the strands, R - radius of the resultant yarn.*

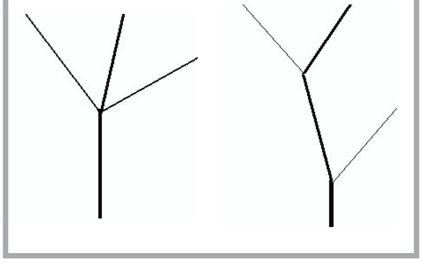

Figure 2. *Three-strand yarn spinning; different methods at joining.*

doing research on three-strand yarn (see Figure 2). The stable working condition for the three-strand yarn is as follows:

$$R^2\rho u = R_1^2\rho_1 u_1 + R_2^2\rho_2 u_2 + R_3^2\rho_3 u_3 . \quad (4)$$

The three-strand yarn can be prepared in a single processing step, and far-reaching implications are emerging for its use in applications including intelligent textile and multi-functional materials. We will discuss how to determine the convergent points and to establish its dynamical model in future papers.

The aim of this paper is to use the variational method to study the nonlinear phenomena in the process of two-strand yarn spinning. For this purpose, a simple mathematical model was developed. We do not present the boundary limits concerned with the type of yarn, linear masses of the strands, spinning velocity and the geometrical dimensions, as it appears that this is not necessary for these commonly-used qualities at this stage of consideration.

Dynamical model and variational principle

Let the ends of the two strands above the convergent point be fixed at a distance **2*L*** apart, and the equilibrium position be ***H*** below. Let ***x***-direction be the direction of strand-spacing, and ***y***-direction be perpendicular to the ***x***-direction as illustrated in Figure 3. The density of strands and spinning velocity are assumed to be constant in the Sirospun system, and the equations of the motion in ***x***- and ***y***-directions are:

$$M\frac{d^2x}{dt^2} + F_1\cos\alpha - F_2\cos\beta = 0, \quad (5)$$

$$M\frac{d^2y}{dt^2} + F_1\sin\alpha + F_2\sin\beta - F = 0. \quad (6)$$

Here ***M*** is the total mass of a fixed control volume, $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, ***F*** - the forces acting in strands and yarn respectively. This control volume is chosen in such a way that the mass centre coincides with the convergent point (***O***) of the two strands.

By a simple manipulation, we simplify Equations (5) and (6) as follows [5]:

$$\frac{d^2x}{dt^2} + \omega_x^2 x + axy = 0, \quad x(0) = A,\ x'(0) = 0, \quad (7)$$

$$\frac{d^2y}{dt^2} + \omega_y^2 y + bx^2 + cy^2 = 0, \quad y(0) = B,\ y'(0) = 0, \quad (8)$$

$$\begin{aligned}
\omega_x^2 &= 2F\left[(L^2+H^2)^{-1/2} - L^2(L^2+H^2)^{-3/2}\right]M^{-1},\\
\omega_y^2 &= 2F\left[(L^2+H^2)^{-1/2} - H^2(L^2+H^2)^{-3/2}\right]M^{-1},\\
a &= 2F\left[-H(L^2+H^2)^{-3/2} + 3L^2H(L^2+H^2)^{-5/2}\right]M^{-1},\\
b &= F\left[-H(L^2+H^2)^{-3/2} + 3L^2H(L^2+H^2)^{-5/2}\right]M^{-1},\\
c &= F\left[-3H(L^2+H^2)^{-3/2} + 3H^3(L^2+H^2)^{-5/2}\right]M^{-1}
\end{aligned} \quad (8')$$

Equations 8'. *The quantities and constants described by Equation 8.*

where the particular quantities (ω_x, ω_y) and constants (a, b, c) are described by Equations (8').

In this paper we will apply the variational method to analysis of Equations (7) and (8). The variational method is widely applied to dealing with nonlinear problems [14 - 17]. Equations (7) and (8) can be derived from a functional as stationary conditions. To search for such a functional, in view of the semi-inverse method [14],

$$J(x,y) = \int \left\{ -\frac{1}{2}\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\omega_x^2 x^2 + \frac{1}{2}ax^2y + \Pi \right\} = 0 \quad (9)$$

$$L = -\frac{1}{2}\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\omega_x^2 x^2 + \frac{1}{2}ax^2y + \Pi\left(y, \frac{dy}{dt}\right) \quad (11)$$

$$\frac{\delta\Pi}{\delta y} = \frac{\partial\Pi}{\partial y} - \frac{\partial}{\partial t}\left(\frac{\partial\Pi}{\partial y'}\right) + \frac{\partial^2}{\partial t^2}\left(\frac{\partial\Pi}{\partial y''}\right) + \cdots \quad (13)$$

$$\frac{\delta\Pi}{\delta y} = -\frac{1}{2}ax^2 = \frac{a}{2b}\left(\frac{d^2y}{dt^2} + \omega_y^2 y + cy^2\right) \quad (14)$$

$$\Pi = -\frac{1}{2}ax^2 = \frac{a}{2b}\left(-\frac{1}{2}\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}\omega_y^2 y^2 + \frac{1}{3}cy^3\right) \quad (15)$$

$$J(x,y) = \int \left\{ -\frac{1}{2}\left(\frac{dx}{dt}\right)^2 - \frac{a}{4b}\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}\omega_x^2x^2 + \frac{a}{4b}\omega_y^2y^2 + \frac{1}{2}ax^2y + \frac{ac}{6b}y^3 \right\} dt \quad (16)$$

$$\frac{1}{2}\left(\frac{dx}{dt}\right)^2 + \frac{a}{4b}\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}\omega_x^2x^2 + \frac{a}{4b}\omega_y^2y^2 + \frac{1}{2}ax^2y + \frac{ac}{6b}y^3 = H \quad (17)$$

$$\left(\frac{dx}{dt}\right)^2 + \frac{a}{2b}\left(\frac{dy}{dt}\right)^2 + \omega_x^2(x^2 - A^2) + \frac{a}{2b}\omega_y^2(y^2 - B^2) + a(x^2y - A^2B) + \frac{ac}{3b}(y^3 - B^3) = 0 \quad (18)$$

$$R(t) = A^2\Omega_x^2\sin^2\Omega_x t + \frac{a}{2b}B^2\Omega_y^2\sin^2\Omega_y t + \omega_x^2(A^2\cos^2\Omega_x t - A^2) + \frac{a}{2b}\omega_y^2(B^2\cos^2\Omega_y t - B^2) + a(A^2B\cos^2\Omega_x t\cos\Omega_y t - A^2B) + \frac{ac}{3b}(B^3\cos^3\Omega_y t - B^3). \quad (21)$$

Equations: 9, 11, 13 - 18, and 21.

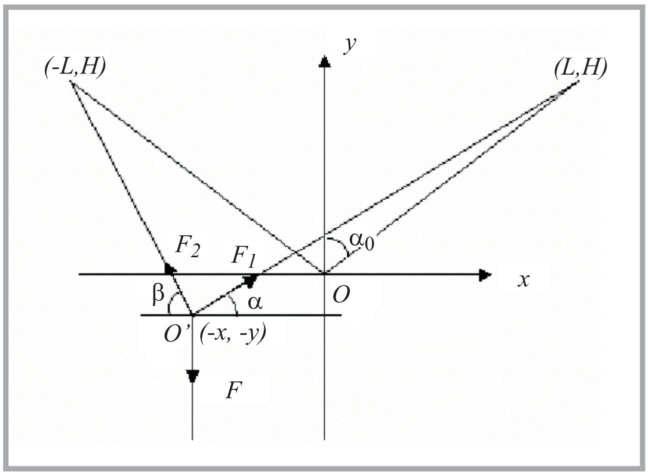

Figure 3. *The dynamical illustration of two-strand spun yarns; F, F_1, F_2 – forces acting in yarn and the both strands; α, β – angles between the strands and the horizontal axis at equilibrium positions and the dynamically displaced position, α_0 - convergence angle; the converqence paint is displacemed from 0(0,0) to 0'(-x,-y).*

we begin with the trial-functional presented by Equation (9), where Π is an unknown function of $\boldsymbol{y}$ and its derivatives. The stationary condition of the above functional (3) with respect to $\boldsymbol{x}$ can be expressed as follows:

$$\frac{\partial L}{\partial x} - \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x'}\right) = 0, \tag{10}$$

where $x' = dx/dt$, and $\boldsymbol{L}$ is the Lagrangian as defined by (11).

Equation (10) reduces to Equation (7) by the substitution of (11) into (10). Now calculating the variation of (9) with respect to $\boldsymbol{y}$, we obtain the following stationary condition:

$$\frac{1}{2}ax^2 + \frac{\delta \Pi}{\delta y} = 0, \tag{12}$$

where $\frac{\delta \Pi}{\delta y}$ is the variational derivative with respect to $\boldsymbol{y}$ defined by Equation (13).

We search for such an $\boldsymbol{F}$ so that Equation (12) is equivalent to Equation (8). To this end, we set (14), from which Π can be identified as (15).

Finally we obtain the following necessary variational formulation presented by (16).

From the functional (16), we obtain the Hamiltonian invariant (17), where $\boldsymbol{H}$ is the conserved constant which can be determined from the initial conditions: $x(0) = A$, $x'(0) = 0$, $y(0) = B$, $y'(0) = 0$, so Equation (17) can be re-written in the form of (18).

The system has a period solution, so we assume that

$$x = A \sin\Omega_x t, \tag{19}$$

$$y = B \sin\Omega_y t, \tag{20}$$

where Ω_x, Ω_y are frequencies in $\boldsymbol{x}$- and $\boldsymbol{y}$-directions respectively.

Substituting (19) and (20) into (18), we obtain the residual (21).

We apply the collocation method to identify the frequencies. Collocating at $\Omega_x t = \pi/4$ and $\Omega_y t = \pi/4$, and putting $R = 0$ gives

$$R\left(t = \frac{\pi}{4\Omega_x}\right) = R_1(\Omega_x, \Omega_y) = 0, \tag{22}$$

$$R\left(t = \frac{\pi}{4\Omega_y}\right) = R_2(\Omega_x, \Omega_y) = 0. \tag{23}$$

From (22) and (23), the frequencies can be approximately determined. It is very clear that the frequencies (Ω_x and Ω_y) depend upon strand-spacing, the convergent angle α_0. For the given parameters ($\boldsymbol{H}$, $\boldsymbol{L}$, $\boldsymbol{A}$ and $\boldsymbol{B}$), the values of the frequencies can be readily determined from Equations (22) and (23) by MatLab. The Lissajou figures are illustrated in Figure 4 for various different values of Ω_x and Ω_y.

Resonance

In order to find the resonance condition of the coupled oscillator (7) and (8), we have written (7) and (8) in the approximately forms of (24) and (25).

Resonance occurs when $\omega_x = \Omega_x + \Omega_y$, or $\omega_x = \Omega_y - \Omega_x$, or $\omega_y = 2\Omega_x$, or $\omega_y = 2\Omega_y$, where Ω_x, Ω_y are solved approximately by collocation of the Hamiltonian invariant, Equation (21).

In case a is small, then $\Omega_x \approx \omega_x$. In such a case, resonance occurs when $2\omega_x = \omega_y$ (i.e. $\boldsymbol{L} = \boldsymbol{2H}$ or the convergence angle $2\alpha_0 = 2 \times 63.43°$).

Conclusion

To conclude, we obtain a Hamiltonian invariant, from which the frequencies are approximately determined by two-point collocation. The frequencies depend upon the strand-spacing ($\boldsymbol{H}$) and the convergent angle (α_0). Note that the convergent angle is determined from the inlet velocities and the forces acting on the strands. So, we obtain a very simple analytical model describing the nonlinear oscillation during the yarn spinning. Resonance occurs when the convergence angle is near 127°, so the design convergence angle should be far from 127°. The linear dynamical model [18] predicts an optimal convergence angle of 90°.

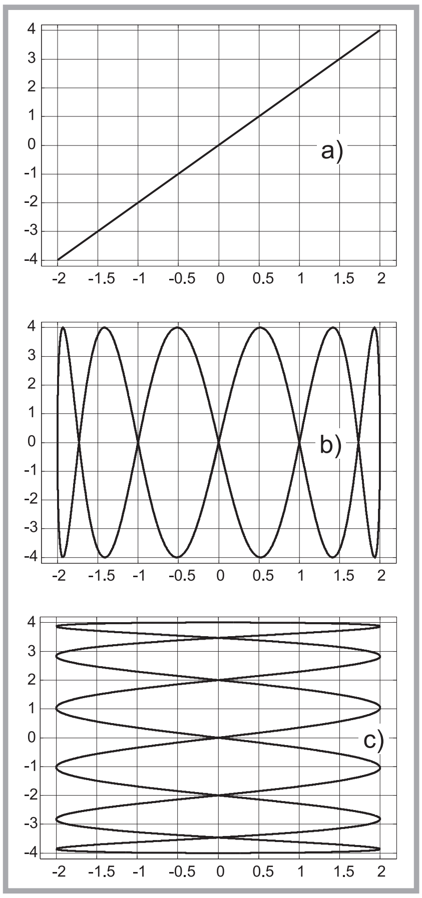

***Figure 4.** Trajectories of the convergent point under different conditions;*
a) $\Omega_x : \Omega_y = 1 : 1$, b) $\Omega_x : \Omega_y = 1 : 6$, c) $\Omega_x : \Omega_y = 6 : 1$.

$$\frac{d^2x}{dt^2} + \omega_x^2 x + \frac{1}{2}aAB\left[\cos(\Omega_x + \Omega_y)t + \cos(\Omega_x - \Omega_y)t\right] = 0$$
$$x(0) = A \qquad x'(0) = 0 \tag{24}$$

$$\frac{d^2y}{dt^2} + \omega_y^2 y + \frac{1}{2}bA^2(1 + \cos 2\Omega_x t) + \frac{1}{2}cB^2(1 + \cos 2\Omega_y t) = 0$$
$$y(0) = B \qquad y'(0) = 0 \tag{25}$$

***Equations:** 24, and 25.*

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The 9th International Cotton Conference

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