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Applying Notions and Methods of Fuzzy Set Theory to Analysis of Microstructure of Cotton Fracture

Abstract
The degree of cotton maturity is a significant parameter characterising the properties of a fibre. Many textile properties depends on the degree of cotton maturity. A frequently applied method for determining this is based on the so-called 'Russian standards' which have been long applied in Poland. This paper proposes another method for determining the degree of cotton maturity, based on a cotton fracture image from a scanning electron microscope. For this analysis of the cotton image, the elements of fuzzy set theory are applied. Since categories of images of cotton fracture do not have clearly defined boundaries, the application of fuzzy set theory for determining the degree of maturity seems justified.

Key words: *cotton fibres, fibre fracture, microstructure, maturity, fuzzy set theory, SEM images.*

Introduction

The properties of textiles, and ultimately their industrial utility, depend among other things on the degree of cotton maturity. Determining this degree enables us to predict textile properties. Literature regarding determining this degree is limited to analyses of macroscopic features of single fibres, or bundles of fibres [1].

A well-known standardised method using polarised light enables us to determine the average maturity of cotton fibre bundles [2]. Also well-known is the AFIS system to determine the degree of maturity, in which a computer analysis of cross-section images of single fibres is applied [3].

The commonly known 'Soviet standards' method is a comparative method, used as a base for the standardised PN method.

This method consists in observing the longitudinal view of the fibre (its transparencies, breaks and numbers of turns) under an optical microscope. The fibre's shape is then comprised with the 'Soviet standard' pattern. This estimation is burdened to a large degree with the subjective factor.

The presented paper introduces a method for determining the degree of cotton maturity based on images from a scanning electron microscope (SEM) of the shape of the cotton fracture. For this purpose, the fuzzy set theory method is applied. We were inspired to use this method by the ambiguity of the characteristic features of cotton fractures as assigned for a given degree of fibre maturity. Whether or not the given features of fracture occur is not entirely unambiguous.

Procedure for cotton maturity identification by the fuzzy set

The procedure for identifying cotton fibre with the fuzzy set approach is shown in Figure 1.

Let us analyse the stages of this procedure. A population of 200 cotton filament fibres is randomly chosen from the cotton samples. For this population a maturity degree has been determined according to a standard PN pattern. This comparative method is based on qualitative analysis of such features of the cotton as transparency, breaks, number of turns and comparison with standard PN pattern. It is obvious that such comparisons are to some degree subjective. The boundaries between individual degrees of maturity are not sharp. For these cotton fibres, after the PN standard determining the maturity degree, according to the PN standard cotton fractures are prepared. Each cotton fibre is stretched on an Instron tensile tester to breaking point.

Images of cotton fractures depend on the cotton's microstructure (crystallinity index, sizes of crystallites, and texture), and consequently on its maturity degree [1, 6]. Detailed analyses of cotton fracture images allow us to distinguish 14 different categories [6]. Examples of this category images together with the assigned number are shown in Figure 2 (set Y). The following categories of fracture, which are described with linguistic variables and numbered, are identified on the broken fibre ends; tensile rubber (1), broken tree (2), cracked untwisted rope (3), cohesion of broken fibre end (4), macro- and micro- fibrils occurring in the broken fibre end (5), ductile callosities (6), shapeless agglomerate (7), folds (8), wrinkles (9), furrows (10), grooves (11), stratification of morphological fibre structure (12), appearance of fibril structure, primary wall (13), brush shape (14).

These features are not defined with completely distinct boundaries of difference, categories are not entirely distinguished among each other, and have been only linguistically denoted. For each maturity degree it has been subjectively evaluated participation of category image broken cotton to maturity degree. This evaluation is comparative nature, burdened subjective factor because of not quite defined boundaries and uncertainty of evaluation decision. For this reasons membership of categories to maturity degree has been evaluated on Zadeh's fuzzy set theory conception [5]. The membership function value has been subjectively evaluated according to the concepts proposed by Zadeh (blurred boundaries) and Sugeno which proposition is based rather on guessing [8]. The membership function value '1' is assigned for the category of 'completely belonging' to a given

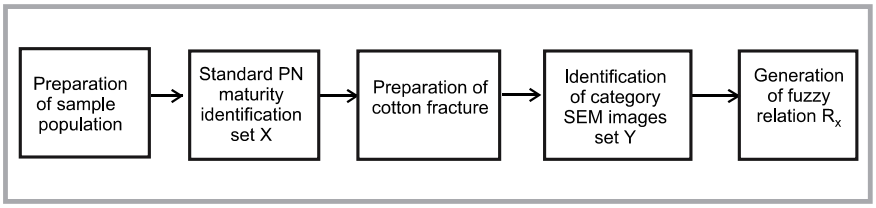

Figure 1. *Procedure for assessing cotton fibre maturity.*

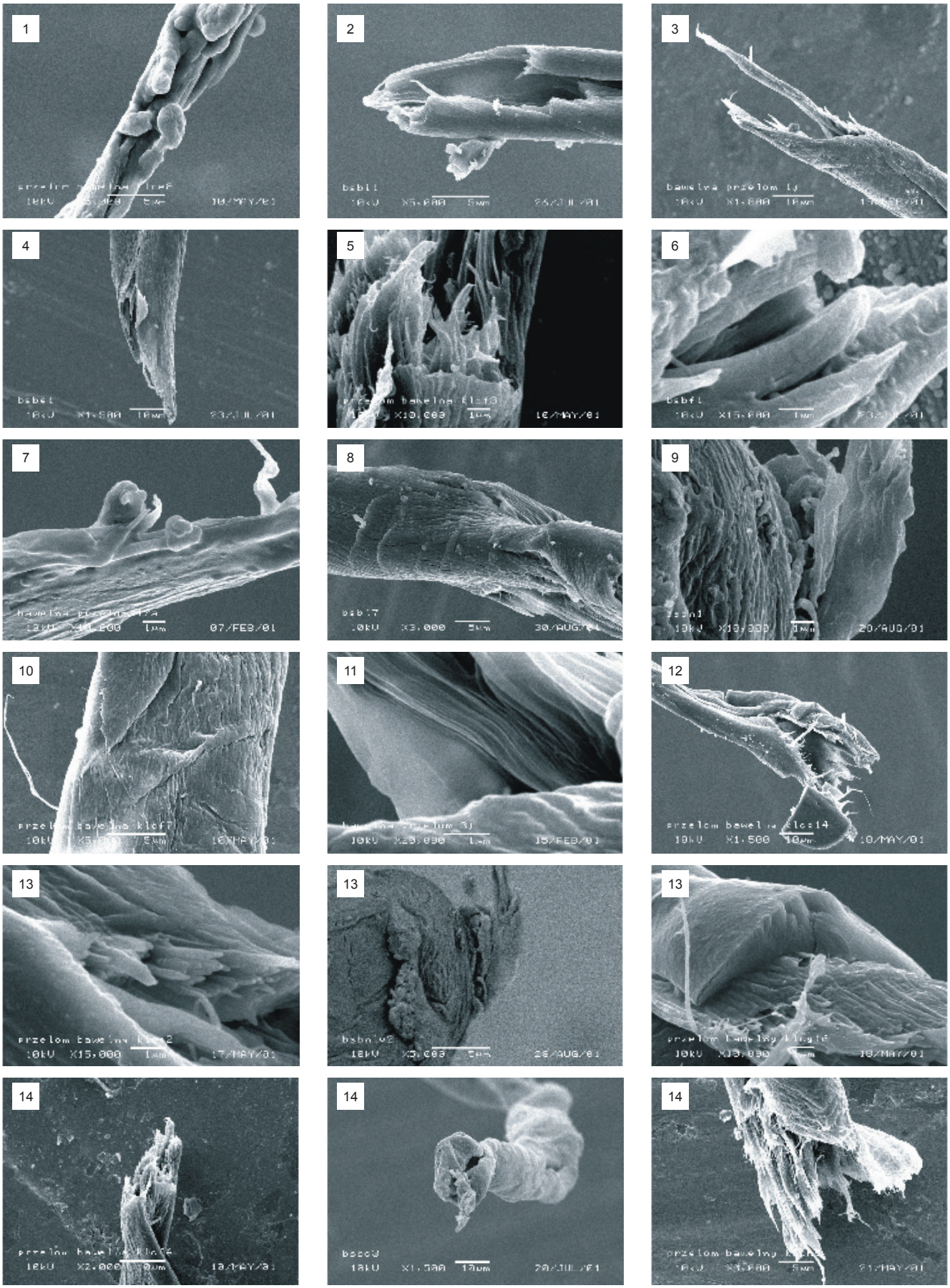

Figure 2. *Category of cotton fracture, visible on the SEM images, and number of category (in brackets): tensile rubber (1), broken tree (2), cracked untwisted rope (3), cohesion of broken fibre end (4), macro- and micro- fibrils occurring in the broken fibre end (5), ductile callosities (6), shapeless agglomerate (7), folds (8), wrinkles (9), furrows (10), grooves (11), stratification of morphological fibre structure (12), appearance of fibril structure, primary wall (13), brush shape (14).*

Table 1. *Discrete membership function values for SEM images of cotton fracture.*

Maturity degree	Number of category													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Plastic fracture				Elastic - plastic fracture				Elastic fracture					
0.5	1	0.5	0.5	0.5	0	1	0.5	0	0	0	0	0	0	0
1.0	1	1	1	1	0	1	1	0.5	0.5	0	0	0	0	0
1.5	0.5	1	1	1	0.5	1	1	1	0.5	0.5	0.5	0	0	0
2.0	0.5	0.5	0.5	1	1	0.5	1	1	1	1	1	0.5	0.5	0.5
2.5	0	0.5	0.5	0.5	1	0.5	0.5	0.5	1	1	1	0.5	0.5	1
3.0	0	0	0	0	1	0	0	0	1	1	1	1	1	1
3.5	0	0	0	0	0.5	0	0	0	0.5	0.5	0.5	1	1	1

maturity degree, and '0' for the case of category 'evidently not belonging' to a maturity degree. The membership function values between 0 and 1 evaluate the intermediate state of category, belonging to a given maturity degree. Fuzziness should be understood as a subjective grade of certainty for the category belonging to a given maturity degree.

Table 1 shows the estimated values of membership function for standard patterns. In this manner a fuzzy relation R_x has been generated as pattern standards based on SEM cotton fracture images.

Formalisation of cotton maturity degree in the fuzzy set theory approach

Let us assume the following givens:

1). Discreet set of degrees of cotton maturity:

$$X : [x_1,.....,x_7] \quad (1)$$

where $x_1 = 0.5$, $x_2 = 1$, $x_3 = 1.5$, $x_4 = 2.0$, $x5 = 2.5$, $x6 = 3.0$ $x_7 = 3.5$

2). Set of 'features' of cotton fracture defined with linguistic variables (see Figure 2)

$$Y : [y_1,.....,y_{14}] \quad (2)$$

3) Let us form the Cartesian product

$$\Gamma : XxY$$

4) Let us define the fuzzy relation R:

$$R = \{f(x,y) \forall (x,y) \in \Gamma | f : \Gamma \rightarrow [0,1]\} \quad (3)$$

The membership function f of the fuzzy relation R assigns a number from the interval [0, 1] to every pair (x, y).

The values of membership function qualify the membership of pairs 'category–maturity degree' to the fuzzy relation (3). These values of membership function for a given category of cotton fracture are set down in Table 1. Therefore, the fuzzy relation formed may be interpreted as a 'micro-scale' pattern for determining the degree of cotton maturity, equivalent to the PN and 'Soviet' standards.

Figure 3 illustrates the 'spatial' extension of fuzzy relation (3). The values of the membership function $f(x,y)$ are extended into intermediate values, because the support of the fuzzy relation has been extended.

Determination of maturity degree on the fuzzy set pattern relation R_x

The above-formed 'fuzzy pattern' for determining the maturity degree may be a tool for determining the maturity degree of newly examined cottons. For this purpose, we suggest using a distinct representation of fuzzy relation (3) using Zadeh's notation [7], namely:

$$A_i = R|x_i = f(x_i, y_1)|y_1 + ... + f(x_i, y_{14})|y_{14} \quad (4)$$

These representations for $x_i \in X$ consist of a sequence of discrete fuzzy sets A_i. Each individual fuzzy set A_i corresponds to a maturity degree. Every maturity degree (x_i) of cotton has a characteristic representation (A_i) identifying this maturity degree.

To assess the maturity degree of the newly-examined cotton fibres, SEM microphotographs of the cotton fracture should be taken, and next the category should be identified according to Table 1 for the SEM images obtained. This identification consists in assigning the values of the membership function $\xi(x..., y...)$ for each category.

Therefore we obtain a new fuzzy set:

$$B = \xi(x_1, y_1)|y_1 + ... + \xi(x_1, y_{14})|y_{14} \quad (5)$$

Let us now form a measure of similarity fuzzy sets A_i and B [8]:

$$S_i = \frac{\|A_i \cap B\|}{\|A_i \cup B\|} \quad (6)$$

where $\|A_i \cap B\|$, $\|A_i \cup B\|$ denotes the responsible norm of conjunction and the sum of fuzzy sets [8]. Determining values of similarity, we measure S_i for each fuzzy set A_i and B, and we obtain a sequence of S_i values. The supreme value of S_i corresponds to the maturity degree of the cotton investigated.

Example

Let the fuzzy set B of the maturity degree be given for the cotton analysed:

$$B = 0/1 + 0/2 + 0.6/3 + 0.8/4 + 0.9/5 + + 0.9/6 + 0.5/7 + 0.5/8 + 0.6/9 + + 1/10 + 1/11 + 0/12 + 0/13 + 0/14$$

We recall Table 1 and the standard pattern of set A:

$$B = 1/1 + 0.5/2 + 0.5/3 + 0.5/4 + 0/5 + + 1/6 + 0.5/7 + 0/8 + 0/9 + 0/10 + + 0/11 + 0/12 + 0/13 + 0/14$$

Let us calculate the similarity index

$$S_1 = \frac{\|A_1 \cap B\|}{\|A_1 \cup B\|}$$

$$A_1 \cap B = 0/1 + 0/2 + 0.5/3 + 0.5/4 + + 0/5 + 0.9/6 + 0.5/7 + 0/8 + + 0/9 + 0/10 + 0/11 + 0/12 + + 0/13 + 0/14$$

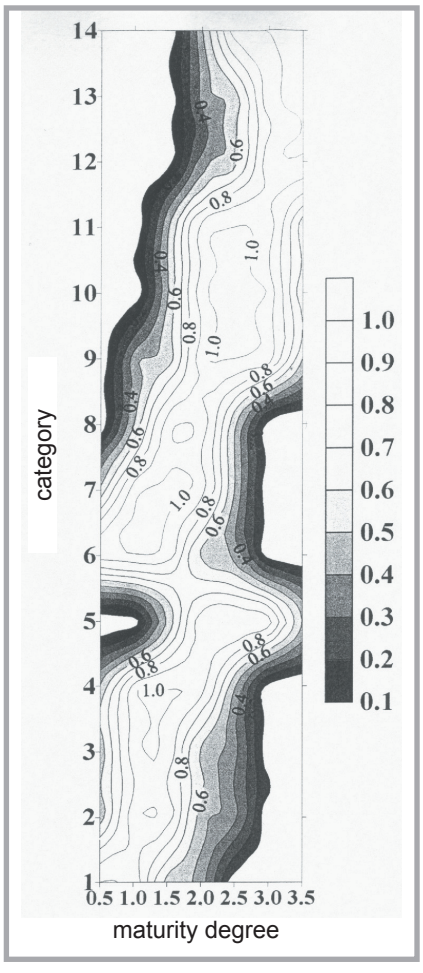

Figure 3. *Dependence of the category of cotton fracture on its manurity degree. Correlation between category of cotton fracture and membership function.*

$$\|A_1 \cap B\| = \frac{0.5+0.5+0.9+0.5}{14} = \frac{2.4}{14}$$

$$A_1 \cup B = 1/1 + 0.5/2 + 0.6/3 + 0.8/4 + \\ + 0.9/5 + 1/6 + 0.5/7 + 0.5/8 + \\ + 0.6/9 + 1/10 + 1/11 + 0/12 + \\ + 0/13 + 0/14$$

$$||A_1 \cup B|| = (1 + 0.5 + 0.6 + 0.8 + \\ + 0.9 + 1 + 0.5 + 0.5 + \\ + 0.6 + 10 + 1)/14 = \\ = 8.4/14$$

Therefore $S_1 = 2.4/8.4 = 0.285$

In the same way we calculate: $S_2 = 0.294$, $S_3 = 0.563$, $S_4 = 0.518$, $S_5 = 0.385$, $S_6 = 0.409$, $S_7 = 0.186$.

The sum of S_i is:
$\sup\{S_1, S_2, S_3, S_4, S_5, S_6, S_7\} = S_3$

The similarity index reaches a maximum value for set A_3 of the fuzzy standard and set B of the cotton fibre analysed. It means that the maturity degree of the cotton analysed is equal to 1.5.

Summary

In this work we have introduced a method for determining the degree of cotton maturity on the basis of SEM images of fibre fracture. To analyse the SEM images of the cotton fracture we applied the notion and method of fuzzy set theory.

The connection of the experimental technique of scanning electron microscopy and the theoretical method of image analysis may prove to be valuable. This initial consideration as we have presented it will be developed in further analysis with the computer method.

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