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Model of the Feeding Process of Anisotropic Warp Knitted Fabrics

Abstract
The model of the feeding process of anisotropic warp knitted fabrics includes a mathematical model of a knitted fabric in which the surface distribution of variable features of the knitted fabric structure is defined in the form of numerical matrixes, the criteria of selecting the feeding method are defined on the basis of the existing technologies and the definition of forcing the feeding system on the basis of the kinematics of loop-forming elements, and finally the gradient of length of the fed and required threads in structural elements of the knitted fabric. Identifying the feeding system dynamics at constant-length thread feeding formulates the assumptions of the physical model. The mathematical model describes time courses of variations in dynamic forces in threads in a stitch repeat. On the basis of the algorithm of calculations, a numerical simulation of thread feeding was carried out for the structure of a jacquard knitted fabric, determining the extreme values of forces in the fed warp threads.

Key words: *warp knitting, anisotropic warp knitting, feeding, surface distribution, mathematical model.*

■ Introduction

A characteristic feature of anisostructural knitted fabrics is the variation in the component stitches surface structure. The stitches form a given combination of threads of structural elements, of different spatial configurations and different lengths. In the process of manufacturing warp anisomorphic knitted fabrics, selecting the optimum parameters for feeding the knitting zone of warp knitting machines with threads is a key factor in programming the technological process [1]. The model of feeding assumes a system of active warp unwinding from warp beams with a constant or variable programmed rotational speed of the beams.

The model of feeding the manufactured anisotropic warp knitted fabric with warp threads is presented in the form of an algorithm with defined successive steps of the model partial solutions (Table 1).

■ Step 1 - a mathematical model of an anisotropic warp knitted fabric described in the form of numerical matrixes of A_i component stitches [2].
■ Step 2 - a geometrical-empirical model of the structural elements of the fabric.
■ Step 3 - a definition of numerical matrixes $L_i(V_i)$ of surface distribution of the thread length in stitch repeat.
■ Step 4 - determining the thread gradient $\Delta l_{r,k}$ according to one of four possible methods of constant-length thread feeding.
■ Step 5 - a condition of selecting the feeding method according to criteria determining that the thread gradient in the last formed course is close to zero, and

that the overfeeds and deficiencies of threads between the lengths of fed warps and thread requirements in structural elements of the knitted fabric have minimum values.
■ Step 6 - defining the forcing of the feeding system $S(t)$, which is the sum of:
$S'(t)$ - forcing conditioned by displacement of knitting elements of the warp-knitting machines, back displacement of threads in the knitting zone, type of stitch, conditions of fabric take-up. The forcing was determined empirically by digital analysis of pictures of the thre-

ad moving between the back rest roller and the needles.
$\Delta S(t)$ - the forcing defined by variation of the thread run-in gradient for the selected feeding method.
■ Steps 7 and 8 - a mathematical model of the feeding process describing the time courses of the back rest roller deflections y(t) and the variability of the dynamic forces in threads forming a stitch repeat. The mathematical model was formulated on the basis of the physical model of the feeding process presented in the following chapter.
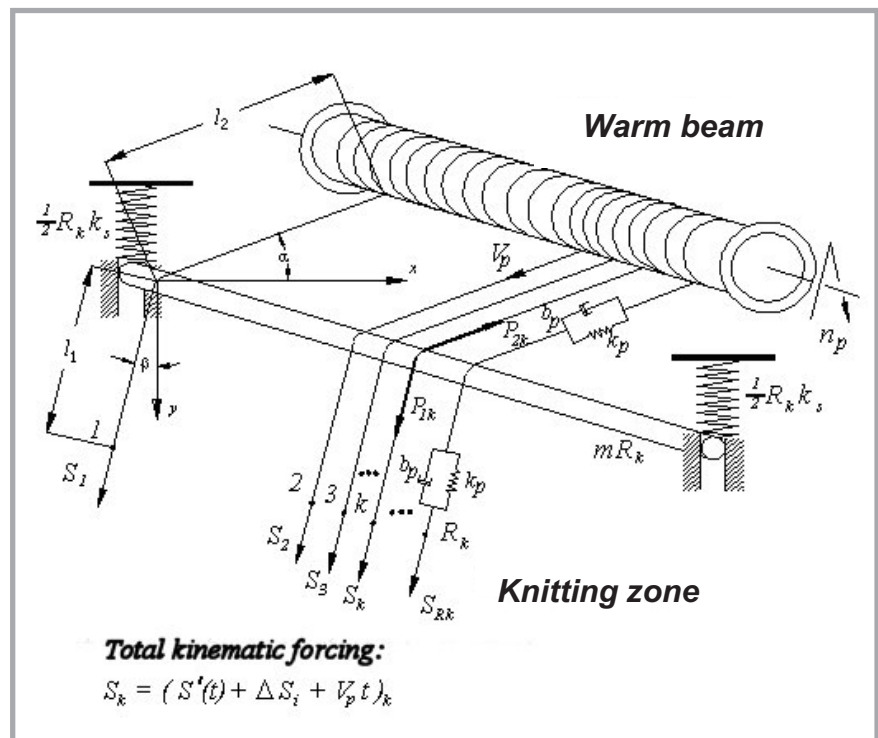

Figure 1. A physical model of a constant-length feeding system.

Table 1. *Algorithm of the feeding model.*

Step 1	Defining numerical matrixes of i-th component stitches. Numerical matrix of the i-th component stitch $A_i = \{a_{r,k}\}$ or $A_i = \{B_j\}$ for $r = 1$ to R_r, $k = 1$ to R_k where R_r, R_k - height and width of a stitch repeat.
Step 2	Assigning an element of matrix A_i to the value of thread length l_i $a_{r,k} = l_i$, where: $l_i = f(A,B,d,k_e)$, A,B,d - dimension parameters of the fabric, k_e - empirical coefficient of the open model
Step 3	Describing numerical matrixes of thread lengths L_i of the i-th component stitch $L_i = \{l_{r,k}\}$ Transformation of matrix L_i into symmetrical matrix L'_i
Step 4	Determining gradients of thread run-in's $\Delta l_{r,k}$ for element $a_{r,k}$ $\Delta l_{r,k} = (l_{r,k})_p - (l_{r,k})_z$: where: $(l_{r,k})_p$ - length of the fed thread, $(l_{r,k})_z$ - length of the required thread, according to a given method of thread feeding: 'A': $(\Delta l_{r,k})_A$ - feeding $\quad l_p = \overline{l_{r,k}} = \sum_{r=1}^{R_r}\sum_{k=1}^{R_k} l_{r,k} = const$ (average requirement l_z within the limits of a stitch repeat) 'B': $(\Delta l_{r,k})_B$ - feeding $\quad l_p = (\overline{l_r})_k = \left(\dfrac{\sum_{r=1}^{R_r} l_{r,k}}{R_r}\right)_k$ (of a variable thread length of $\overline{l_z}$ in a group of threads at the height of the stitch repeat) 'C': $(\Delta l_{r,k})_C$ - feeding $\quad l_p = (\overline{l_k})_r = \left(\dfrac{\sum_{k=1}^{R_k} l_{r,k}}{R_k}\right)_r$ (of a variable thread length of $\overline{l_z}$ in the fabric course) "D": $(\Delta l_{r,k})_{B+C}$ - feeding a variable thread length in a group of threads of l_p, equal to the average requirement l_z in courses of a separated group of threads.
Step 5	Selecting a feeding method 'A', 'B', 'C', 'D' according to the criteria meeting the following conditions: $\Delta l_{r,k} \cong 0$ for $r = R_r$ (for the last course of the stitch repeat), $\left\lvert +\Delta l_{r,k}\right\rvert_{min}$ - thread overfeed in element $a_{r,k}$, $\left\lvert -\Delta l_{r,k}\right\rvert_{min}$ - thread deficiency in element $a_{r,k}$
Step 6	Defining the function of thread forcing $S(t)$ in the feeding process $S(t) = (S'(t) + \Delta S_r)_k$ for threads $k = 1$ to R_k, $S'(t)$ -forcing conditioned by the displacement of knitting elements in the warp-knitting machine, $\Delta S_{r,k} = -\Delta l_{r,k}$ a component variable of forcing defined by the gradient of thread run-in's
Step 7	Determining dislocations of the back rest roller $y(t)$ on the basis of identification of the feeding system dynamics $R_k \cdot m \cdot \dfrac{d^2 y}{dt^2} + b_{zr} \cdot \dfrac{dy}{dt} + k_{zr} \cdot y = a_1\left[k_p \cdot \sum_{k=1}^{N_1} S_1(t) + b_p \cdot \sum_{k=1}^{N_1} \dfrac{dS_1}{dt}\right] +$ $+ a_2\left[k_p \sum_{k=1}^{N_2} S_2(t) + b_p \sum_{k=1}^{N_2} \dfrac{dS_2}{dt}\right] + a_3\left[k_p \sum_{k=1}^{N_3} S_3(t) + b_p \sum_{k=1}^{N_3} \dfrac{dS_3}{dt}\right]$ where: a_1, a_2, a_3 - coefficients describing the construction of the feeding system, b_{zr} - coefficient of the system attenuation k_{zr} - coefficient of the system elasticity
Step 8	Determining the dynamic loads in threads $P_{1k}(t)$: $P_{1k}(t) = \dfrac{1}{l_2 e^{\pm\mu\cdot\rho} + l_1}\left[k_p S_k(t) + b_p\dfrac{dS_k}{dt} - k_p(\cos\beta - \sin\alpha)y - b_p(\cos\beta - \sin\alpha)\dfrac{dy}{dt}\right]$

Identification of the Feeding System Dynamics at Constant-length Thread Feeding

A physical model of the feeding system in a warp-knitting machine at constant-length thread feeding in the process of manufacturing anisotropic structures (Figure 1) was defined for the following assumptions:

- a quasi-flat system referred to k warp threads in the width of the stitch repeat R_k,
- parameters of the system described by stiffness, the reduced mass of the back rest roller, elasticity and attenuation of threads. Threads are treated as weightless viscoelastic bodies,
- the assumed physical model reflects the geometry of the warp thread feeding zone,
- at a given direction of thread movement on the back rest roller, the physical model is a system subject to variable kinematic forcing $S_k(t)$,
- the physical model allows for the hysteresis of thread loads caused by their friction against the back rest roller,
- the back rest roller is treated as an inflexible element, elastically supported in such a way that it has one degree of freedom of movement.

Symbols used in Figure 1:

P_{1k} - tension in k-th thread between the back rest roller and the knitting zone,

P_{2k} - tension in k-th thread between the warp beam and the back rest roller,

m - point, reduced to one thread, mass of the back rest roller,

k_s - coefficient of the back rest roller stiffness reduced to one thread,

k_p - coefficient of thread elasticity,
b_p - coefficient of thread attenuation,

l_1, l_2 - geometrical parameters of the system,

α, β - geometrical parameters of the system,

n_p, v_p - rotational and linear speed of thread feeding (n_p=const. and v_p=const. for constant-length warp unwinding),

R_k - wale repeat of the stitch describing the number of threads affecting the back rest roller,

S_k - total kinematic forcing of the feeding system,

$S'(t)$ - cyclic forcing during formation of one course of fabric,

ΔS_i - growth of forcing equal to the gradient of thread run-ins.

Mathematical Model of the Feeding System

The physical model of the feeding system dynamics (Figure 1) can be described by the equation of motion (1) where:
$e^{\pm\mu\rho}$ - the quantity describing the interrelation between forces P_{1k} and P_{2k};

$$P_{2k} = P_{2k} \cdot e^{\pm\mu\rho} \qquad (2)$$

μ - the coefficient of friction between the back rest roller and the thread moving on the back rest roller,
ρ - the angle of encirclement of the back rest roller with thread.

Taking into account the properties of warp threads in the physical model, as described by the Kelvin-Voigt model, forces P_1 and P_2 can be presented by equation (3) and (4), where λ_{1k}, λ_{2k} - the thread elongation after and before the back rest roller. Substituting relations (3) and (4) to equation (2), we obtain equation (5).

From the condition of warp thread continuity

$$S(t)_k = \left(S'(t) + \Delta S_i\right)_k = \qquad (6)$$
$$= \lambda_{1k} + \lambda_{2k} + y(\cos\beta - \sin\alpha)$$

we can determine the quantity:

$$\lambda_{2k} = S(t)_k - \lambda_{1k} - y(\cos\beta - \sin\alpha) \qquad (7)$$

Placing relation (7) in equation (5), we obtain equation (8). After transformations and grouping of terms, relation (8) can be written as equation (9).

The left side of relation (9) is the value of force P_{1k} (equation (1)), hence by placing the above relation in the equation of motion (1) and rearranging it we obtain equation (10), where:
b_π - the coefficient of the system attenuation
k_π - the coefficient of the system elasticity,
a_1, a_2, a_3 - coefficients describing the geometry of the feeding system,
N_1 - the number of threads moving on the back rest roller in the direction of the warp beam, when condition 1 is fulfilled:

$$\Delta S - V \cdot \Delta t - \Delta\lambda_1 - \Delta\lambda_2 < 0 \qquad (11)$$

for:
ΔS - the growth of the kinematic forcing of the system,
$\Delta\lambda_1, \Delta\lambda_2$ - growth of thread elongation in the arms of thread l_1 and l_2,
V - the speed of thread delivery,
Δt - the step for discrete calculations after time.
N_2 - the number of threads moving on the back rest roller in the direction of needles, when condition 2 is fulfilled:

$$\Delta S - V \cdot \Delta t - \Delta\lambda_1 - \Delta\lambda_2 > 0 \qquad (12)$$

N_3 - the number of threads not moving on the back rest roller, when condition 3 is fulfilled:

$$\Delta S - V \cdot \Delta t - \Delta\lambda_1 - \Delta\lambda_2 = 0 \qquad (13)$$
$$N_1 + N_2 + N_3 = R_k$$

Depending on one of the three defined conditions of thread movement on the back rest roller, coefficients a_1, a_2 and a_3 have the values:
- for the group of threads N_1 (condition 1)

$$a_1 = \frac{\cos\beta - e^{\mu\rho} \cdot \sin\alpha}{l_2 \cdot e^{\mu\rho} + l_1} \qquad (14)$$

- for the group of threads N_2 (condition 2)

$$a_2 = \frac{\cos\beta - e^{-\mu\rho} \cdot \sin\alpha}{l_2 \cdot e^{-\mu\rho} + l_1} \qquad (15)$$

- for the group of threads N_3 (condition 3)

$$a_3 = \frac{\cos\beta - \sin\alpha}{l_2 + l_1} \qquad (16)$$

Coefficients b_π and k_π are determined from relations (17) and (18).
Since:
h - the relative coefficient of system attenuation,

$$h = \frac{b_{zr}}{2mR_k} \qquad (19)$$

and ω_o - the frequency of free vibration of the non-attenuated system,

$$\omega_0 = \sqrt{\frac{k_{zr}}{m \cdot R_k}} \qquad (20)$$

equation (10) has the form presented in (21).

According to conditions (11), (12) and (13), the forces in threads $P_{1k}(t)$ for the successive steps of calculations are determined from the following relations:
- for condition 1 - the relation (22),
- for condition 2 - the relation (23), and
- for condition 3 - the relation (24).

Realisation of the Feeding Model Algorithm for the Structure of a Jacquard Warp Knitted Fabric

The subject of analysis is the structure of a jacquard warp knitted fabric constructed from two component stitches. The first component stitch forms a repeatable set of open chain loops; the other is the weft stitch of varied configuration and length of threads in successively formed courses. Due to the repeatability of elements within its stitch repeat, the chain stitch is an isomorphous loop structure. The weft stitch consists of three repeatable elements $a_{r,k}$ - type '1', '2' and '3' arranged in the

$$R_k \cdot m \cdot \frac{d^2 y}{dt^2} + R_k \cdot k_s \cdot y = \sum_{k=1}^{R_k} \left(\cos\beta - e^{\pm\mu\rho} \cdot \sin\alpha\right) \cdot P_{1k} \qquad (1)$$

$$P_{1k} = k_p \cdot \frac{\lambda_{1k}}{l_1} + \frac{b_p}{l_1} \cdot \frac{d\lambda_{1k}}{dt} \qquad (3)$$

$$P_{2k} = k_p \cdot \frac{\lambda_{1k}}{l_2} + \frac{b_p}{l_2} \cdot \frac{d\lambda_{2k}}{dt} \qquad (4)$$

$$\frac{k_p}{l_1} \cdot \lambda_{1k} \cdot e^{\pm\mu\rho} + \frac{b_p \cdot e^{\pm\mu\rho}}{l_1} \cdot \frac{d\lambda_{1k}}{dt} = \frac{k_p}{l_2} \cdot \lambda_{2k} + \frac{b_p}{l_2} \cdot \frac{d\lambda_{2k}}{dt} \qquad (5)$$

$$\frac{k_p}{l_1} \cdot \lambda_{1k} \cdot e^{\pm\mu\rho} + \frac{b_p \cdot e^{\pm\mu\rho}}{l_1} \cdot \frac{d\lambda_1}{dt} = \frac{k_p}{l_2}\left[S(t)_k - \lambda_{1k} - y(\cos\beta - \sin\alpha)\right] +$$
$$+ \frac{b_p}{l_2} \cdot \left[\frac{dS(t)_k}{dt} - \frac{d\lambda_{1k}}{dt} - \frac{dy}{dt}(\cos\beta - \sin\alpha)\right] \qquad (8)$$

$$\frac{k_p}{l_1} \cdot \lambda_{1k} + \frac{b_p}{l_1} \cdot \frac{d\lambda_{1k}}{dt} = \frac{k_p}{l_2 \cdot e^{\pm\mu\rho} + l_1} \cdot S(t)_k + \frac{b_p}{l_2 \cdot e^{\pm\mu\rho} + l_1} \cdot \frac{dS(t)_k}{dt} -$$
$$- \frac{k_p \cdot (\cos\beta - \sin\alpha)}{l_2 \cdot e^{\pm\mu\rho} + l_1} \cdot y - \frac{b_p \cdot (\cos\beta - \sin\alpha)}{l_2 \cdot e^{\pm\mu\rho} + l_1} \cdot \frac{dy}{dt} \qquad (9)$$

$$R_k \cdot m \cdot \frac{d^2 y}{dt^2} + b_{2r} \cdot \frac{dy}{dt} + k_{2r} \cdot y = a_1 \cdot \left[k_p \cdot \sum_{k=1}^{N_1} S(t)_{k(1)} + b_p \cdot \sum_{k=1}^{N_1} \frac{dS(t)_{k(1)}}{dt}\right] +$$
$$+ a_2 \cdot \left[k_p \cdot \sum_{k=1}^{N_2} S(t)_{k(2)} + b_p \cdot \sum_{k=1}^{N_2} \frac{dS(t)_{k(2)}}{dt}\right] + a_3 \cdot \left[k_p \cdot \sum_{k=1}^{N_3} S(t)_{k(3)} + b_p \cdot \sum_{k=1}^{N_3} \frac{dS(t)_{k(2)}}{dt}\right] \qquad (10)$$

Equations 1, 3, 4, 5, 8, 9, and 10.

repeat surface according to the plastic design of the fabric pattern (Figure 2a).

The pattern recorded in the form of coloured squares (c - red, z - green, b - white) contains information on the surface distribution of weft threads in one wale (element '1') or joining two (element '2') or three wales (element '3') of the first component stitch. An anisostructural system of single elements of the weft stitch structure was recorded in the form of a numerical matrix A_2 (Figure 2b), on the basis of the defined principles of stitch coding. The numerical matrix A_2 clearly defines the anisotropic character of the fabric structure, and is the key parameter in identifying the variability of the fabric features as well as the technology of its production. The algorithm (Table 1) parameters of the fabric structure were defined according to step 2: course height B=0.95 mm, wale width A=2.80 mm, chain stitch thread diameter d_f=0.14 mm (polyester yarn of linear density 13.3 tex), weft thread diameter d_w=0.2 mm (polyester yarn of linear density 29.4 tex). The thread lengths in chain loops and in wefts, calculated according to an open geometrical model, are: l_f=3.4 mm, l_{w1}=1.2 mm, l_{w2}=3.9 mm, l_{w3}=7 mm (relative error εl=1.9%). By assigning the values of thread lengths in elements ($a_{r,k}=l_{r,k}$) to the elements $a_{r,k}$ of matrix A_2, we obtain matrix L_2.

For method 'A', that is feeding a constant thread length equal to the average requirement l_z in the stitch repeat, the gradients of the thread run-ins $\Delta l_{r,k}$ were determined. The quantities

$$\Delta w_{r,k} = \frac{\Delta r_{r,k}}{B}$$

were shown in the lined fields of the thread run-in matrix V'2 (Figure 2c). For step 6, the algorithm Table 1 components of kinematic forcing $\Delta=S_{r,k}$ were determined, where:

$$\Delta S_{r,k} = -\Delta l_{r,k}.$$

Thus, for the successive four weft threads $\Delta S_{r,k}$ expressed in mm:
- for k_1: $\Delta S_{1,1}$= -2.3, $\Delta S_{2,1}$= -2.2, $\Delta S_{3,1}$= -2.1, $\Delta S_{4,1}$= -2.0, $\Delta S_{5,1}$= 1.0, $\Delta S_{6,1}$= 1.1
- for k_2: $\Delta S_{1,2}$= 0.1, $\Delta S_{2,2}$= 0.2, $\Delta S_{3,2}$= 3.2, $\Delta S_{4,2}$= 3.3, $\Delta S_{5,2}$= 1.0, $\Delta S_{6,2}$= 1.1
- for k_3: $\Delta S_{1,3}$= 3.0, $\Delta S_{2,3}$= 3.1, $\Delta S_{3,3}$= 0.8, $\Delta S_{4,3}$= 0.9, $\Delta S_{5,3}$= 1.0, $\Delta S_{6,3}$= 1.1

$$b_{zr} = b_p(\cos\beta - \sin\alpha)\cdot(a_1\cdot N_1 + a_2\cdot N_2 + a_3\cdot N_3) \tag{17}$$

$$k_{rz} = N_1\cdot[k_s + a_1\cdot(\cos\beta - \sin\alpha)\cdot k_p] + N_2\cdot[k_s + a_2(\cos\beta - \sin\alpha)\cdot k_p] + N_3\cdot[k_s + a_3\cdot(\cos\beta - \sin\alpha)\cdot k_p] \tag{18}$$

$$\frac{d^2y}{dt^2} + 2h\cdot\frac{dy}{dt} + \omega_0^2\cdot y = \frac{1}{R_k\cdot m}\cdot\left\{\begin{array}{l} a_1\left[k_p\cdot\sum_{k=1}^{N_1}S(t)_{k(1)} + b_p\cdot\sum_{k=1}^{N_1}\frac{dS(t)_{k(1)}}{dt}\right] + \\ + a_2\left[k_p\cdot\sum_{k=1}^{N_2}S(t)_{k(2)} + b_p\cdot\sum_{k=1}^{N_2}\frac{dS(t)_{k(2)}}{dt}\right] + \\ + a_3\left[k_p\cdot\sum_{k=1}^{N_3}S(t)_{k(3)} + b_p\cdot\sum_{k=1}^{N_3}\frac{dS(t)_{k(3)}}{dt}\right] \end{array}\right\} \tag{21}$$

$$P_1(t) = \frac{1}{l_2\cdot e^{\mu\rho} + l_1}\cdot\left[\begin{array}{l} k_p\cdot S(t)_{k(1)} + b_p\cdot\frac{dS(t)_{k(1)}}{dt} - k_p\cdot(\cos\beta - \sin\alpha)\cdot y - \\ - b_p\cdot(\cos\beta - \sin\alpha)\cdot\frac{dy}{dt} \end{array}\right] \tag{22}$$

$$P_1(t)_{(2)} = \frac{1}{l_2\cdot e^{-\mu\rho} + l_1}\cdot\left[\begin{array}{l} k_p\cdot S(t)_{k(2)} + b_p\cdot\frac{dS(t)_{k(2)}}{dt} - k_p\cdot(\cos\beta - \sin\alpha)\cdot y - \\ - b_p\cdot(\cos\beta - \sin\alpha)\cdot\frac{dy}{dt} \end{array}\right] \tag{23}$$

$$P_1(t)_{(3)} = \frac{1}{l_2 + l_1}\cdot\left[\begin{array}{l} k_p\cdot S(t)_{k(3)} + b_p\cdot\frac{dS(t)_{k(3)}}{dt} - k_p\cdot(\cos\beta - \sin\alpha)\cdot y - \\ - b_p\cdot(\cos\beta - \sin\alpha)\cdot\frac{dy}{dt} \end{array}\right] \tag{24}$$

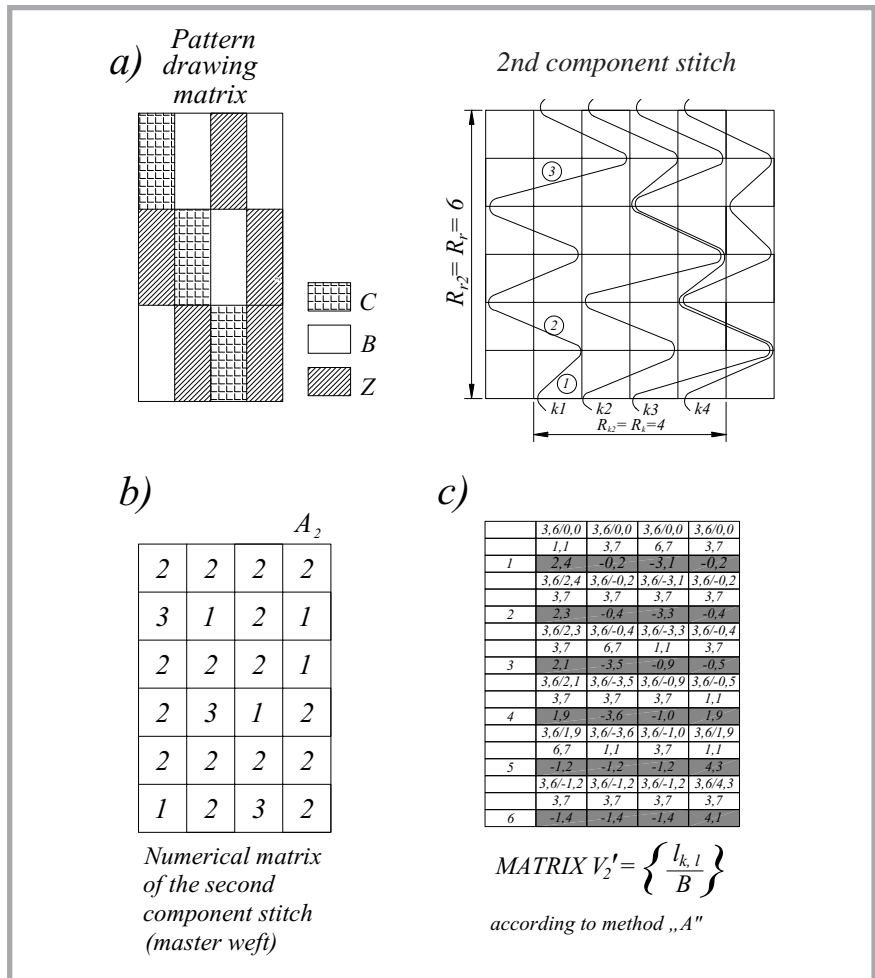**Equations 17, 18, 21, 22, 23, and 24.**

Figure 2. A mathematical model of fabric structure.

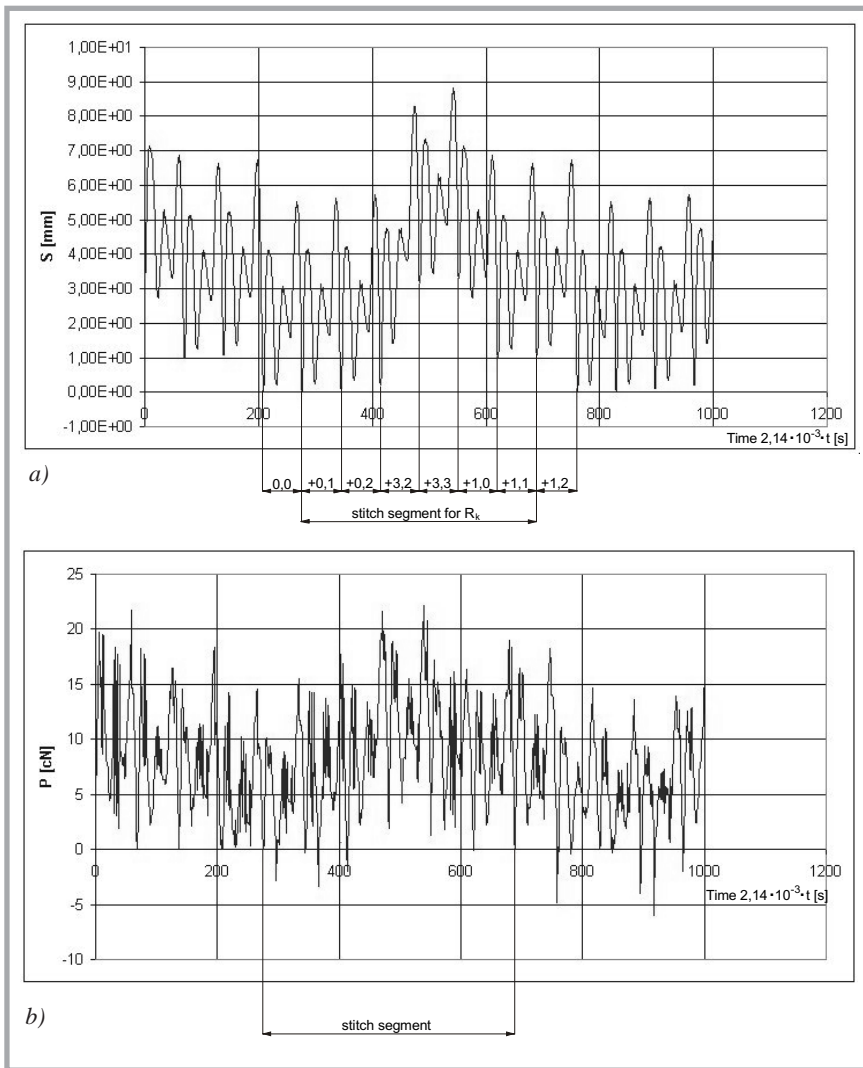**Figure 3.** *a) Forcing S(t) for the weft thread k_2; b) A time course of dynamic forces in thread k_2.*

- for k_4: $\Delta S_{1,4}= 0.1$, $\Delta S_{2,4}= 0.2$, $\Delta S_{3,4}= 0.3$, $\Delta S_{4,4}= -2.0$, $\Delta S_{5,4}= -4.3$, $\Delta S_{6,4}= 4.2$.

For calculations of the back rest roller displacements $y(t)$ and dynamic loads of threads $P_k(t)$, the following input data was taken (symbols as in Figure 1): m=3.7 g; k_s=0.51 cN/mm; k_p=4680 cN; b_p=2100 cNms; μ=0.2; l_1=723 mm; l_2=1344 mm; α=46º; β=34º; knitting speed n=400 courses/min; Nu=18/2"E. The final result, in the form of an example time course of forces in the thread k_2 for forcing $S(t)$, is presented in Figures 3a and 3b.

Conclusions

The mathematical model of the feeding process for anisostructural warp knitted fabrics, with defined assumptions of the dynamic system of constant-length thread feeding for the forcing reflecting variable parameters of the fabric structure and features of the knitting process on warp-knit-

ting machines, should be helpful in selecting optimum conditions for the knitting process. This optimisation is defined by the limiting values of forces in threads P_{max} and P_{min}. Beyond these values, especially for $P_{min}>P>P_{max}$, disturbances of the knitting process occur, which further leads to unfavourable changes in the fabric structure.

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